TEXAS MATH SOLUTION

How to support your student as they learn about

Transforming Geometric Objects

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when seeing new concepts in this module.

Module Introduction

In this module your student will develop their understanding of congruence and similarity. There are 3 topics in this module: *Rigid Motion Transformations, Similarity*, and *Line and Angle Relationships*. Your student will use what they already know about geometric objects in this module.

Academic Glossary

Each module will highlight an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Analyze
Definition	• To study or look closely for patterns. • To break a concept down into smaller parts to gain a better understanding of it.
Questions to Ask Your Student	• Do you see any patterns? • Have you seen something like this before? • What happens if the shape, model, or numbers change?
Related Phrases	• Examine • Evaluate • Determine • Observe • Consider • Investigate • What do you notice?

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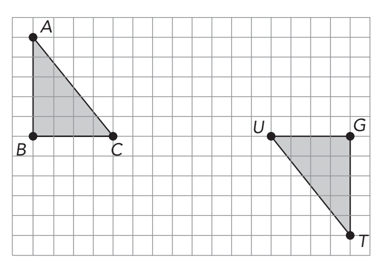
Example: Topic 1 Lesson 6

Analyze the two congruent triangles. Can you determine a way to map one triangle onto the other in a single transformation?



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Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for different ways to solve problems.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way

Our Instructional Approach

Carnegie Learning's instructional approach is based on how people learn and real-world understandings. It is based on three key components:

ENGAGE	DEVELOP	DEMONSTRATE
Purpose: Provide an introduction that creates curiosity and uses what students already know and have experienced. **Questions to Ask:** How does this problem look like something you did in class?	**Purpose:** Build a deep understanding of mathematics through different activities. **Questions to Ask:** Do you know another way to solve this problem? Does your answer make sense?	**Purpose:** Reflect on and evaluate what was learned. **Questions to Ask:** Is there anything you do not understand?

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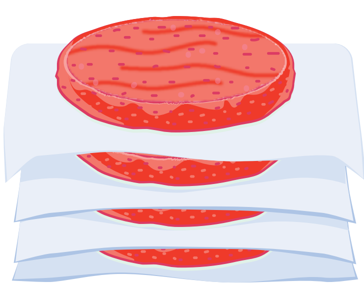
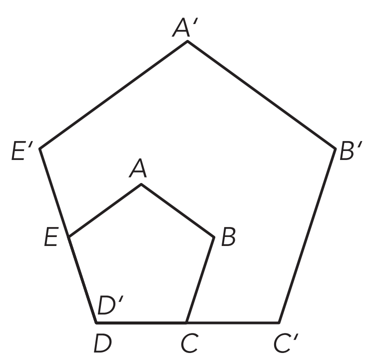
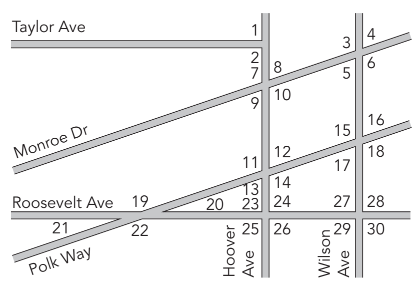
Module Overview

TOPIC 1	TOPIC 2	TOPIC 3
Rigid Motion Transformations	**Similarity**	**Line and Angle Relationships**
20 Days	10 Days	10 Days
Your student will use patty paper and the coordinate plane to study the creation of congruent figures with translations, reflections, and rotations.	**Your student will study dilations and similarity.**	**Your student will use their knowledge of transformations, congruence, and similarity to understand the Triangle Sum Theorem, the Exterior Angle Theorem, relationships between angles formed when a transversal cuts parallel lines, and the Angle-Angle Similarity Theorem.**
Did you know that?  Patty paper separates patties of meat! Little did the inventors know that it could also serve as a powerful geometric tool. You can write on it, trace with it, and see creases when you fold it.	Did you know that?  A **dilation** is a transformation that produces a figure that is the same shape as the original figure, but not necessarily the same size.	What in the world?  Many city streets are parallel to each other. When another street or multiple streets cross through the parallel roads, special angle relationships are formed. We see these angles at the intersection of the streets. Visualize a 3- or 4-way stop.

