3

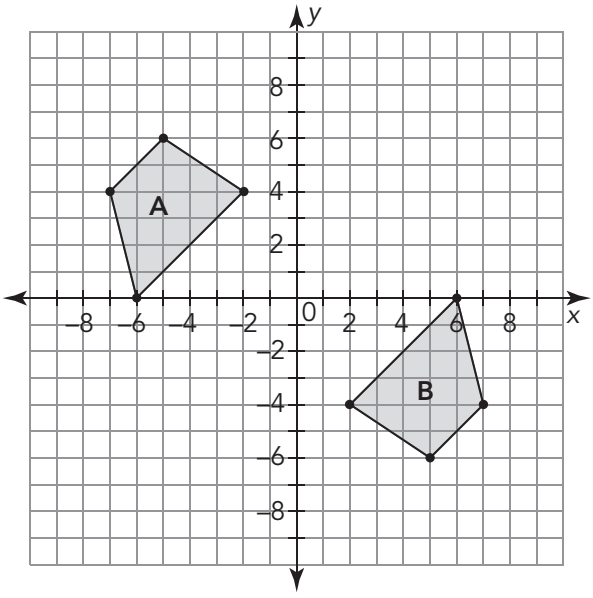
From Here to There

Mapping Similar Figures Using Transformations

WARM UP

1. Describe at least two different single transformations or sequences of transformations that map Figure *A* to Figure *B*.

2. Describe the geometric relationships between the figures.



LEARNING GOALS

- Describe a single dilation that maps a two-dimensional figure onto a similar figure.
- Determine a sequence of transformations that maps a two-dimensional figure onto a similar figure.
- Determine the relationship between images of the same pre-image.

You have used sequences of translations, reflections, and rotations to verify that two images are congruent. How can you use transformations to determine if two images are similar and/or congruent?

Same Figure or Same Shape?

When two figures are similar, the same scale factor can be applied to all side lengths to map one figure to the other.

We often say that dilations preserve shape and that rigid motions preserve both size and shape. As a result, it is common to state that similar figures have the same shape, and congruent figures have the same size and shape. However, what does it mean for two figures to have the same shape in this context? Are all rectangles similar? Are all triangles similar?

Use the definition of similar figures to determine which figures are similar.

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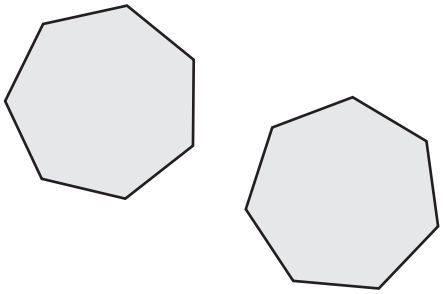

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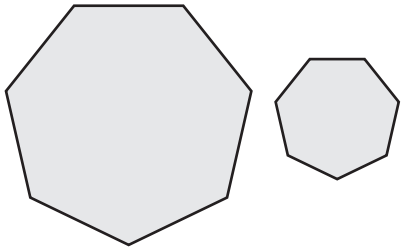

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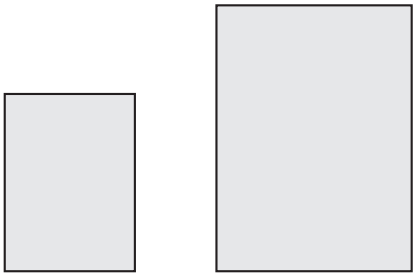

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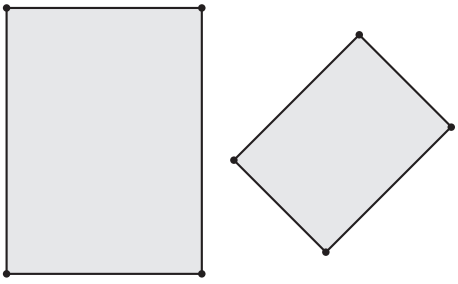

