Running, Rising, Stepping, Scaling

2

Dilating Figures on the Coordinate Plane

MATERIALS

Rulers
Protractors

Lesson Overview

Students build dilations on the coordinate plane as repeated geometric translations, using the origin as the center of dilation. Throughout, students create and modify conjectures about the effect of dilations with the origin as the center on the coordinates, perimeter, and area of a figure. They use dilations and transformations they learned previously to verify that two figures are similar.

Grade 8
Proportionality

(3) The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to:

(B) compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane.
(C) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.

Two-Dimensional Shapes

(10) The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(B) differentiate between transformations that preserve congruence and those that do not.
(D) model the effect on linear and area measurements of dilated two-dimensional shapes.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- Dilation is a transformation that produces images that are the same shape as the pre-image, but not the same size.
- When a figure is dilated on the coordinate plane, the resulting figure is similar to the original figure.
- Each coordinate (x, y) point on a figure is multiplied by the scale factor when the figure is dilated using the origin as the center of dilation.
- When a figure is dilated using the origin as the center of dilation, the perimeter of the new figure is the scale factor multiplied by the perimeter of the original figure. The area of the new figure is the square of the scale factor multiplied by the area of the original figure.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: The Escalator or the Stairs

Students begin by representing the path of an escalator using a line drawn from the origin. They then draw a sequence of congruent steps that reach the same location as the escalator. Students use equivalent ratios, scaling up, and scaling down to discuss how different step designs can accomplish the same goal of reaching a location.

Develop

Activity 2.1: Scaling Up and Down on the Coordinate Plane

Students learn from two Worked Examples how to dilate a figure by a sequence of repeated translations starting from the origin. They use the horizontal and vertical translations of each point as the unit and then scale accordingly for each point. Students conjecture about how the coordinates of a figure are affected by dilations using the origin as the center of dilation.

Day 2

Activity 2.2: Using the Origin as the Center of Dilation

Students dilate a variety of figures on the coordinate plane, using the origin as the center of dilation and a variety of scale factors. They verify that these figures are similar using corresponding angle measures and the ratios of corresponding side lengths. Students explore the effects of dilations on the perimeter and perimeter of the original figure. Students monitor their conjectures about how dilations affect the coordinates, perimeter, and area of a figure. They also write algebraic rules for dilations using the origin as the center.

Demonstrate

Talk the Talk: Location, Location, Location

Students summarize what they learned in this lesson and the previous lesson by describing the different strategies they can use to perform a dilation on and off the coordinate plane. They practice writing the algebraic rules for a dilation with the origin as the center. Finally, students describe how to verify similarity on the coordinate plane using just the coordinates of the figures.

ENGAGE

Getting Started: The Escalator or the Stairs

Facilitation Notes

In this activity, students are given a scenario that uses coordinate planes and equivalent ratios.

Have students work with a partner or in a group to complete Questions 1 through 3. Emphasize to students that when comparing, the order of length and height in the ratio matters. Share responses as a class.

As students work, look for

Students that draw the steps starting with the horizontal length and students that draw the steps starting with the vertical height. Either way is acceptable.

Questions to ask

- How did you locate the point (12, 8)?
- What two points did you connect to form the line?
- Do the steps and the escalator go to the same location?
- Which graph describes a smooth path?
- Which graph describes a series of horizontal and vertical translations?
- How would you describe the vertical to horizontal ratio formed by the steps?
- How would you describe the horizontal to vertical ratio formed by the steps?

Summary

Scaling up, scaling down, and equivalent ratios can be represented using a coordinate plane.

DEVELOP

Activity 2.1
Scaling Up and Down on the Coordinate Plane

Facilitation Notes

In this activity, students dilate a figure on the coordinate plane by a sequence of repeated horizontal and vertical translations. The origin is the center of dilation.

Remember that the similarity of figures can always be verified using rulers and protractors.

Ask a student to read the introduction aloud. Analyze the Worked Example as a class. Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What are the coordinates of point *B*?
- Why isn't point *B'* located at (6, 6)?
- Why is point *B'* located at (9, 9)?
- What are the coordinates of point *C*?
- Why isn't point *C'* located at (8, 2)?
- Is point *C'* located at (3, 12) or (12, 3)?
- Why is point *C'* located at (12, 3)?
- Are the corresponding angles of $\triangle ABC$ and $\triangle A'B'C'$ congruent?
- Are the corresponding side ratios of $\triangle ABC$ and $\triangle A'B'C'$ equal?
- Are $\triangle ABC$ and $\triangle A'B'C'$ the same shape?

