From Here to There

3

Mapping Similar Figures Using Transformations

MATERIALS

Centimeter rulers (optional)
Protractors (optional)
Patty paper (optional)

Lesson Overview

Students determine if figures are similar through transformations. They explore what is meant by "same shape" when referring to similar figures. Students determine similarity using a single dilation and verify similarity of a variety of figures through a sequence of transformations. They then explore the relationship between images of a common pre-image under different conditions and the relationship between figures that are similar. Finally, students summarize the relationships between transformations and congruent and similar figures.

Grade 8

Proportionality

(3) The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to:

(C) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.

Two-Dimensional Shapes

(10) The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.
(B) differentiate between transformations that preserve congruence and those that do not.
(C) explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- If two figures are similar, one can be mapped onto the other through a sequence of transformations.
- Images created from the same pre-image are similar.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Same Figure or Same Shape?

Students decide if two figures are similar using the definition of similar figures. They consider what it means for similar figures to have the "same shape."

Develop

Activity 3.1: Proving Similarity Through Dilations

Students determine if two triangles on the coordinate plane are similar. If they are similar, students state the scale factor and the center of dilation used to map from one similar figure onto the other.

Activity 3.2: Proving Similarity Through Transformations

Students determine a sequence of transformations that map a figure onto a similar figure. They may use translations, rotations, reflections, and dilations. Students notice that the order of the sequence of transformations does not matter.

Day 2

Activity 3.3: Comparing Images

Students compare the similar and congruent figures created through transformations. They investigate the relationship between images formed by applying different scale factors but the same center of dilation. Finally, students map from congruent figures onto a related similar figure.

Demonstrate

Talk the Talk: Summing Up Similar Figures

Students answer *sometimes, always,* or *never* questions about sequences of transformations, images from transformations, and similar figures.

Getting Started: Same Figure or Same Shape?

ENGAGE

Facilitation Notes

In this activity, students use the definition of *similar figures* to identify examples of similarity.

Provide measuring tools such as rulers and protractors to verify similarity.

Ask a student to read the introduction aloud. Discuss as a class. Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

Questions to ask

- Are the pairs of corresponding angles congruent?
- Are the ratios formed by the corresponding sides equal?
- Do the two figures appear to have the same shape?
- Are the two figures the same size?
- Do similar figures always have the same shape? Same size?

Summary

If two figures are similar, the same scale factor can be applied to all side lengths to map one figure onto the other.

Activity 3.1
Proving Similarity through Dilations

DEVELOP

Facilitation Notes

In this activity, students determine the similarity of two figures drawn on a coordinate plane and identify the scale factor and center of dilation.

Students may wish to use a centimeter ruler or a non-standard measuring tool to compare the side lengths of the figures.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- What are the coordinates of the pre-image? The image?
- Did you draw lines connecting each of the corresponding vertices?
- Where did the lines intersect? What are the coordinates of the point of intersection?
- Was the point of intersection at the origin?
- How are the coordinates of the pre-image and the coordinates of the image used to calculate the scale factor?
- What horizontal and vertical translations occur to move from the center of dilation to a vertex of the pre-image?
- What repeated translations occur to move from the center of dilation to the corresponding vertex of the image?
- What horizontal and vertical translations occur to move from the center of dilation to a different vertex of the pre-image?
- What relationship exists between the translations used to move from the center of dilation to a vertex of the pre-image and the translations used to move from the center of dilation to the corresponding vertex of the image?
- Are all of the ratios formed using the lengths of the corresponding sides of the figures equal?

Summary

The center of dilation of similar figures can be determined by locating the intersection of all lines drawn through the corresponding vertices of the image and pre-image. The scale factor of similar figures can be determined by calculating ratios formed by the horizontal and vertical translations that occur when moving from the center of dilation to vertices of the image and vertices of the pre-image.

Activity 3.2

Proving Similarity through Transformations

Facilitation Notes

In this activity, students identify the combination of transformations (translations, rotations, reflections, and dilations) needed to map a figure onto a similar figure.

Emphasize that the order of the sequence of transformations does not matter.