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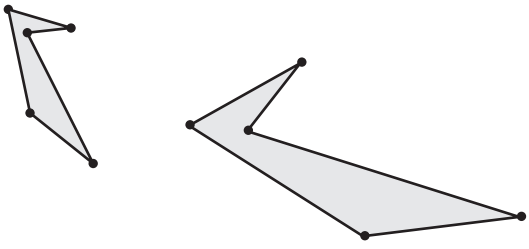

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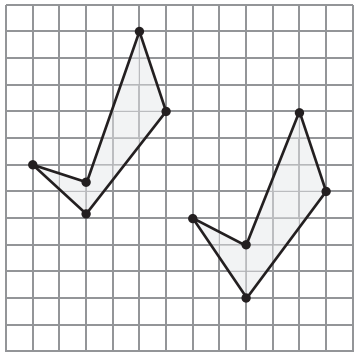


ACTIVITY
3.1

Proving Similarity Through Dilations

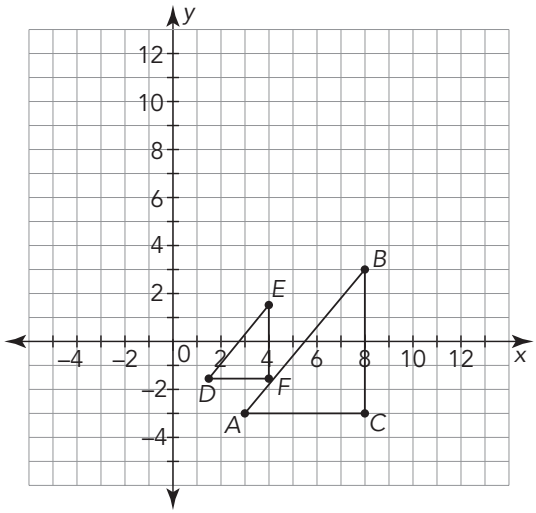
NOTES

In this activity, you will use what you know about dilations to determine if figures are similar.

1. **Determine if the figures are similar. If they are similar, state the scale factor and the center of dilation that maps Figure 1 onto Figure 2.**