Analyze the Worked Example following Question 2 as a class. Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

Questions to ask

- How did you determine the location of point *E'*?
- Point *E* is four repeated translations of what movements?
- Is point *E* four repeated translations of right 2 and up 2? How is this helpful when determining the location of point *E'*?
- How did you determine the location of point *F'*?
- Point *F* is four repeated translations of what movements?
- Is point *F* four repeated translations of right 3 and up 1? How is this helpful when determining the location of point *F'*?
- Are the corresponding angles of $\triangle DEF$ and $\triangle D'E'F'$ congruent?
- Are the corresponding side ratios of $\triangle DEF$ and $\triangle D'E'F'$ equal?
- Are $\triangle DEF$ and $\triangle D'E'F'$ the same shape?

Summary

Repeated horizontal and vertical translations on the coordinate plane can be used to dilate figures.

Activity 2.2
Using the Origin as the Center of Dilation

Facilitation Notes

In this activity, students dilate figures on the coordinate plane, using the origin as the center of dilation. Corresponding angle measures and ratios of corresponding side lengths are used to verify similarity. Students explore the effects of dilations on the perimeter and perimeter of the original figure.

Provide students with measuring tools. Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- Is the image an enlargement or a reduction? How do you know?
- How did you determine the location of point *A*′?
- Point *A*′ is three repeated translations of what movements?
- Is point *A*′ three repeated translations of right 2 and up 3?
- What are the coordinates of point *B*?
- How did you locate point *B*′?
- Point *B*′ is three repeated translations of what movements?
- Are the corresponding angles of Quadrilateral *ABCD* and Quadrilateral *A*′*B*′*C*′*D*′ congruent?
- Are the corresponding side ratios of Quadrilateral *ABCD* and Quadrilateral *A*′*B*′*C*′*D*′ equal?
- Are Quadrilateral *ABCD* and Quadrilateral *A*′*B*′*C*′*D*′ the same shape?
- How did you determine the side lengths of the figure on the coordinate plane?
- How did you determine the perimeter of the figure?
- How did you determine the area of the figure?
- How does the perimeter of the new figure compare to the perimeter of the original figure?
- How does the area of the new figure compare to the area of the original figure?

Have students work with a partner or in a group to complete Questions 5 and 6. Share responses as a class.

Questions to ask

- What do you notice about the relationship between the perimeter of the original figure and the new figure?
- What do you notice about the relationship between the area of the original figure and the new figure?
- How can you determine if your prediction is correct?
- Can you determine how a dilation would affect the area and perimeter of a figure without graphing?
- Does your conjecture hold true for all figures? Explain your reasoning.

Have students work with a partner or in a group to complete Questions 7 through 10. Share responses as a class.

Questions to ask

- Is the image an enlargement or a reduction? How do you know?
- How did you determine the location of point *W'*?
- Point *W'* is one translation of what movements?
- Is point *W'* one translation of left 4 and down 5?
- What are the coordinates of point *X'*?
- How did you locate point *X'*?
- Point *X'* is one translation of what movements?
- Are the corresponding angles of Quadrilateral *WXYZ* and Quadrilateral *W'X'Y'Z'* congruent?
- Are the corresponding side ratios of Quadrilateral *WXYZ* and Quadrilateral *W'X'Y'Z'* equal?
- Are Quadrilateral *WXYZ* and Quadrilateral *W'X'Y'Z'* the same shape?

Summary

When a figure is dilated on the coordinate plane, the resulting figure is similar to the original figure. The perimeter of the new figure is the scale factor multiplied by the perimeter of the original figure. The area of the new figure is the square of the scale factor multiplied by the area of the original figure.

DEMONSTRATE

Talk the Talk: Location, Location, Location

Facilitation Notes

In this activity, students describe the process of dilating figures with the origin as the center, and how that affects the coordinates of the dilated figure.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- When are measurement tools helpful?
- How are measurement tools helpful?
- How can you use the value of the scale factor to determine whether the dilation is an enlargement or a reduction of the original figure?

Summary

The dilation of all the points (x, y) of an original figure will be (sx, sy), where the center of dilation is the origin and the scale factor is s.

Running, Rising, Stepping, Scaling

2

Dilating Figures on the Coordinate Plane

WARM UP

Scale up or scale down to determine the value of the variable in each equivalent ratio.

