1

Pinch-Zoom Geometry

Dilations of Figures

WARM UP

A billboard advertises a watch. The face of the watch is 2 meters wide on the billboard. The face of the actual watch is 2 centimeters wide. What scale factor was used to create the billboard?

LEARNING GOALS

- Dilate figures given a center of dilation and scale factor such that the resulting dilation is an enlargement or a reduction of the original figure.
- Identify the scale factor used in a dilation of a figure.
- Determine whether a two-dimensional figure is similar to another by obtaining one from the other using a sequence of dilations.
- Describe a sequence of dilations that demonstrates that two figures are similar.

KEY TERMS

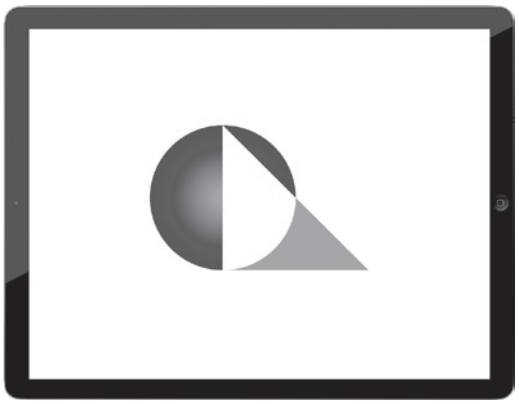
- dilation
- center of dilation
- scale factor
- enlargement
- reduction
- similar

You have learned about geometric transformations that preserve the size and shape of figures. You also know how to use scale factors to produce scale drawings. Is there a geometric transformation that changes the scale of a figure?

Scale Drawing by Doing

Recall that a scale drawing is a representation of a real object or place that is in proportion to the real object or place it represents. The ratios of corresponding side lengths between the drawing and the object are all the same.

Consider the logo shown on the tablet screen.



1. **When the logo on the tablet screen appears on the smartphone screen, it will be reduced by a scale factor of $\frac{1}{2}$. Sketch the logo on the smartphone screen and explain your process.**

2. When the logo on the tablet screen appears on the desktop screen, it will be enlarged by a scale factor of 2. Sketch the logo on the desktop screen and explain your process.

ACTIVITY
1.1

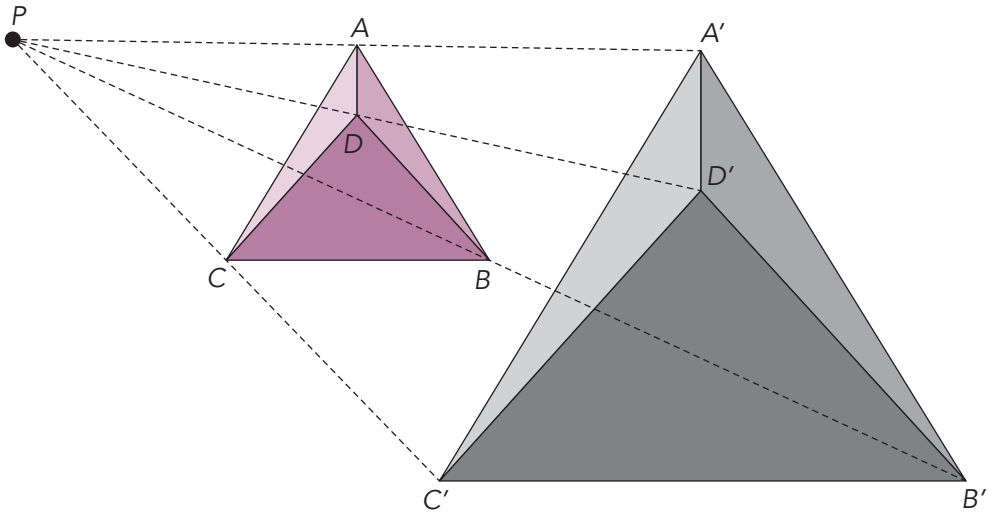
Dilating Figures with a Scale Factor Greater Than 1

Dilations are transformations that produce figures that are the same shape as the original figure, but not necessarily the same size. Each point on the original figure is moved along a straight line, and the straight line is drawn from a fixed point known as the **center of dilation**. The distance each point moves is determined by the *scale factor* used.

The **scale factor** is the ratio of the distance of the new figure from the center of dilation to the distance of the original figure from the center of dilation. When the scale factor is greater than 1, the new figure is called an **enlargement**.

WORKED EXAMPLE

This image of a logo was dilated to produce an enlargement using point *P* as the center of dilation.