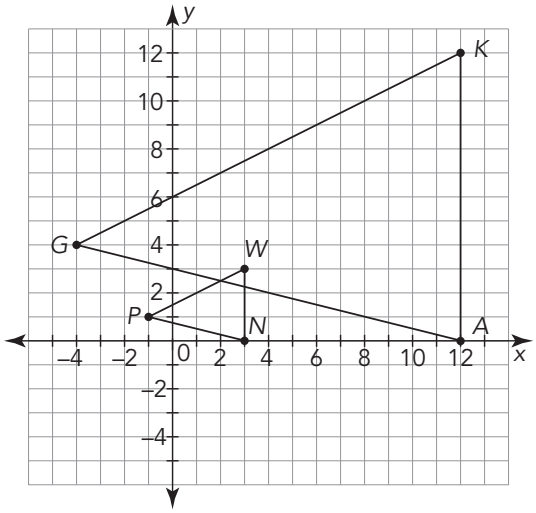
   a. **Figure 1: △*ABC***

      Figure 2: △*DEF*



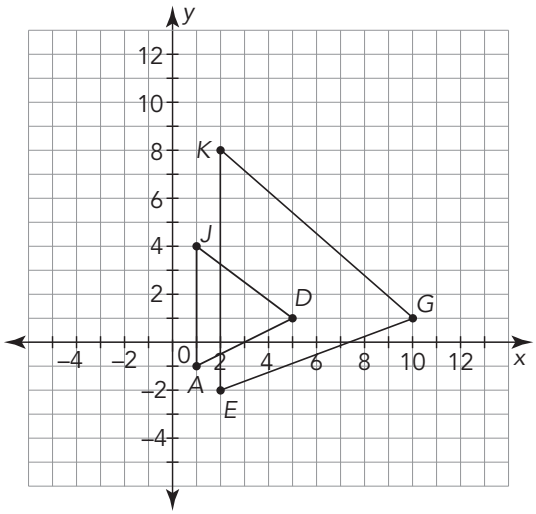
   b. **Figure 1: △*PWN***

      Figure 2: △*GKA*



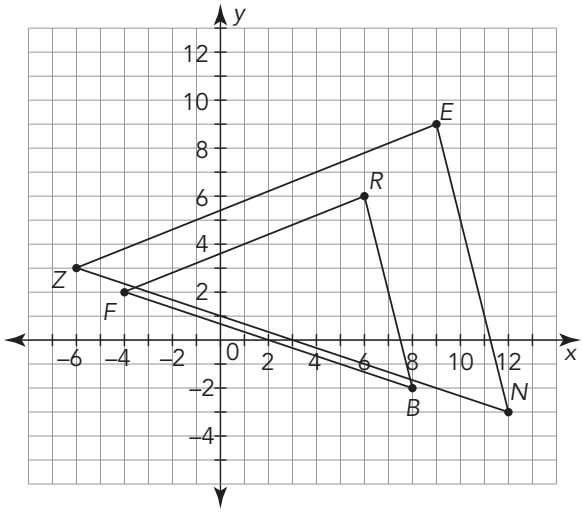
   c. **Figure 1: △*JDA***

      Figure 2: △*KGE*



d. Figure 1: △*ZEN*

Figure 2: △*FRB*

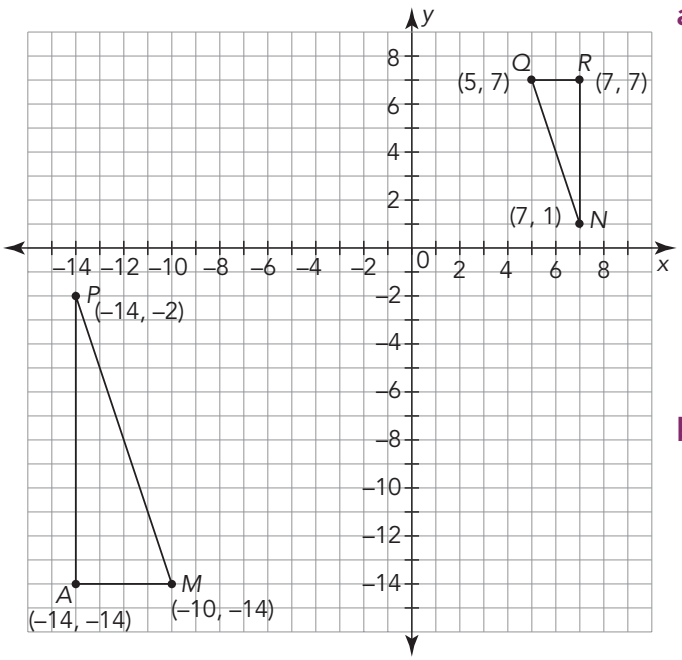


ACTIVITY
3.2

Proving Similarity Through Transformations

Sometimes similar figures cannot be mapped from one to another using only a dilation. You may need a combination of translations, reflections, rotations, and dilations to map a figure onto a similar figure.

1. Triangle *MAP* is the image of Triangle *QRN* after undergoing at least one transformation.

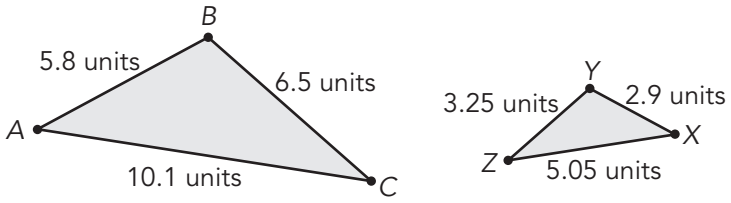


a. Determine a possible sequence of transformations to map △*QRN* onto △*MAP*.

