Rigid Motion Transformations
Topic 1 Overview

How is *Rigid Motion Transformations* organized?

In this topic, students use patty paper and the coordinate plane to investigate congruent figures. Throughout the topic, students are expected to make conjectures, investigate conjectures, and justify true results about transformations.

The topic begins with an introduction to congruent figures through the use of patty paper. Students use informal language to describe how one figure can be mapped to a congruent figure. In the second lesson, students formalize the language of rigid motion transformations (translations, reflections, and rotations), and describe how a single rigid motion maps between congruent figures. They also learn that rigid motions preserve the size and shape of a figure, but that reflections change the orientation of the vertices of a figure.

The next three lessons involve investigating each of the three rigid motion transformations on the coordinate plane. Students describe the effect of translations, reflections, and rotations on two-dimensional figures using coordinates. As the lessons progress, students identify a sequence of transformations that map one figure onto another, first using only translations, then using translations and reflections, and finally using all three rigid motion transformations. In the final lesson, students define congruent line segments and angles, and write congruence statements for triangles, angles, and segments. They also determine a possible sequence of transformations that map a figure onto a congruent figure, and they generalize the effects of rigid motion transformations on the coordinates of figures.

What is the entry point for students?

Students use intuition and patty paper to establish what it means for two objects to be congruent. They learn, or review, how to use patty paper to compare side lengths and angle measures and how to locate the midpoint of a segment. Students use what they have learned about shapes to describe geometric figures. They sort figures to establish an understanding that congruent figures have the same shape and the same size. While students begin with informal terms for transformations, they formalize the language of transformations throughout the topic.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Define and identify translations, rotations, and reflections.
- Translate, reflect, and rotate geometric figures using patty paper and on the coordinate plane.
- Verify congruence of figures by measuring and comparing the properties of the geometric figures after undergoing a translation, reflection, or rotation.
- Verify that lines and line segments, angles, and parallel lines remain the same length after undergoing a translation, reflection, or rotation.
- Explain that a two-dimensional figure is congruent to another if the second figure can be made from the first by translations, reflections, and rotations.
- Verify that rigid motions preserve the size and shape of a figure, but reflections change the orientation of the vertices of a figure.
- Describe the effects of rigid motion transformations to the *x*- and *y*-coordinates of a figure using algebraic representations.

Why is *Rigid Motion Transformations* important?

This topic begins the study of congruence and sets the stage for similarity. Through informal and formal methods, students investigate properties of rigid motion transformations and their effects on lines, line segments, angles, and parallel lines. These properties will be contrasted with properties of dilations in the next topic.

The terminology and notation used in this topic will be used again in the next topic. Students will use transformations to prove the angle relationships that are formed when parallel lines are cut by a transversal. When studying linear relationships in this course, students will use rigid motions to transform lines on the coordinate plane. In high school, students will continue to formalize their knowledge of congruent triangles and use congruence to prove a wide variety of geometric theorems and justify constructions.

How do the activities in *Rigid Motion Transformations* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

The standards of this topic link directly to the process standards of reasoning about mathematics, making and testing conjectures, generalizing patterns, using tools, and practicing precision. By reasoning about their explorations with tools, students make and test conjectures about the relationships between corresponding sides and angles after applying transformations.

After sufficient work with a transformation on the coordinate plane, students make generalizations about the coordinates of the images of the transformation. They use patty paper to make, test, and verify conjectures about congruent figures. Students must apply their knowledge of transformations as they determine specific sequences and order of transformations that map images onto each other.

New Tools and Notation Used

- Patty Paper: a sheet of thin, transparent paper useful for exploring mathematical ideas, particularly geometric ideas. Patty paper can be purchased from educational outlets or restaurant supply stores. Some stores may also donate patty paper to schools. If students do not have access to patty paper, they may use tracing paper, parchment paper, or plain white paper.
- When a figure (pre-image) is transformed, the newly formed figure is called the *image*. Corresponding vertices are indicated with a "prime." For example, $\triangle ABC$ is translated to form $\triangle A'B'C'$. The pre-image is $\triangle ABC$ and the image is $\Delta A'B'C'$.

