Rigid Motion Transformations Summary

KEY TERMS

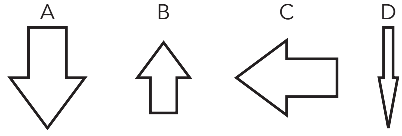
- congruent figures
- corresponding sides
- corresponding angles
- plane
- transformation
- rigid motion
- pre-image
- image
- translation
- reflection
- line of reflection
- rotation
- center of rotation
- angle of rotation
- congruent line segments
- congruent angles

LESSON 1 Patty Paper, Patty Paper

Figures that have the same size and shape are **congruent figures**. If two figures are congruent, all corresponding sides and all corresponding angles have the same measures. **Corresponding sides** are sides that have the same relative position in geometric figures. **Corresponding angles** are angles that have the same relative position in geometric figures.

If two figures are congruent, you can obtain one figure by a combination of sliding, flipping, and spinning the figure until it lies on the other figure.

For example, Figure A is congruent to Figure C, but it is not congruent to Figure B or Figure D.



LESSON 2

Slides, Flips, and Spins

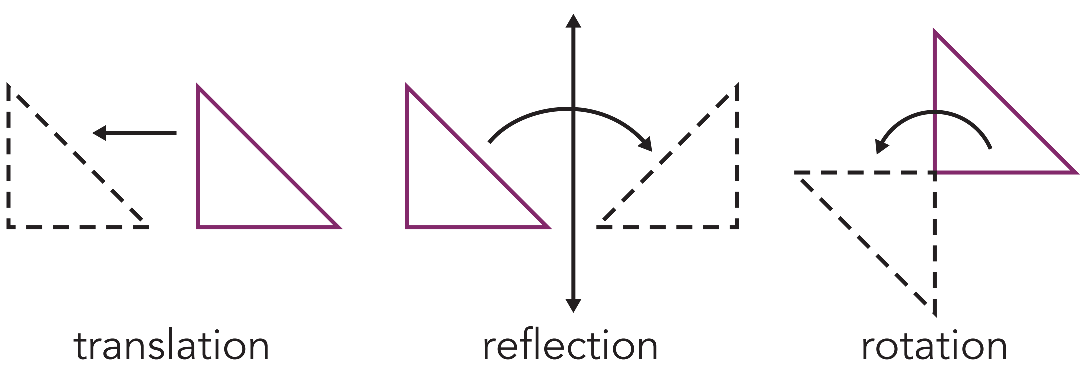
A **plane** extends infinitely in all directions in two dimensions and has no thickness. A **transformation** is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of each figure.

The original figure on the plane is called the **pre-image**, and the new figure that results from a transformation is called the **image**. The labels for the vertices of an image use the symbol (′), which is read as "prime."

A **translation** is a rigid motion transformation that slides each point of a figure the same distance and direction along a line. A figure can be translated in any direction. Two special translations are vertical and horizontal translations. Sliding a figure left or right is a horizontal translation, and sliding it up or down is a vertical translation.

A **reflection** is a rigid motion transformation that flips a figure across a line of reflection. A **line of reflection** is a line that acts as a mirror so that corresponding points are the same distance from the line.

A **rotation** is a rigid motion transformation that turns a figure on a plane about a fixed point, called the **center of rotation**, through a given angle, called the **angle of rotation**. The center of rotation can be a point outside of the figure, inside of the figure, or on the figure itself. Rotation can be clockwise or counterclockwise.



LESSON 3

Lateral Moves

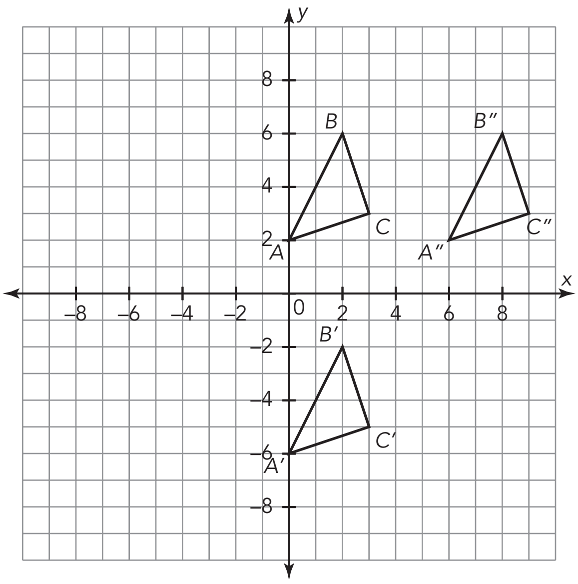
A translation slides an image on the coordinate plane. When an image is horizontally translated *c* units on the coordinate plane, the value of the *x*-coordinates change by *c* units. When an image is vertically translated *c* units on the coordinate plane, the value of the *y*-coordinate changes by *c*-units. The coordinates of an image after a translation are summarized in the table.

Original Point	Horizontal Translation to the Left	Horizontal Translation to the Right	Vertical Translation Up	Vertical Translation Down
(x, y)	$(x - c, y)$	$(x + c, y)$	$(x, y + c)$	$(x, y - c)$

For example, the coordinates of ΔABC are A (0, 2), B (2, 6), and C (3, 3).

When ΔABC is translated down 8 units, the coordinates of the image are A' (0, −6), B' (2, −2), and C' (3, −5).

When ΔABC is translated right 6 units, the coordinates of the image are A'' (6, 2), B'' (8, 6), and C'' (9, 3).