1. 3 : 1 = 25.5 : z

2. 2 : 5 = a : 30

3. 1 : 4 = x : 80

4. 9.9 : 10 = 99 : p

LEARNING GOALS

- Dilate figures on a coordinate plane.
- Understand the dilation of a figure on the coordinate plane as a scaling up or scaling down of the coordinates of the figure.
- Describe how a dilation of a figure on a coordinate plane affects the coordinates of the figure.

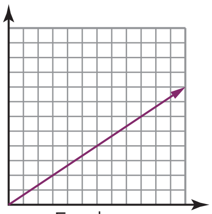
You have used transformations called dilations to create similar figures. How can you use coordinates to determine whether two figures are similar?

Warm Up Answers

1. $z = 8.5$
2. $a = 12$
3. $x = 20$
4. $p = 100$

Answers

1a.



1b. Answers will vary. Check students' graphs.

2. Both the steps and the escalator go to the same location of (12, 8). The escalator is a smooth path from the origin, while the steps are repeated up-and-over moves from the origin.

3. Check students' answers. Congruent steps that reach the same location of (12, 8) will have the same ratio of vertical-to-horizontal (or horizontal-to-vertical) translations. These ratios will all be equal to 12 : 8 (or 8 : 12 when starting with the vertical). Sample step shapes: (+6, +4), (+3, +2).

Getting Started

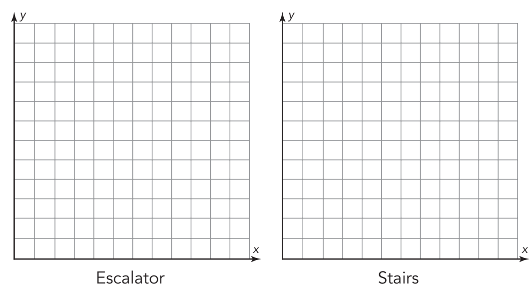
The Escalator or the Stairs

Bob is riding an escalator. The escalator starts at (0, 0) and drops Bob off at (12, 8).

1. Use the coordinate planes given to represent Bob's journey.

a. Draw a line to show Bob's path on the escalator.

b. Alice takes the stairs. Draw steps starting at the origin that will take Alice to the same location as Bob. Make all of the steps the same.



2. How is taking the stairs similar to riding the escalator? How is it different? Explain your reasoning.

3. Compare the steps that you designed for Alice with your classmates' steps. How are these steps similar to your steps?

ELL Tip

English Language Learners may not know what an *escalator* is. Explain to students what an *escalator* is and provide a visual, either a picture or a video of an *escalator*. Ask students to name places where they might find an *escalator*.

ACTIVITY **2.1**

Scaling Up and Down on the Coordinate Plane

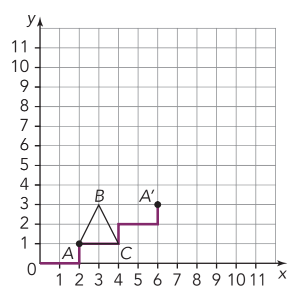
NOTES

You know that a translation moves a point along a line. A sequence of repeated horizontal and/or vertical translations also moves a point along a line. You can use this fact to dilate figures.

WORKED EXAMPLE

Dilate △*ABC* by a scale factor of 3 using the origin as the center of dilation.

Let's start by dilating Point *A*, which is located at (2, 1). In other words, Point *A* is translated from the origin 2 units right and 1 unit up.



To dilate point *A* by a scale factor of 3, translate Point *A* by three repeated sequences: 2 units right and 1 unit up from the origin.

1. Describe the repeated translations you can use to scale point *B* and point *C*. Then plot point *B*' and point *C*' on the coordinate plane in the Worked Example.

a. point *B* to point *B*′

b. point *C* to point *C*′

2. Draw △*A*′*B*′*C*′ on the coordinate plane in the example. Is △*ABC* similar to △*A*′*B*′*C*′? Explain your reasoning.

Answers

1a. Point *B* is translated from the origin right 3 and up 3. Three repeated translations of right 3 and up 3 from the origin is (+3, +3), (+6, +6), (+9, +9). So, point *B*′ is at (9, 9).

1b. Point *C* is translated from the origin right 4 and up 1. Three repeated translations of right 4 and up 1 from the origin is (+4, +1), (+8, +2), (+12, +3). So, point *C*′ is at (12, 3).