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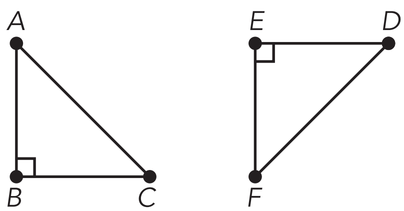
Topic 1: Rigid Motion Transformations

Key Terms

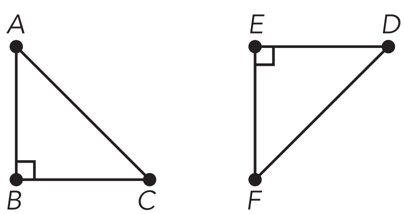
- congruent figures
- corresponding sides
- corresponding angles
- plane
- transformation
- rigid motion
- pre-image
- image
- translation
- reflection
- line of reflection
- rotation
- center of rotation
- angle of rotation
- congruent line segments
- congruent angles

Corresponding angles are angles that have the same relative positions in geometric figures.



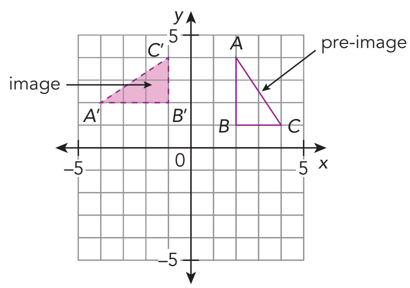
Angle *B* and Angle *E* are corresponding angles.

Corresponding sides are sides that have the same relative positions in geometric figures.

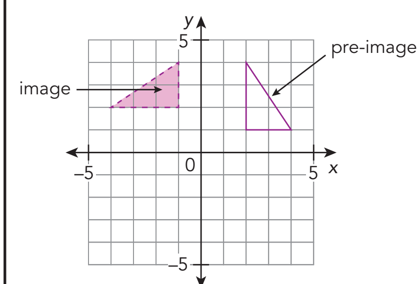


Sides *AB* and *DE* are corresponding sides.

The new figure created from a **transformation** is the **image**.



The **center of rotation** is the point around which you rotate a figure. The center of rotation can be a point on the figure, inside the figure, or outside the figure.



The image is a rotation of the **pre-image** 90° counterclockwise about the center of rotation, which is the origin (0, 0).

Follow the link to access the Mathematics Glossary:
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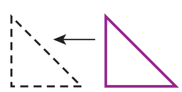
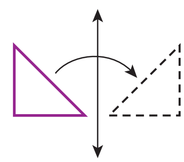
In this topic, students will use everyday language like *slide*, *flip*, and *turn* to describe how to map, or move, one figure onto another. They then use the mathematical vocabulary of **rigid motion** transformations—translations, reflections, and rotations—and describe how a single rigid motion makes the same change between **congruent figures**. Students also learn that rigid motions preserve, or keep, the size and shape of a figure but that reflections change the orientation, or position/direction, of a figure's vertices.

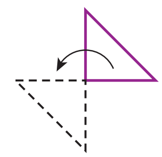
Transformations



translation



reflection



rotation

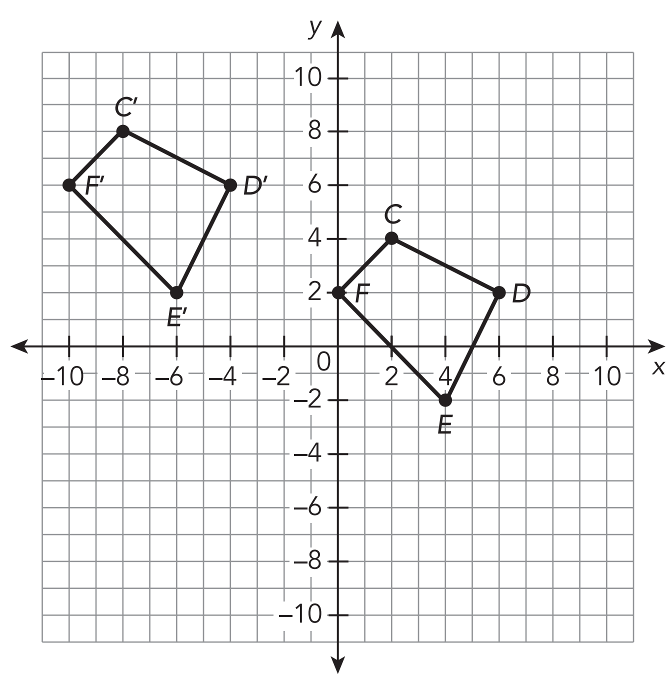
A **translation** is a rigid motion transformation that slides each point of a figure the same distance and direction along a line.