Materials Needed

- Patty paper
- Protractors
- Centimeter rulers
- Scissors
- Straightedges

Learning Together

ELPS: 1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

Lesson	Lesson Name	TEKS	Days	Highlights
1	Patty Paper, Patty Paper: Introduction to Congruent Figures	8.10A	2	Students use patty paper to indirectly measure segments and angles and use folds to make observations about a figure. They determine if figures are the same size and shape. The term *congruent figures* is defined. Students use patty paper to determine if figures are congruent. They then make conjectures about congruence, investigate their conjectures, and justify their conjectures using informal transformation language.
2	Slides, Flips, and Spins: Introduction to Rigid Motions	8.10A 8.10B	3	Students develop a formal understanding of translations, rotations, and reflections in the plane. The terminology of transformations is introduced, including *pre-image, image, translation, reflection, line of reflection, rotation, center of rotation*, and *angle of rotation*. Students use patty paper to investigate each transformation, create images from pre-images, and determine the properties of each tranformation. They learn that each rigid motion transformation preserves the size and shape of the original figure, and that translations and rotations also preserve the orientation of the figure. At the end of the lesson, students state the formal name for transformations that carry figures onto congruent figures and reason that an image of an image of a pre-image is congruent to the pre-image.
3	Lateral Moves: Translations of Figures on the Coordinate Plane	8.10A 8.10C	2	Students use patty paper to explore translations of various figures on a coordinate plane. They then generalize about the effects of translating a figure on its coordinates. Students verify that two figures are congruent by describing a sequence of translations that map one figure onto another.

Lesson	Lesson Name	TEKS	Days	Highlights
4	Mirror, Mirror: Reflections of Figures on the Coordinate Plane	8.10A 8.10C	2	Students use patty paper to explore reflections of various figures on a coordinate plane. They then generalize about the effects reflecting a figure has on its coordinates. Students verify that two figures are congruent by describing a sequence of translations and reflections that map one figure onto another.
5	Half Turns and Quarter Turns: Rotations of Figures on the Coordinate Plane	8.10A 8.10C	2	Students use patty paper to explore rotations of various figures on a coordinate plane. They then generalize about the effects of rotating a figure on its coordinates. Students verify that two figures are congruent by describing a sequence of rigid motions that map one figure onto another.
6	Every Which Way: Combining Rigid Motions	8.10A 8.10C	2	Students use coordinates to determine the rigid motion used to map one congruent figure onto another. They learn about and write congruence statements for congruent triangles. Using figures on a grid, students investigate and determine a sequence of transformations that can be used to verify figures are congruent. They then generalize the effects of rigid motions on the coordinates of figures.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 8.10A **LESSON 1** **Patty Paper, Patty Paper** GETTING STARTED ACTIVITY 1	**LESSON 1** continued ACTIVITY 2 TALK THE TALK	TEKS: 8.10A, 8.10B **LESSON 2** **Slides, Flips, and Spins** GETTING STARTED ACTIVITY 1	**LESSON 2** continued ACTIVITY 2 ACTIVITY 3	**LESSON 2** continued ACTIVITY 4 TALK THE TALK
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 8.10A, 8.10C **LESSON 3** **Lateral Moves** GETTING STARTED ACTIVITY 1	**LESSON 3** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**MID-TOPIC ASSESSMENT**
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
TEKS: 8.10A, 8.10C **LESSON 4** **MIrror, Mirror** GETTING STARTED ACTIVITY 1	**LESSON 4** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 8.10A, 8.10C **LESSON 5** **Half Turns and Quarter Turns** GETTING STARTED ACTIVITY 1	**LESSON 5** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK
Day 16	**Day 17**	**Day 18**	**Day 19**	**Day 20**
MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 8.10A, 8.10C **LESSON 6** **Every Which Way** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 6** continued ACTIVITY 3 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**

Assessments

There are two assessments aligned to this topic: a Mid-Topic Assessment and an End of Topic Assessment.