Differentiation strategies

- Assign different groups of students different problems, but make the entire class accountable for all questions by sharing their solutions with the class.
- Have students try the transformations in different orders to check for correct mappings. When the image and pre-image are not identified in the situation, have students work the situation both ways.

Have students work with a partner or in a group to complete Questions 1 through 5. Some students may want to use patty paper as they work. Share responses as a class.

Questions to ask

- Did you draw lines connecting each of the corresponding vertices?
- Where did the lines intersect? What are the coordinates of the point of intersection?
- How are the coordinates of the pre-image and the coordinates of the image used to calculate the scale factor?
- What horizontal and vertical translations occur to move from the center of dilation to a vertex of the pre-image?
- What repeated translations occur to move from the center of dilation to the corresponding vertex of the image?
- What horizontal and vertical translations occur to move from the center of dilation to a different vertex of the pre-image?
- What relationship exists between the translations used to move from the center of dilation to a vertex of the pre-image and the translations used to move from the center of dilation to the corresponding vertex of the image?
- Are all of the ratios formed using the lengths of the corresponding sides of the figures equal?
- How many transformations were involved in this situation?
- Is there another sequence of transformations that will satisfy this situation?
- If two figures are similar, what must be true?
- If two figures are congruent, what must be true?
- What is the difference between congruence and similarity?
- If the triangles are congruent, should the corresponding sides be congruent?

Summary

Identifying a sequence of transformations that map one figure onto another is a method for proving two figures are similar.

Activity 3.3
Comparing Images

Facilitation Notes

In this activity, students use similar and congruent figures created through a sequence of transformations. They investigate the relationship between images formed by applying different scale factors but the same center of dilation. Finally, students map from congruent figures onto a related similar figure.

Have a student read the introduction to the activity. Then have students make conjectures as a class before they begin the problems.

Questions to ask

- How do you think the images will be related to each other?
- Will the images be congruent to each other?
- Will the images be similar to each other?

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Which figures are the images?
- Which figure is the pre-image?
- How did you determine the scale factor used to map one figure onto another?
- What horizontal and vertical translations occurred to create each of these mappings?
- Did you draw lines through corresponding vertices to locate the center of dilation?
- Can a different sequence of transformations be used in this situation?
- What is a different sequence of transformations that could be used in this situation?

Summary

Scale factors and centers of dilation are used to identify similar figures.

Talk the Talk: Summing Up Similar Figures

Facilitation Notes

In this activity, students answer questions related to sequences of transformations and similarity.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- What is true about the corresponding side lengths of congruent figures?
- What is true about the corresponding side lengths of similar figures?
- What are the conditions of congruence?
- Are dilating by 8 and dilating by $\frac{1}{8}$ inverse dilations? Will the sides of the final image be the same length as the pre-image after using an inverse dilation?
- If the scale factor is 1, is the dilation a congruent figure?
- Can a scale factor have a value of 0?

Summary

Similar figures can be described as a result of one or more transformations such as translations, reflections, rotations, and dilations.

NOTES

From Here to There

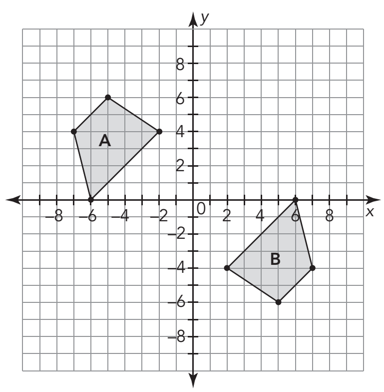
3

Mapping Similar Figures Using Transformations

WARM UP

1. Describe at least two different single transformations or sequences of transformations that map Figure *A* to Figure *B*.

2. Describe the geometric relationships between the figures.



LEARNING GOALS

- Describe a single dilation that maps a two-dimensional figure onto a similar figure.
- Determine a sequence of transformations that maps a two-dimensional figure onto a similar figure.
- Determine the relationship between images of the same pre-image.

You have used sequences of translations, reflections, and rotations to verify that two images are congruent. How can you use transformations to determine if two images are similar and/or congruent?

Warm Up Answers

1. Figure *A* can be reflected across both axes to map onto Figure *B*. Figure *A* can be rotated 180° around the origin to map onto Figure *B*.
2. The figures are congruent and similar.