b. Are the triangles congruent? Are they similar? Explain your reasoning.

c. **Reverse the order of the sequence of transformations you described in part (b). What do you notice?**

2. **Triangle *XYZ* is the image of Triangle *ABC* after undergoing at least one transformation.**

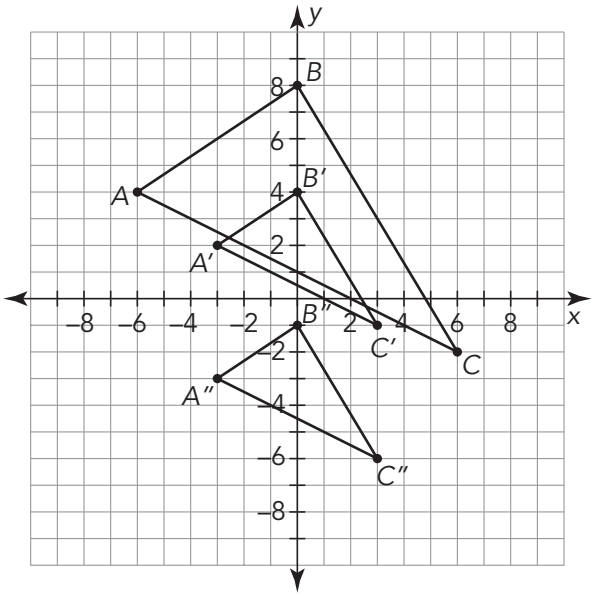


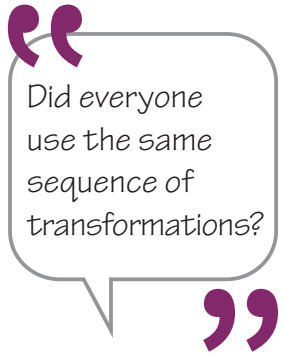
a. **List the corresponding sides and angles for △*ABC* and △*XYZ*.**

b. **Determine a possible sequence of transformations to map △*ABC* onto △*XYZ*.**

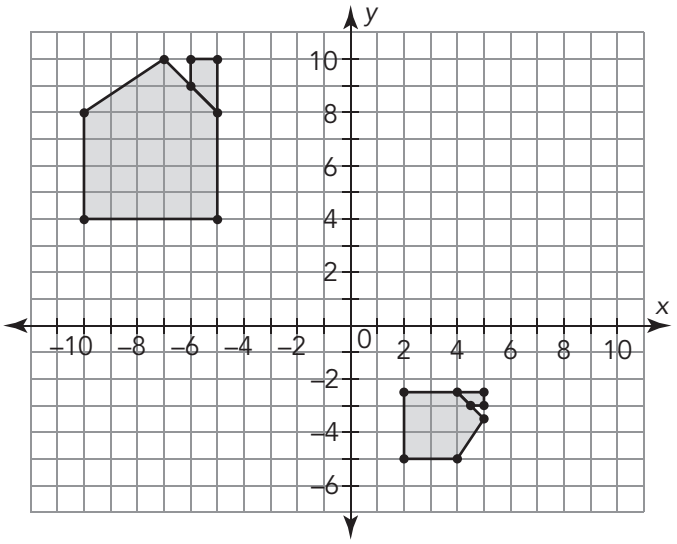
c. **Reverse the order of the sequence of transformations you described in part (b). What do you notice?**

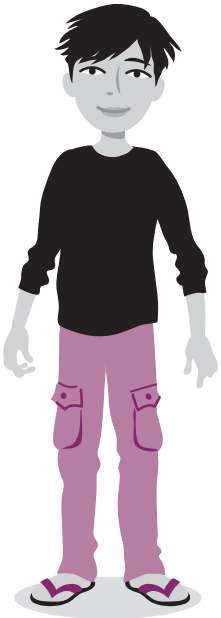
3. Triangle *ABC* was dilated to create Triangle *A′B′C′*. Then Triangle *A′B′C′* was dilated to create Triangle *A″B″C″*. Describe a series of transformations that map △*ABC* onto △*A″B″C″*.