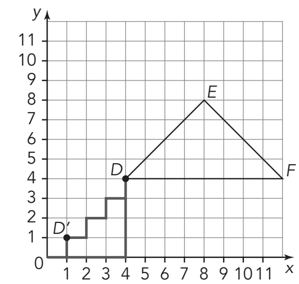
2. Yes. Triangle *ABC* is similar to Triangle *A*′*B*′*C*′.

Answers

3. Point E is translated from the origin right 8 and up 8. Four repeated translations of right 2 and up 2 is the same as (+8, +8), so point E' is at (2, 2).
   Point F is translated from the origin right 12 and up 4. Four repeated translations of right 3 and up 1 is the same as (+12, +4), so point F' is at (3, 1).
4. Yes. Triangle DEF is similar to Triangle $D'E'F'$.
5. Answers will vary.

WORKED EXAMPLE

Dilate ΔDEF by a scale factor of $\frac{1}{4}$ using the origin as the center of dilation.



Point D is translated from the origin 4 units right and 4 units up (4, 4). This is the same as four translations of 1 unit right and 1 unit up.

Therefore, scaling point D to (1, 1) represents a dilation by a scale factor of $\frac{1}{4}$.

3. Determine the coordinates of points E' and F'. Explain how you determined your answers. Then, draw $\Delta D'E'F'$ on the coordinate plane in the example.

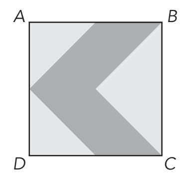
4. Is ΔDEF similar to $\Delta D'E'F'$? Explain your reasoning.

5. How does dilating a figure, using the origin as the center of dilation, affect the coordinates of the original figure? Make a conjecture using the examples in this activity.

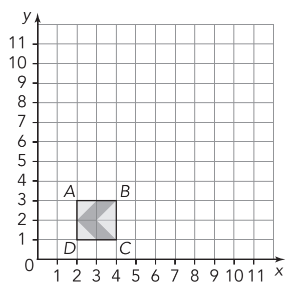
ACTIVITY 2.2

Using the Origin as the Center of Dilation

Road signs maintain a constant scale, regardless of whether they are on the road or in the drivers' manual. This sign indicates that the road is bending to the left.



1. Dilate the figure on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of 3 to form a new figure.



2. List the ordered pairs for the original figure and for the new figure. How are the values in the ordered pairs affected by the dilation?

3. Compare and contrast the corresponding angles and corresponding side lengths of the new figure and the original figure.

Answers

1. Check students' graphs.
2. The coordinates of Quadrilateral *ABCD* are *A* (2, 3), *B* (4, 3) *C* (4, 1), *D* (2, 1).
   The coordinates of Quadrilateral *A′B′C′D′* are *A′* (6, 9), *B′* (12, 9), *C′* (12, 3) and *D′* (6, 3).
   The *x*- and *y*-coordinates of the ordered pairs are all multiplied by the scale factor.
3. The corresponding angles are all congruent, and the corresponding side lengths form equivalent ratios of 3 : 1 (or 1 : 3).

Answers

4. See table below.

4a. The perimeter of the new figure is three times larger than the perimeter of the original road sign. The perimeter of the original road sign is multiplied by the scale factor to get the perimeter of the new figure.

4b. The area of the new figure is nine times larger than the area of the original road sign. The area of the original road sign is multiplied by the square of the scale factor to get the area of the new figure.

5a. Sample answer. The dilation will cause the new figure to become smaller, so the perimeter will also become smaller. The new figure will have a perimeter that is $\frac{1}{2}$ of the perimeter of the original road sign.

5b. Sample answer. The dilation will cause the new figure to become smaller, so the area will also become smaller. The new figure will have an area that is $\frac{1}{4}$ of the area of the original road sign.

4. Determine the perimeter and area of the original figure and the new figure.

	Perimeter	Area
Original Figure		
New Figure		

a. How is the perimeter affected by the dilation?

b. How is the area affected by the dilation?

5. A road sign is represented by the coordinates *A* (2, 1), *B* (2, 12), *C* (6, 12), and *D* (6, 1). Suppose you were to dilate the figure by a scale factor of $\frac{1}{2}$ using the origin as the center of dilation.

a. Predict how the perimeter of the figure will be affected by the dilation.