Answers

1. These figures are similar. All of the corresponding sides have the same measure, so the ratios are all equal to 1.
2. These figures are similar. All of the sides of the smaller heptagon are half the length of the corresponding sides of the larger heptagon.
3. These figures are similar. Each side of the larger rectangle is about 1.5 times the length of the corresponding side of the smaller rectangle.
4. These figures are similar. Each side of the smaller rectangle is about $\frac{2}{3}$ or 0.67 times the length of the corresponding side of the larger rectangle.
5. These figures are similar. Each side of the larger convex pentagon is twice the length of the corresponding side of the smaller convex pentagon.
6. The figures are not similar. The ratio of the corresponding side lengths is not constant.

Getting Started

Same Figure or Same Shape?

When two figures are similar, the same scale factor can be applied to all side lengths to map one figure to the other.

We often say that dilations preserve shape and that rigid motions preserve both size and shape. As a result, it is common to state that similar figures have the same shape, and congruent figures have the same size and shape. However, what does it mean for two figures to have the same shape in this context? Are all rectangles similar? Are all triangles similar?

Use the definition of similar figures to determine which figures are similar.

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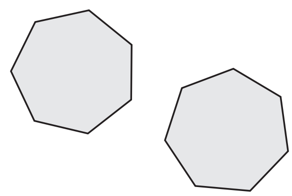

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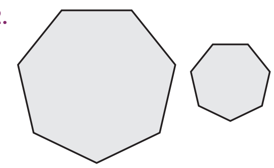

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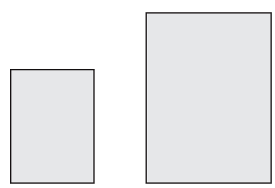

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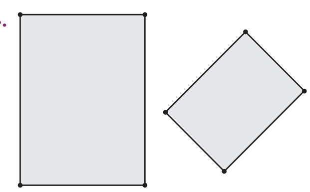

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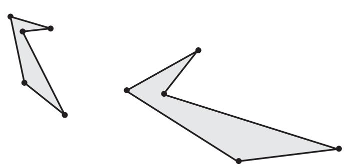

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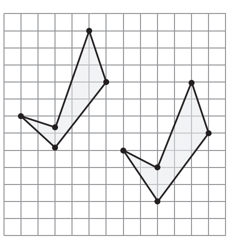


ACTIVITY 3.1

Proving Similarity Through Dilations

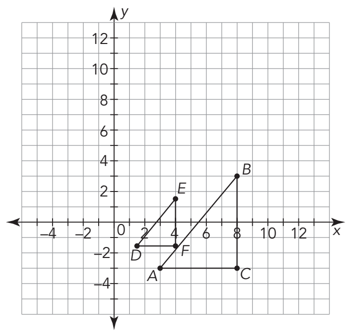
NOTES

In this activity, you will use what you know about dilations to determine if figures are similar.

1. Determine if the figures are similar. If they are similar, state the scale factor and the center of dilation that maps Figure 1 onto Figure 2.

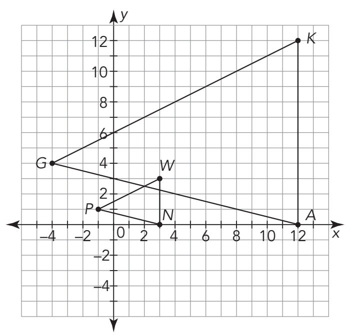
a. Figure 1: $\triangle ABC$

Figure 2: $\triangle DEF$



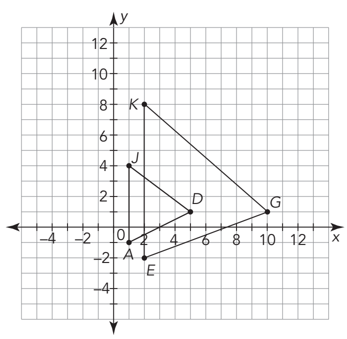
b. Figure 1: $\triangle PWN$

Figure 2: $\triangle GKA$



c. Figure 1: $\triangle JDA$

Figure 2: $\triangle KGE$



Answers

1a. The triangles are similar. $\triangle DEF$ is created from $\triangle ABC$ by a dilation with a scale factor of $\frac{1}{2}$ and a center of dilation at the origin.