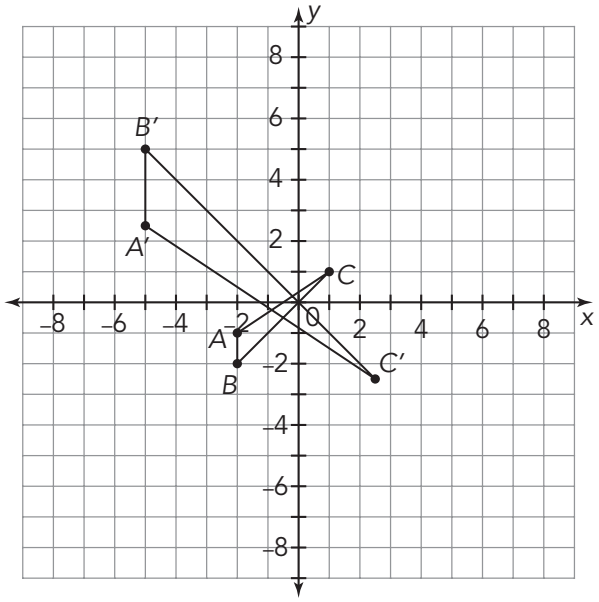
4. Verify that the two houses are similar by describing a sequence of transformations that maps one figure onto the other.



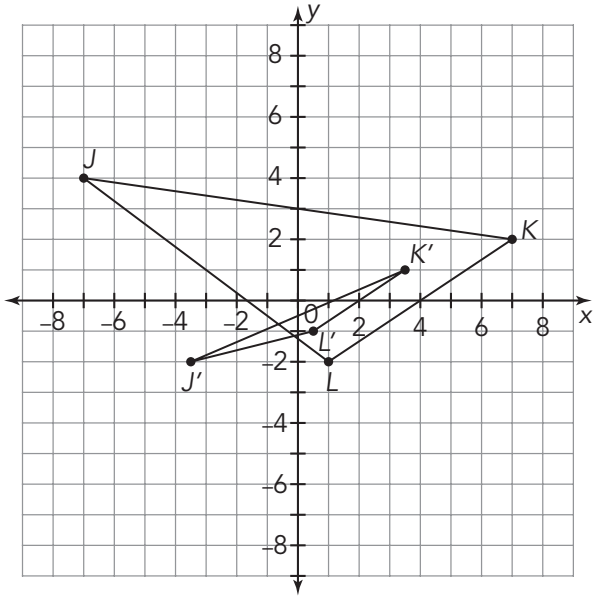


5. **Use dilations and other transformations to determine if the triangles represented by the coordinates are similar. Show your work and explain your reasoning.**

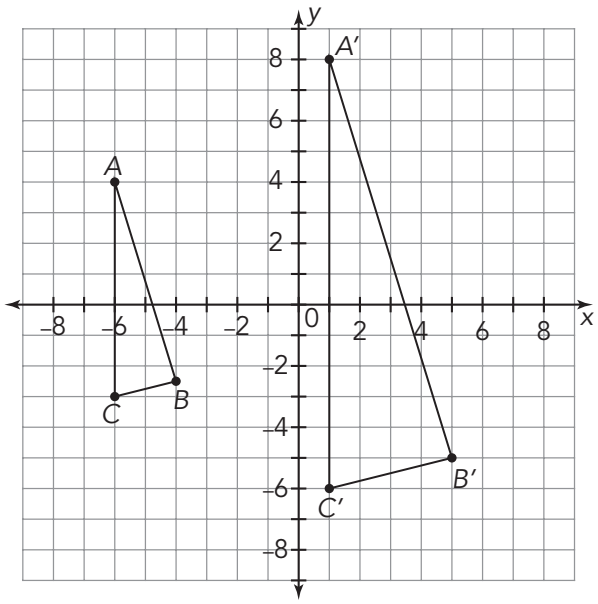
   a. ***A*** **(−2, −1)** ***B*** **(−2, −2)** ***C*** **(1, 1)**
   A′ **(−5, 2.5)** ***B′*** **(−5, 5)** ***C′*** **(2.5, −2.5)**



