The Vanishing Point

3

The Angle-Angle Similarity Theorem

MATERIALS

Ruler
Protractor

Lesson Overview

The Angle-Angle Similarity Theorem can be used to show that two triangles are similar. From previous lessons, students should already recognize that two similar triangles have congruent corresponding angles and proportional corresponding sides. The Angle-Angle Similarity Theorem allows students to show that two triangles are similar without comparing the measures of the six parts of each triangle.

Grade 8
Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to:

(D) use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

ELPS

1.A; 1.C; 1.E; 1.F; 1.G; 2.C; 2.E; 2.I; 3.D; 3.E; 4.B; 4.C; 5.B; 5.F; 5.G

Essential Ideas

- The Angle-Angle (AA) Similarity Theorem states that if two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.
- The Angle-Angle Similarity Theorem can be applied to identify similar triangles in problem situations.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Vanishing Point

Students are given a drawing created using a vanishing point, which is a point where all the parallel lines in the drawing should appear to meet. A vanishing point is sometimes used by graphic artists to create perspective in their art. This also leads to similar figures being created—the vanishing point is like a center of dilation. Students demonstrate that two triangles in the drawing are similar using dilations or line and angle relationships. They then create another similar triangle and tree using the appropriate scale factor.

Develop

Activity 3.1: Exploring the Angle-Angle Similarity Theorem

Students draw a triangle, measure two angles, and draw a second triangle using the same measurements. They use a ruler to determine the length of the sides of both triangles and a protractor to measure the angles of the second triangle. Upon completion of these measurements, students conclude that the two triangles are similar. The *Angle-Angle (AA) Similarity Theorem* is formally stated.

Activity 3.2: Using the Angle-Angle Similarity Theorem

Students determine how triangles are similar by the Angle-Angle Similarity Theorem.

Activity 3.3: Reasoning with the Angle-Angle Similarity Theorem

Students use the Angle-Angle Similarity Theorem to determine if a diagram containing parallel and perpendicular lines form similar triangles.

Demonstrate

Talk the Talk: Bow-Tie Triangles

Students investigate whether bow-tie triangles—triangles with two intersecting segments and a pair of parallel lines—are always similar.

Getting Started: Vanishing Point

ENGAGE

Facilitation Notes

In this activity, students show that two triangles in a drawing are similar using dilations or line and angle relationships.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Are the two triangles formed the same shape? How do you know?
- Are the two triangles formed the same size? How do you know?
- What is true about all similar triangles?
- What is the relationship between corresponding angles of two similar triangles?
- What is the relationship between corresponding sides of two similar triangles?
- Could the vanishing point, *A*, be used as the center of dilation?

Summary

The characteristics of similar triangles can be used to solve problem situations.

Activity 3.1
Exploring the Angle-Angle Similarity Theorem

DEVELOP

Facilitation Notes

In this activity, students informally prove the Angle-Angle Similarity Theorem.

Ask a student to read the introduction aloud. Discuss as a class. Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- Is the triangle you drew an acute, obtuse, or right triangle?
- Is the triangle you drew an isosceles, scalene, or equilateral triangle?
- How does your triangle compare to your classmates' triangles?

- Do the corresponding angles in $\triangle ABC$ appear to be congruent to the corresponding angles in $\triangle DEF$?
- If the three pairs of corresponding sides are proportional and the three pairs of corresponding angles are congruent, are the triangles similar?

Ask a student to read the information and theorem following Question 1. Discuss as a class.

Questions to ask

- Why don't you need to check the measure of the third angle each time?
- Why don't you need to check that corresponding sides are proportional each time?

Summary

The Angle-Angle (AA) Similarity Theorem states that if two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.

Activity 3.2

Using the Angle-Angle Similarity Theorem

Facilitation Notes

In this activity, students identify similar triangles using the Angle-Angle Similarity Theorem.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

- To scaffold support,
  - Draw the two triangles being compared separately so students can visualize what they are comparing. Students may even want to orient them the same way.
  - Use the same color to mark congruent angles to assist in writing similarity statements.
  - Provide one half of the similarity statement, and have students complete the corresponding portion.
- To extend the activity,
  - Have students write two different similarity statements for Question 4.