1b. The triangles are similar. $\triangle GKA$ is created from $\triangle PWN$ by a dilation with a scale factor of 4 and a center of dilation at the origin.

1c. These triangles are not similar. Sample explanation. When I draw a line through each of the corresponding vertices they do not intersect in a single point; therefore there is no center of dilation.

Answers

1d. These triangles are similar. $\triangle FRB$ is created from $\triangle ZEN$ by a dilation with a scale factor of $\frac{2}{3}$ and a center of dilation at the origin.

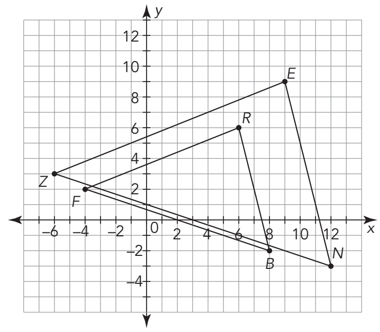
Answers

1a. Sample answer. Using the origin as the center of rotation and center of dilation, I can rotate $\triangle QRN$ 180° and then dilate by a scale factor of 2.

1b. The triangles are similar but they are not congruent.

d. Figure 1: $\triangle ZEN$

Figure 2: $\triangle FRB$

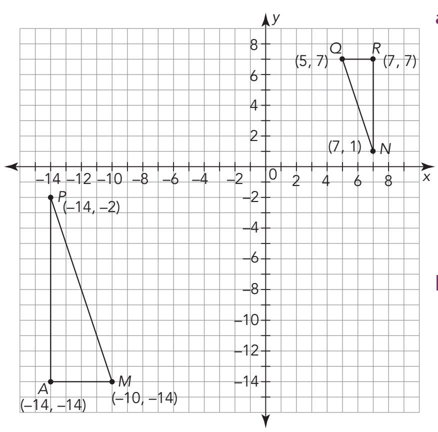


ACTIVITY 3.2

Proving Similarity Through Transformations

Sometimes similar figures cannot be mapped from one to another using only a dilation. You may need a combination of translations, reflections, rotations, and dilations to map a figure onto a similar figure.

1. Triangle *MAP* is the image of Triangle *QRN* after undergoing at least one transformation.

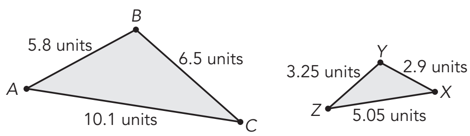


a. Determine a possible sequence of transformations to map $\triangle QRN$ onto $\triangle MAP$.

b. Are the triangles congruent? Are they similar? Explain your reasoning.

c. **Reverse the order of the sequence of transformations you described in part (b). What do you notice?**

2. **Triangle *XYZ* is the image of Triangle *ABC* after undergoing at least one transformation.**



a. **List the corresponding sides and angles for $\triangle ABC$ and $\triangle XYZ$.**

b. **Determine a possible sequence of transformations to map $\triangle ABC$ onto $\triangle XYZ$.**

c. **Reverse the order of the sequence of transformations you described in part (b). What do you notice?**

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Answers

1c. Sample answer. When I first dilate by a scale factor of 2 and then rotate 180°, $\triangle QRN$ still maps onto $\triangle MAP$. The order of the transformations does not matter.

2a. $\angle A$ corresponds to $\angle X$, $\angle B$ corresponds to $\angle Y$, and $\angle C$ corresponds to $\angle Z$. $\overline{AB}$ corresponds to $\overline{XY}$, $\overline{BC}$ corresponds to $\overline{YZ}$, and $\overline{CA}$ corresponds to $\overline{ZX}$.

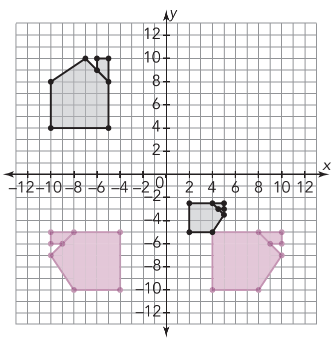
2b. Sample answer. Reflect $\triangle ABC$ across a line halfway between the triangles and then dilate by a factor of $\frac{1}{2}$ to create $\triangle XYZ$.