   b. ***J*** **(−7, 4)** ***K*** **(7, 2)** ***L*** **(1, −2)**
   J′ **(−3.5, −2)** ***K′*** **(3.5, 1)** ***L′*** **(0.5, −1)**



c. $A(-6, 4)$ $B(-4, -2.5)$ $C(-6, -3)$
$A'(1, 8)$ $B'(5, -5)$ $C'(1, -6)$



ACTIVITY
3.3

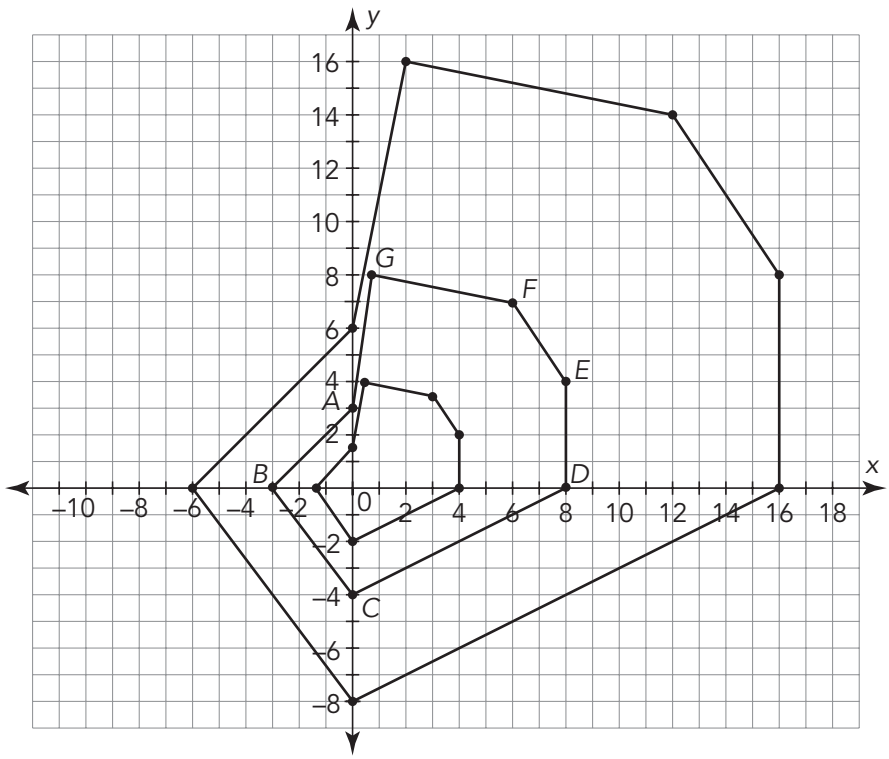
Comparing Images

You know that similar figures can be mapped from one to another using a sequence of transformations. How are the images of the same pre-image related to each other?

Let's investigate!