Questions to ask

- Which triangle contains Angle *J*?
- Is Angle *J* shared by both triangles?
- Is Angle *J* congruent to itself?
- What is the relationship between Angle *C* and Angle *HGJ*? Are they congruent? Why?
- What kind of angles are Angle *W* and Angle *Y*?
- What do you know about all right angles?
- Are all right angles congruent?
- Do the intersecting lines form a pair of vertical angles?
- Is there a pair of vertical angles in this diagram? Where?
- What do you know about the measure of vertical angles?
- Are vertical angles always congruent?
- Do these triangles share a common angle?
- Which angle is common to both triangles?

Summary

Triangles can be proven similar using the Angle-Angle Similarity Theorem.

Activity 3.3

Reasoning with the Angle-Angle Similarity Theorem

Facilitation Notes

In this activity, students identify similar triangles using the relationships of special angle pairs formed by a transversal intersecting parallel lines and the Angle-Angle Similarity Theorem.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

Differentiation strategy

To scaffold support, provide five copies of a template for guidance, one for each problem. The template should include:

- A copy of the diagram so students may color code it appropriately,
- Space to list the three congruent angle pairs,
- Space to list reasoning for two of the three angle pairs,
- If necessary, a list of possible reasons (corresponding angles, alternate interior angles, common angles, vertical angles) for students to choose from.

Questions to ask

- How many triangles are in the diagram?
- Do the two triangles share a common angle? Which angle?
- What special angle pair associated with parallel lines is $\angle ABC$ and $\angle AHG$?
- What special angle pair associated with parallel lines is $\angle ACB$ and $\angle AGH$?
- Which angles are vertical angles in this situation?
- Are all vertical angles congruent?
- Can you use alternate interior angles in this situation?
- Can you use corresponding angles in this situation?

Summary

Triangles can be proven similar using the Angle-Angle Similarity Theorem.

DEMONSTRATE

Talk the Talk: Bow-Tie Triangles

Facilitation Notes

In this activity, students determine that bow-tie triangles—triangles drawn sharing a pair of vertical angles and having parallel bases—are always similar.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Differentiation strategy

To scaffold support, suggest that students extend the sides with dotted lines so the diagram looks more familiar. Explain that this is a strategy that can always be used to recognize parallel lines and transversals.

Questions to ask

- Do bow-tie triangles always contain a pair of vertical angles?
- Do these triangles share a common angle?
- Do these triangles share a common side?
- What is the relationship between vertical angles?
- Are corresponding angles helpful in this situation?
- Which lines are transversals?
- Are alternate interior angles helpful in this situation?
- Are alternate exterior angles helpful in this situation?

Summary

Triangles can be proven similar using the Angle-Angle Similarity Theorem.

The Vanishing Point

3

The Angle-Angle Similarity Theorem

WARM UP

Suppose $\triangle BHX$ is similar to $\triangle KRC$.

1. List the corresponding angles.

2. Write the ratios to identify the proportional side lengths.

LEARNING GOALS

- Develop the minimum criteria to show that two triangles are similar.
- Use informal arguments to establish facts about the angle-angle criterion for similarity of triangles.
- Use the Angle-Angle Similarity Theorem to identify similar triangles.

KEY TERM

- Angle-Angle (AA) Similarity Theorem

You have determined that when two triangles are similar, the corresponding angles are congruent and the corresponding sides are proportional. How can you show that two triangles are similar without measuring all of the angles and side lengths?

Warm Up Answers

1. $\angle B$ and $\angle K$,
   $\angle H$ and $\angle R$,
   $\angle X$ and $\angle C$
2. $\frac{BH}{KR} = \frac{HX}{RC} = \frac{BX}{KC}$

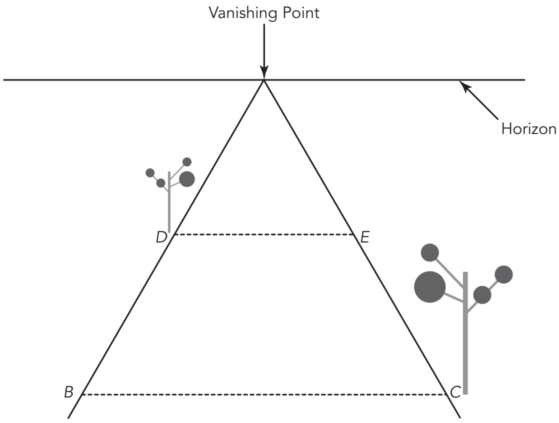
Answers

1. The vanishing point, *A*, could be used as the center of dilation. Then, demonstrate that the corresponding angles are congruent ($\angle E$ is congruent to $\angle C$ and $\angle D$ is congruent to $\angle B$). The parallel lines and transversals can also be used to demonstrate that the corresponding angles are congruent.
2. Check students' drawings.