2c. Sample answer. When I first dilate by a scale factor of $\frac{1}{2}$ and then reflect, $\triangle ABC$ still maps onto $\triangle XYZ$.

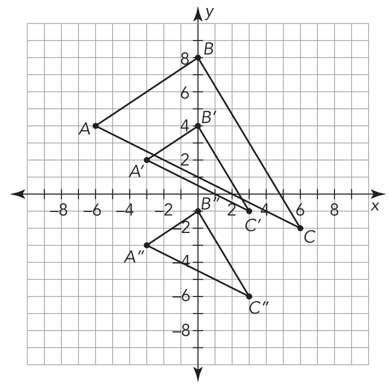
Answers

3. Using the origin as the center of dilation, I can dilate $\triangle ABC$ by a scale factor of $\frac{1}{2}$, and then I can translate triangle $A'B'C'$ down 5 units to map onto $\triangle A''B''C''$.

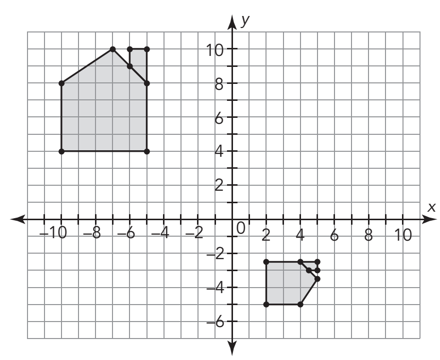
4. Sample answer.
   One sequence would be to rotate the image 90° counterclockwise (center of rotation is the origin), reflect the image across the y-axis, and finally dilate by a scale factor of $\frac{1}{2}$ with the origin as the center of dilation.



3. **Triangle *ABC* was dilated to create Triangle *A′B′C′*. Then Triangle *A′B′C′* was dilated to create Triangle *A″B″C″*. Describe a series of transformations that map $\triangle ABC$ onto $\triangle A''B''C''$.**



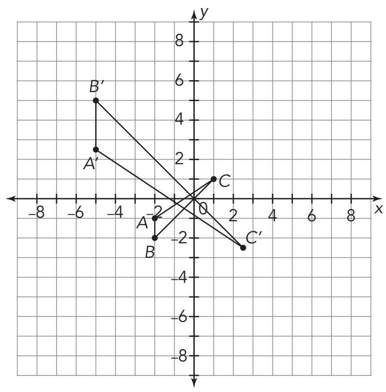
4. **Verify that the two houses are similar by describing a sequence of transformations that maps one figure onto the other.**



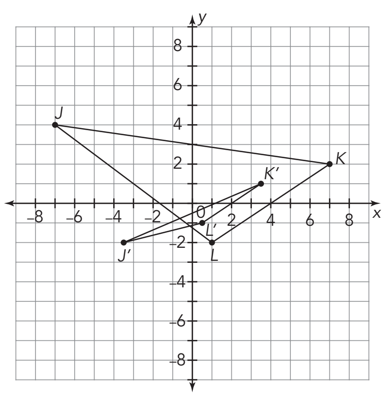
6 • TOPIC 2: Similarity

5. **Use dilations and other transformations to determine if the triangles represented by the coordinates are similar. Show your work and explain your reasoning.**

   a. ***A*** **(−2, −1)** ***B*** **(−2, −2)** ***C*** **(1, 1)**
   A′ **(−5, 2.5)** ***B′*** **(−5, 5)** ***C′*** **(2.5, −2.5)**

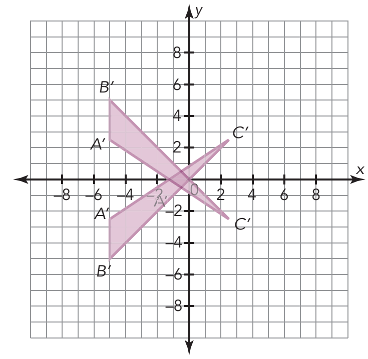


   b. ***J*** **(−7, 4)** ***K*** **(7, 2)** ***L*** **(1, −2)**
   J′ **(−3.5, −2)** ***K′*** **(3.5, 1)** ***L′*** **(0.5, −1)**