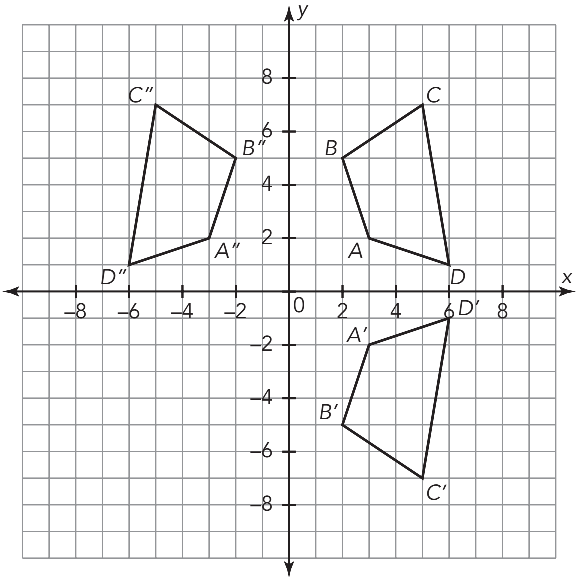


LESSON 4 Mirror, Mirror

A reflection flips an image across a line of reflection. When an image on the coordinate plane is reflected across the *y*-axis, the value of the *x*-coordinate of the image is opposite the *x*-coordinate of the pre-image. When an image on the coordinate plane is reflected across the *x*-axis, the value of the *y*-coordinate of the image is opposite the *y*-coordinate of the pre-image. The coordinates of an image after a reflection on the coordinate plane are summarized in the table.

Original Point	Reflection Over *x*-Axis	Reflection Over *y*-Axis
(x, y)	$(x, -y)$	$(-x, y)$

For example, the coordinates of Quadrilateral *ABCD* are *A* (3, 2), *B* (2, 5), *C* (5, 7), and *D* (6, 1).



When Quadrilateral *ABCD* is reflected across the *x*-axis, the coordinates of the image are *A*′ (3, –2), *B*′ (2, –5), *C*′ (5, –7), and *D*′ (6, –1).

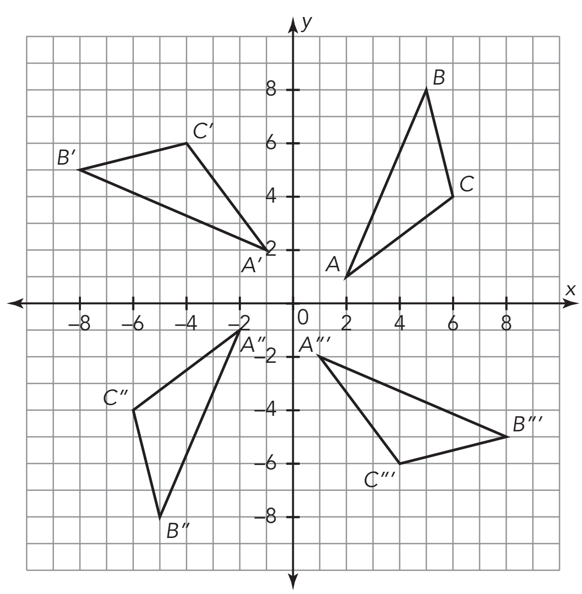
When Quadrilateral *ABCD* is reflected across the *y*-axis, the coordinates of the image are *A*″ (–3, 2), *B*″ (–2, 5), *C*″ (–5, 7), and *D*″ (–6, 1).

LESSON 5
Half Turns and Quarter Turns

A rotation turns a figure about a point through an angle of rotation. When the center of rotation is at the origin (0, 0), and the angle of rotation is 90°, 180°, 270°, or 360° the coordinates of an image can be determined using the rules summarized in the table.

Original Point and Rotation About the Origin 360°	Rotation About the Origin 90° Counterclockwise and 270° Clockwise	Rotation About the Origin 90° Clockwise and 270° Counterclockwise	Rotation About the Origin 180°
(x, y)	$(-y, x)$	$(y, -x)$	$(-x, -y)$

For example, the coordinates of ΔABC are A (2, 1), B (5, 8), and C (6, 4).



When ΔABC is rotated 90° counterclockwise or 270° clockwise about the origin, the coordinates of the image are A' (−1, 2), B' (−8, 5), and C' (−4, 6).

When ΔABC is rotated 180° about the origin, the coordinates of the image are A'' (−2, −1), B'' (−5, −8), and C'' (−6, −4).

When ΔABC is rotated 90° clockwise or 270° counterclockwise about the origin, the coordinates of the image are A'' (1, −2), B'' (8, −5), and C'' (4, −6).

When ΔABC is rotated 360° about the origin, the coordinates are the same as the coordinates of the original triangle.

LESSON 6

Every Which Way

Because rigid motions maintain the size and shape of an image, you can use a sequence of translations, reflections, and rotations to verify that two figures are congruent.

In congruent figures, the corresponding sides are congruent line segments. **Congruent line segments** are line segments that have the same length. Likewise, if corresponding angles have the same measure, they are congruent angles. **Congruent angles** are angles that are equal in measure.

For example, if the sides of two different figures are equal in length, such that the length of side *AB* in Triangle *ABC* is equal to the length of side *DE* in Triangle *DEF*, these sides are said to be congruent.

$\overline{AB} \cong \overline{DE}$ is read "line segment *AB* is congruent to line segment *DE*."

Likewise, if the angles of two different figures are equal in measure, such that the measure of angle *A* in Triangle *ABC* is equal to the measure of angle *D* in Triangle *DEF*, these angles are said to be congruent.

$\angle A \cong \angle D$ is read "angle *A* is congruent to angle *D*."

There is often more than one sequence of transformations that can be used to verify that two figures are congruent.