Getting Started

Vanishing Point

Graphic artists use knowledge about similarity to create realistic perspective drawings. Choose where the horizon should be and a vanishing point—a point where all the parallel lines in the drawing should appear to meet—and you too can create a perspective drawing.



The symbol ~ means "is similar to."

1. Suppose the vanishing point is point *A* and that $\overline{DE} \parallel \overline{BC}$. How could you demonstrate that $\triangle ABC \sim \triangle ADE$?

2. Draw a horizontal line in the path to create another similar triangle. Then sketch a tree at that line using the appropriate scale factor.

2 • TOPIC 3: Line and Angle Relationships

ELL Tip

English Language Learners may not understand what a graphic artist is. Ask students if they know what a graphic artist is and ask if any of the students have thought about studying graphic arts later on in life. After a short classroom discussion, show students examples of things that a graphic artist would create.

ACTIVITY 3.1

Exploring the Angle-Angle Similarity Theorem

NOTES

You have determined that when two triangles are similar, the corresponding angles are congruent and the corresponding sides are proportional. To show that two triangles are similar, do you need to show that all of the corresponding sides are proportional and all of the corresponding angles are congruent?

Let's explore an efficient method to determine if two triangles are similar.

1. If the measures of two angles of a triangle are known, is that enough information to draw a similar triangle? Let's explore this possibility.

a. Use a straightedge to draw $\triangle ABC$ in the space provided.

b. Use a protractor to measure $\angle A$ and $\angle B$ of $\triangle ABC$ and record the measurements.

m$\angle A$ = ______ m$\angle B$ = ______

c. Use the Triangle Sum Theorem to determine m$\angle C$.

Answers

1a. Check students' drawings.

1b. Answers will vary.

1c. A protractor is not needed to determine the measure of Angle *C*, because it is the third angle of a triangle and must be equal to 180° minus the sum of the measures of Angles *A* and *B*.

Answers

1d. Check students' drawings.

1e. The measure of the third pair of corresponding angles is needed, and we need to know if the corresponding sides are proportional to determine if the two triangles are similar.

1f. Yes. The third pair of corresponding angles are congruent, and the corresponding sides are proportional, so I can conclude that the triangles are similar.

NOTES

d. Draw a second triangle, △*DEF*, in the space provided using the angle measurements from part (b).

e. Based on your knowledge, what other information is needed to determine if the two triangles are similar, and how can you acquire that information?

f. Determine the measurements to get the additional information needed and decide if the two triangles are similar.

You have just shown that given the measures of two pairs of congruent corresponding angles of two triangles, it is possible to determine that two triangles are similar. In the study of geometry, this is expressed as a theorem.

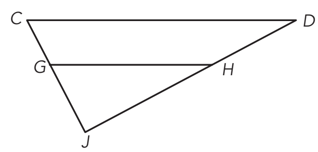
The **Angle-Angle (AA) Similarity Theorem** states that if two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.

ACTIVITY 3.2

Using the Angle-Angle Similarity Theorem

Identify the triangles that are similar by the AA Similarity Theorem. Explain how you know that the triangles are similar.

1. $\overline{CD} \parallel \overline{GH}$



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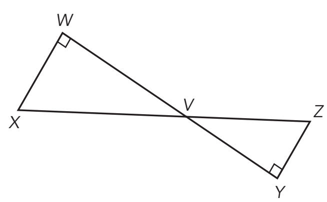

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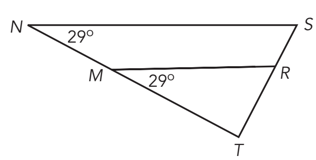

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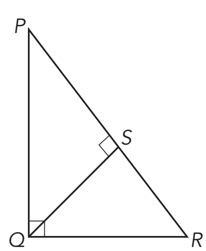


Answers

1. Angles *JHG* and *JDC* are corresponding angles, so they are congruent, and Angle *J* is the same in both triangles. So, $\triangle JHG \sim \triangle JDC$.
2. Angles *ZVY* and *XVW* are vertical angles, so they are congruent. Angle *Y* is congruent to Angle *W* because they are both right angles. So, $\triangle VYZ \sim \triangle VWX$.
3. Angles *TMR* and *TNS* are corresponding angles, so they are congruent, and Angle *T* is the same in both triangles. So, $\triangle TMR \sim \triangle TNS$.
4. Angles *PQR* and *PSQ* are congruent because they are both right angles. Angle *P* is the same in both triangles. So, $\triangle PQR \sim \triangle PSQ$.