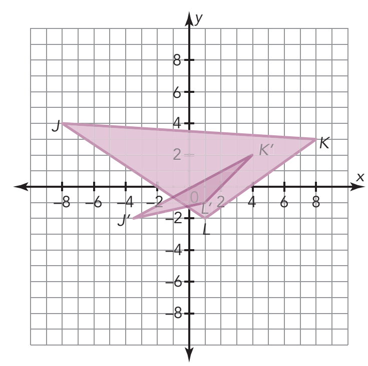
Answers

5a. Sample answers.



Yes. The triangles are similar. I can dilate △*ABC* using the origin as the center of dilation and a scale factor of 2.5 to get new coordinates *A′* (−5, −2.5), *B′* (−5, −5), *C′* (2.5, 2.5).
Then I can reflect that dilation across the *x*-axis to get coordinates *A′* (−5, 2.5), *B′* (−5, 5), *C′* (2.5, −2.5).

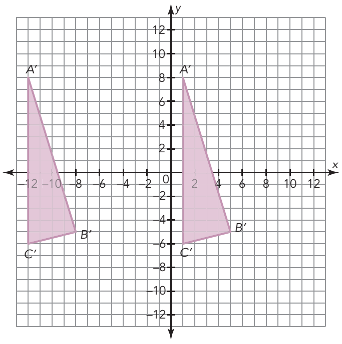
5b.



No. The triangles are not similar. Points *K* and *L* are dilated by a scale factor of $\frac{1}{2}$ from the origin, but point *J* is not. Point *J′* should be at (−3.5, 2) for the two triangles to be similar.

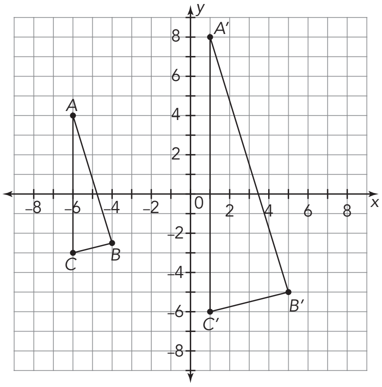
Answers

5c.



Yes. The triangles are similar. I can dilate △*ABC* using the origin as the center of dilation and a scale factor of 2 to get new coordinates *A′* (−12, 8), *B′* (−8, −5), *C′* (−12, −6). Then I can translate the triangle 13 units to the right to get coordinates *A′* (1, 8), *B′* (5, −5), *C′* (1, −6).

c. ***A*** **(−6, 4)** ***B*** **(−4, −2.5)** ***C*** **(−6, −3)**
A′ **(1, 8)** ***B′*** **(5, −5)** ***C′*** **(1, −6)**



ACTIVITY 3.3

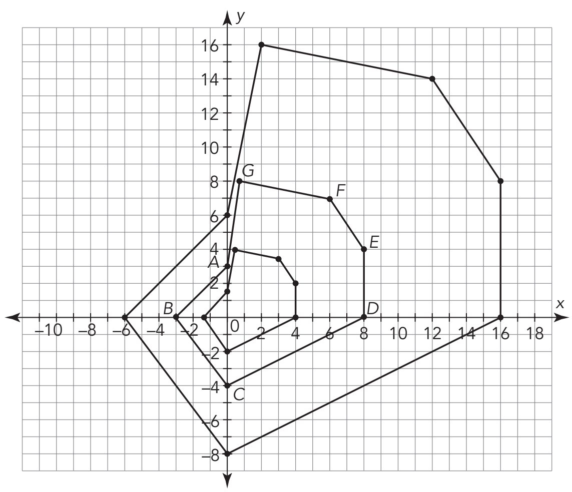
Comparing Images

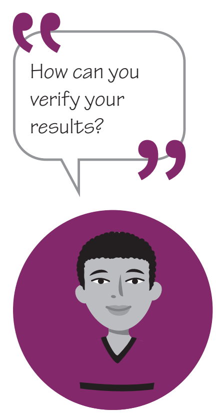
You know that similar figures can be mapped from one to another using a sequence of transformations. How are the images of the same pre-image related to each other?