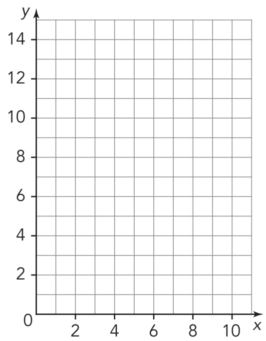
b. Predict how the area of the figure will be affected by the dilation.

6 • TOPIC 2: Similarity

4.

	Perimeter	Area
Original Figure	8 units	4 square units
New Figure	24 units	36 square units

c. Test your prediction by graphing the original figure and the new figure on the coordinate plane.

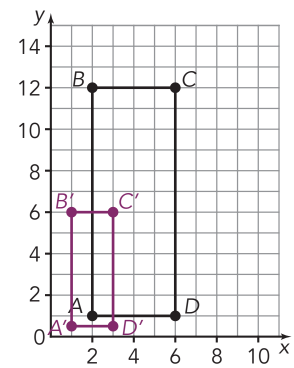


d. Describe your conclusion.

6. How does dilating a figure, using the origin as the center of dilation, affect the perimeter and area of the new figure? Make a conjecture using the examples in this activity.

Answers

5c. Check students' graphs.



5d. The perimeter of the original road sign is 30 units. The perimeter of the new figure is 15 units. $30 \times \frac{1}{2} = 15$, so the new figure has a perimeter that is $\frac{1}{2}$ of the original road sign. The area of the original road sign is 44 square units. The area of the new figure is 11 square units. $44 \times \left(\frac{1}{2}\right)^2 = 44 \times \frac{1}{4} = 11$, so the new figure has an area that is $\frac{1}{4}$ of the original road sign.

6. When a figure is dilated using the origin as the center of dilation, the perimeter of the new figure is equal to the scale factor multiplied by the perimeter of the original figure. The area of the new figure is equal to the square of the scale factor multiplied by the area of the original figure.

Answers

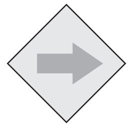
7. Check students' graphs.
8. The coordinates of Quadrilateral *WXYZ* are *W* (8, 10), *X* (10, 8), *Y* (8, 6), *Z* (6, 8). The coordinates of *W′X′Y′Z′* are *W′* (4, 5) *X′* (5, 4) *Y′* (4, 3) *Z′* (3, 4). The *x*- and *y*-coordinates of the ordered pairs are all multiplied by the scale factor.
9. The corresponding angles are all congruent, and the corresponding side lengths form equivalent ratios of 1 : 2 (or 2 : 1).

10a. $\left(\frac{1}{2}x, \frac{1}{2}y\right)$

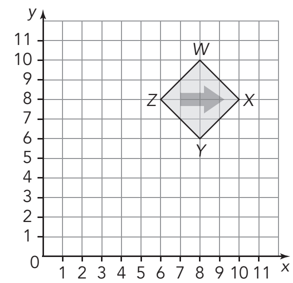
10b. $(5x, 5y)$

10c. $\left(\frac{4}{3}x, \frac{4}{3}y\right)$

Let's consider a different road sign. This sign indicates that the road proceeds to the right.



7. Dilate the figure on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of $\frac{1}{2}$ to form a new figure.



8. List the ordered pairs for the original figure and for the new figure. How are the values in the ordered pairs affected by the dilation?

9. Compare and contrast the corresponding angles and corresponding side lengths of the original figure and the new figure. If the dilation of a figure is centered at the origin, you can multiply the coordinates of the points of the original figure by the scale factor to determine the coordinates of the new figure.

10. Suppose a point is located at (*x*, *y*). Write the ordered pair of the point after a dilation by each given scale factor.

a. $\frac{1}{2}$ **b.** 5

c. $\frac{4}{3}$

NOTES

TALK the TALK

Location, Location, Location

Answer each question to summarize what you know about dilating figures on the coordinate plane. Use your answers to plan a presentation for your classmates that demonstrates what you learned in this lesson.

1. **What strategies can you use to determine if two figures are similar when they are:**

   a. **located on a coordinate plane?**

   b. **not located on a coordinate plane?**

2. **A polygon is graphed on a coordinate plane with (x, y) representing the location of a certain point on the polygon. The polygon is transformed using the rule $(x, y) \rightarrow (ax, ay)$.**

   a. **What will be the impact on the original figure if a is greater than 1?**

   b. **What will be the impact on the original figure if a is between 0 and 1?**

Answers

1. Answers will vary.

2a. The figure will become larger.

2b. The figure will become smaller.