   Angles *PQR* and *QSR* are congruent because they are both right angles. Angle *R* is the same in both triangles. So, $\triangle PQR \sim \triangle QSR$.

Answers

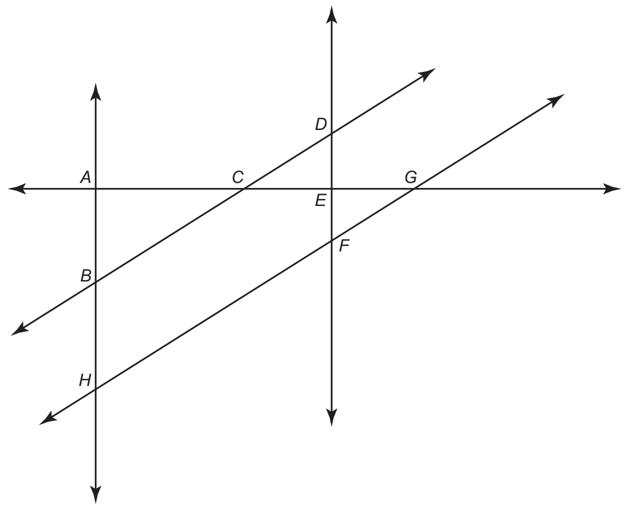
1. Yes, the triangles are similar. Angle *ABC* is congruent to Angle *AHG* and Angle *ACB* is congruent to Angle *AGH* because they are pairs of corresponding angles formed by parallel lines.

ACTIVITY 3.3

Reasoning with the Angle-Angle Similarity Theorem

Use what you have learned about triangle similarity to answer each question.

Given: $\overleftrightarrow{BD} \parallel \overleftrightarrow{HG}$, $\overleftrightarrow{AH} \parallel \overleftrightarrow{DF}$, $\overleftrightarrow{AH} \perp \overleftrightarrow{AG}$, $\overleftrightarrow{DF} \perp \overleftrightarrow{AG}$



1. Is $\triangle ABC \sim \triangle AHG$? Explain your reasoning.

2. Is $\triangle ABC \sim \triangle EDC$? Explain your reasoning.

3. Is $\triangle EDC \sim \triangle EFG$? Explain your reasoning.

4. Is $\triangle ABC \sim \triangle EFG$? Explain your reasoning.

5. Is $\triangle AHG \sim \triangle EFG$? Explain your reasoning.

Answers

2. Yes, the triangles are similar. Angle *CAB* is congruent to Angle *CED* and Angle *CBA* is congruent to Angle *CDE* because they are pairs of alternate interior angles formed by parallel lines.
3. Yes, the triangles are similar. Angle *EDC* is congruent to Angle *EFG* and Angle *DCE* is congruent to Angle *FGE* because they are pairs of alternate interior angles formed by parallel lines.
4. Yes, the triangles are similar. Angle *ACB* is congruent to Angle *EGF* and Angle *CAB* is congruent to Angle *GEF* because they are pairs of corresponding angles formed by parallel lines.
5. Yes, the triangles are similar. Angle *AHG* is congruent to Angle *EFG* and Angle *HAG* is congruent to Angle *FEG* because they are pairs of corresponding angles formed by parallel lines.

Answers

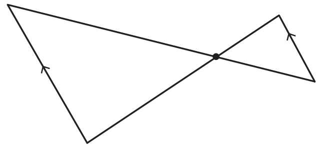
1. Bow-tie triangles with parallel bases are always similar. Students may use measurements, dilations, line and angle relationships, and/or the Angle-Angle Similarity Theorem to justify this conclusion.

NOTES

TALK the TALK

Bow-Tie Triangles

You can draw special triangles known as bow-tie triangles. First, draw a pair of parallel line segements. Then, connect the pairs of endpoints with line segments so that the line segments intersect, like this:



1. **Are bow-tie triangles always similar? Show your work and explain your reasoning. Then, compare your work with your classmates' work.**