Let's investigate!

1. The labeled figure is the pre-image used to create the other two figures using dilations.

a. Determine the scale factor to map the pre-image to each of the other figures. Explain your reasoning.





b. Are the images similar? Are they congruent? Explain your reasoning.

Answers

1a. The scale factors are $\frac{1}{2}$ and 2.

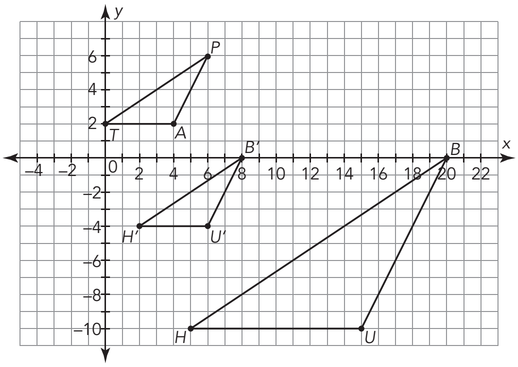
1b. Reasoning will vary. The images are similar but they are not congruent.

Answers

2a. $\triangle HUB \sim \triangle TAP$

2b. Sample answer. After dilating $\triangle HUB$ by a scale factor of $\frac{2}{5}$, I need to translate $\triangle H'U'B'$ up 6 units and left 2 units.

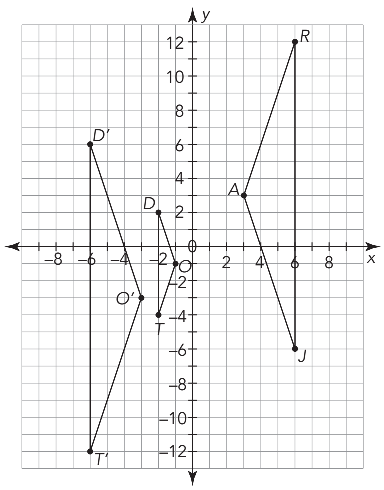
2. Triangle *HUB* was dilated from the origin by a scale factor of $\frac{2}{5}$ to create $\triangle H'U'B'$, and $\triangle H'U'B' \cong \triangle TAP$.



a. What is the relationship between $\triangle HUB$ and $\triangle TAP$? Justify your answer.

b. Determine a possible sequence of transformations that maps $\triangle HUB$ onto $\triangle TAP$.

3. **Triangle *DOT* was dilated from the origin by a scale factor of 3 to create △*D′O′T′*, and △*D′O′T′* ≅ △*JAR*. Determine a possible sequence of transformations that maps △*JAR* onto △*DOT*.**



Answers

3. Sample answer. Rotate △*JAR* 180° around the origin, and dilate △*J′A′R′* by a scale factor of $\frac{1}{3}$ with the origin as the center of dilation.

Answers

1. This statement is never true. If the scale factors are all different, the corresponding side lengths will be different, violating a condition of congruence. The triangles will all be similar to each other.
2. The final image will always be congruent to $\triangle HIP$. Dilating by 8 and dilating by $0.125 = \frac{1}{8}$ are inverse dilations, so the final image's sides will be the same length as the pre-image's sides.
3. This statement is sometimes true. If the scale factor is 1, a dilation will create congruent figures.
4. This statement is always true. Similar figures are created by a sequence of transformations.

NOTES

TALK the TALK

Summing Up Similar Figures

Determine if each statement is *always*, *sometimes*, or *never* true. Provide a justification for each answer.

1. **Triangle *ABC* is dilated four times with different scale factors. The four images are congruent.**

2. **Triangle *HIP* is dilated by a scale factor of 8, followed by a scale factor of 0.125. The final image is congruent to $\triangle HIP$.**

3. **Dilations are used to create congruent figures.**

4. **Transformations are used to create similar figures.**

ELL Tip

When completing the Talk the Talk activity, English Language Learners may not understand what a *justification* is. Explain to students that a *justification* is a reason or explanation why you chose the answer that you did. Observe students as they work, to make sure they are giving a *justification* for their answers.