The scale factor can be expressed as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

1. **In the Worked Example, the scale factor is represented by 4 equivalent ratios. What distances are represented by each part of those ratios? Is the scale factor less than 1, equal to 1, or greater than 1? Explain your reasoning.**

2. **Measure the segment lengths of the original logo in millimeters.**

$m\overline{AB}$ = _____ $m\overline{AC}$ = _____

$m\overline{BC}$ = _____ $m\overline{AD}$ = _____

3. **Measure the segment lengths of the new logo in millimeters.**

$m\overline{A'B'}$ = _____ $m\overline{A'C'}$ = _____

$m\overline{B'C'}$ = _____ $m\overline{A'D'}$ = _____

The notation $\overline{AB}$ means "segment *AB*." The notation *AB* means "the length of segment *AB*."

4. **Measure each line segment in millimeters.**

$m\overline{A'P}$ = _____ $m\overline{AP}$ = _____

$m\overline{B'P}$ = _____ $m\overline{BP}$ = _____

$m\overline{C'P}$ = _____ $m\overline{CP}$ = _____

$m\overline{D'P}$ = _____ $m\overline{DP}$ = _____

To indicate the measure of the segment, you can write *AB* or $m\overline{AB}$.

5. **Determine each ratio.**

$\frac{A'P}{AP}$ = _____ $\frac{B'P}{BP}$ = _____

$\frac{C'P}{CP}$ = _____ $\frac{D'P}{DP}$ = _____

$\frac{B'C'}{BC}$ = _____ $\frac{A'B'}{AB}$ = _____

$\frac{A'D'}{AD}$ = _____ $\frac{A'C'}{AC}$ = _____

6. **How do you think the angle measures of the new logo will compare with those of the old logo? Make a conjecture. Then, test your conjecture by measuring various angles in the original and new logos. Describe your conclusion.**

7. **Compare the original logo and the new logo. What do you notice?**

ACTIVITY
1.2

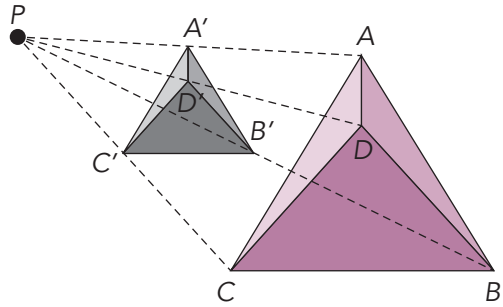
Dilating Figures with a Scale Factor Less Than 1

When the scale factor is less than 1, the new figure is called a **reduction**.

The size of the logo and its distance from point *P* are the same as the Worked Example showing an enlargement of the logo.

WORKED EXAMPLE

The original logo was dilated to produce a reduction using point *P* as the center of dilation.



The scale factor can be expressed as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

1. In the Worked Example, the scale factor is represented by 4 equivalent ratios. What distances are represented by each part of those ratios? Is the scale factor less than 1, equal to 1, or greater than 1? Explain your reasoning.

2. Measure the segment lengths of the new logo in millimeters.

$m\overline{A'B'}$ = _____ $m\overline{A'C'}$ = _____

$m\overline{B'C'}$ = _____ $m\overline{A'D'}$ = _____

3. Measure each line segment in millimeters.

$m\overline{A'P}$ = _____ $m\overline{B'P}$ = _____

$m\overline{C'P}$ = _____ $m\overline{D'P}$ = _____

4. Determine each ratio.

$\frac{A'P}{AP}$ = _____ $\frac{B'P}{BP}$ = _____

$\frac{C'P}{CP}$ = _____ $\frac{D'P}{DP}$ = _____

$\frac{B'C'}{BC}$ = _____ $\frac{A'B'}{AB}$ = _____

$\frac{A'D'}{AD}$ = _____ $\frac{A'C'}{AC}$ = _____

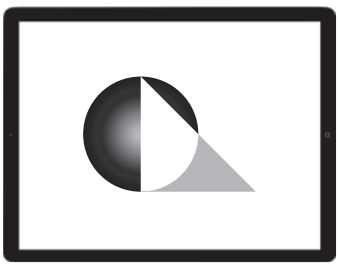
5. **How do you think the angle measures of the new logo will compare with those of the old logo? Make a conjecture. Then, test your conjecture by measuring various angles in the original and new logos. Describe your conclusion.**

6. **Compare the original logo and the new logo. What do you notice?**

ACTIVITY
1.3

Creating and Verifying Similar Figures

When working with images on a computer, the size of the images can be changed by dragging a corner or side of the image. How you drag the images determines whether or not the scale of the image is maintained.