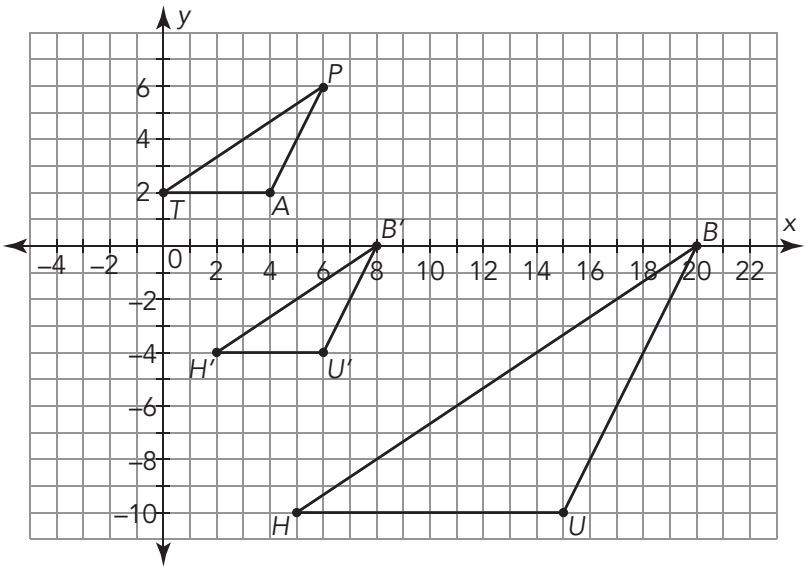
1. The labeled figure is the pre-image used to create the other two figures using dilations.

a. Determine the scale factor to map the pre-image to each of the other figures. Explain your reasoning.



b. Are the images similar? Are they congruent? Explain your reasoning.

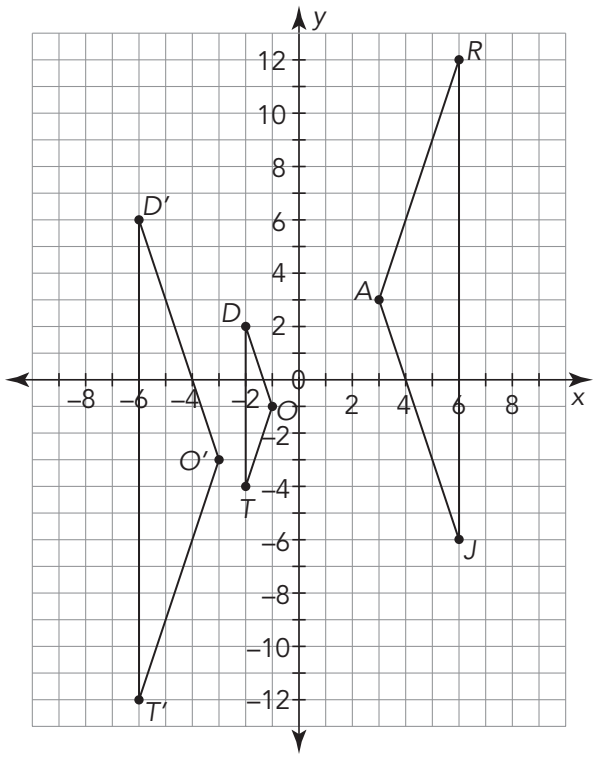
2. Triangle *HUB* was dilated from the origin by a scale factor of $\frac{2}{5}$ to create $\triangle H'U'B'$, and $\triangle H'U'B' \cong \triangle TAP$.



a. What is the relationship between $\triangle HUB$ and $\triangle TAP$? Justify your answer.

b. Determine a possible sequence of transformations that maps $\triangle HUB$ onto $\triangle TAP$.

3. **Triangle *DOT* was dilated from the origin by a scale factor of 3 to create $\triangle D'O'T'$, and $\triangle D'O'T' \cong \triangle JAR$. Determine a possible sequence of transformations that maps $\triangle JAR$ onto $\triangle DOT$.**



NOTES

TALK the TALK

Summing Up Similar Figures

Determine if each statement is *always*, *sometimes*, or *never* true. Provide a justification for each answer.

1. **Triangle *ABC* is dilated four times with different scale factors. The four images are congruent.**

2. **Triangle *HIP* is dilated by a scale factor of 8, followed by a scale factor of 0.125. The final image is congruent to $\triangle HIP$.**

3. **Dilations are used to create congruent figures.**

4. **Transformations are used to create similar figures.**