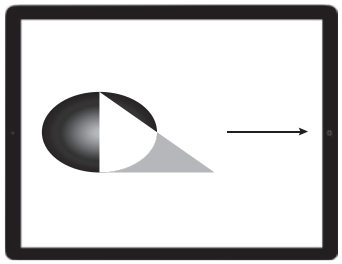
Anne needs to adjust the original logo to use on different web pages. She plays around with the image to determine how she can adjust the logo and still maintain the same scale.

Each image contains an arrow that indicates how Anne adjusts the logo and the resulting logo.

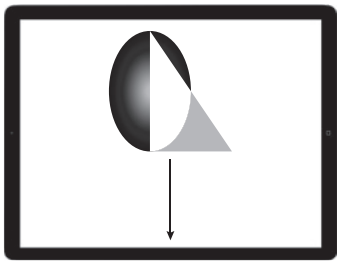
NOTES

1. **Which of the adjusted logos do you think are dilations of the original? Which are not? Explain your thinking.**

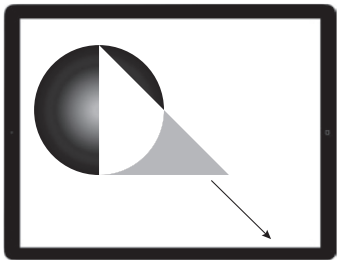
A.



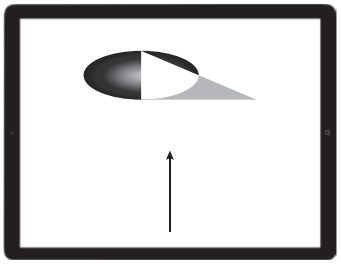
B.



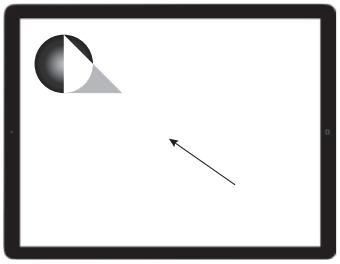
C.



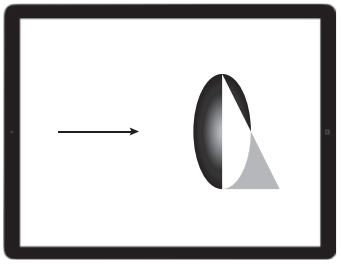
D.



E.



F.



When you dilate a figure, you create a *similar* figure. When two figures are **similar**, the ratios of their corresponding side lengths are equal. This means that you can create a similar figure by multiplying or dividing all of the side lengths of a figure by the same scale factor (except 0). You can multiply or divide by 1 to create a similar figure, too. In that case, the similar figures are congruent figures. Corresponding angles in similar figures are congruent.

NOTES

Many word processing and graphics software programs allow users to change the sizes of images.

WORKED EXAMPLE

Consider the images shown. The height of the original image is 2.66 inches, and the width is 3.48 inches. The original image is then dilated to create a reduction.

2. Are the two images similar? Explain how you know.

3. What scale factor was used to reduce the image? Describe two different ways you can determine the scale factor.

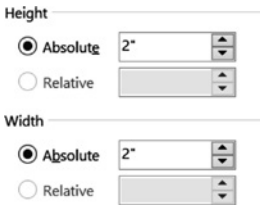
4. How can you tell that a height of 2.66 in. and a width of 3.48 in. are the original dimensions of the image?

5. **Consider each set of new dimensions or scale percents that show adjustments to this original image. Describe how the image changed and whether the new image is similar to the original. Show your work and explain your reasoning.**

a. Scale
Height: 225 % Width: 225 %

b. Scale
Height: 90 % Width: 110 %

c.


d.


6. **Explain why Jed's reasoning is not correct. Draw examples to illustrate your explanation.**

Jed

I can dilate a rectangular figure by adding the same value to its length and width.

NOTES

TALK the TALK

It's a Cloud

1. **Dilate the figure shown using scale factors of $\frac{4}{3}$ and $\frac{3}{4}$ and point Q as the center of dilation.**

2. **Describe the relationship between the corresponding sides in an original figure and the new figure resulting from a dilation.**

3. **Describe the relationship between the corresponding angles in an original figure and the new figure resulting from a dilation.**

Determine if each statement is true or false. If a statement is false, include a counterexample. Explain your reasoning.

4. **True False All similar figures are also congruent figures.**

5. **True False All congruent figures are also similar figures.**