A **reflection** is a rigid motion transformation that flips a figure across a **line of reflection**.

A **rotation** is a rigid motion transformation that turns a figure on a **plane** about a fixed point.

Verifying Congruence Using Translations

A translation "slides" a geometric figure in some direction. Translations can be used to verify, or check, that two figures are congruent. For example, Quadrilateral *CDEF* can be translated up 4 units and left 10 units. This will show that it is congruent to Quadrilateral $C'D'E'F'$.



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Jigsaw Transformations

Because rigid motions maintain the size and shape of an image, you can use a sequence, or a patterned order, of translations, reflections, and rotations to show that two figures are congruent.

MATH PROCESS STANDARDS

How do the activities in *Rigid Motion Transformations* promote student expertise in the math process standards?

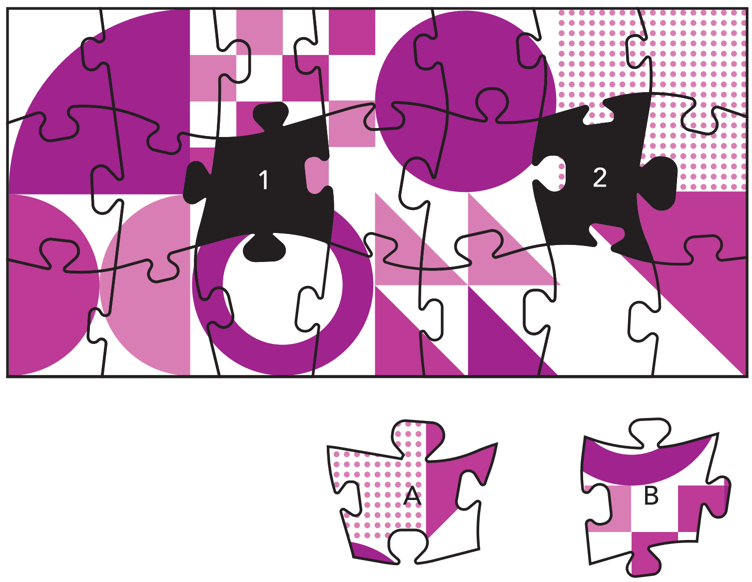
NOTE: This is an example of the math process standard:

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

- I can communicate and defend my own mathematical understanding.

Refer to page 2 for more "I can" statements.

There are just two pieces left to complete the jigsaw puzzle.



Which puzzle piece fills the missing spot at 1?
Which puzzle piece fills the missing spot at 2?

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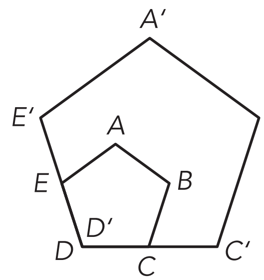
Topic 2: Similarity

Key Terms

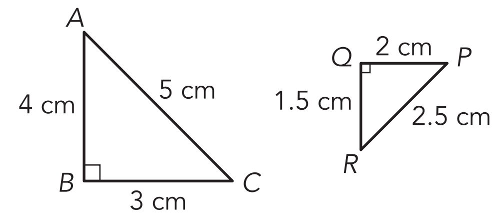
- dilation
- center of dilation
- scale factor
- enlargement
- reduction
- similar

In a **dilation**, the **scale factor** is the ratio of the distance of the new figure from the **center of dilation** to the distance of the original figure from the center of dilation.



ABCDE has been dilated by a scale factor of 2 to create Pentagon *A′B′C′D′E′*.

When two figures are **similar**, the ratios of their corresponding side lengths are equal.



Triangle *ABC* is similar to Triangle *PQR*.

$\overline{AB} : \overline{PQ}$	$\overline{BC} : \overline{QR}$	$\overline{AC} : \overline{RP}$
4cm : 2cm	3cm : 1.5cm	5cm : 2.5cm
2cm : 1cm	2cm : 1cm	2cm : 1cm

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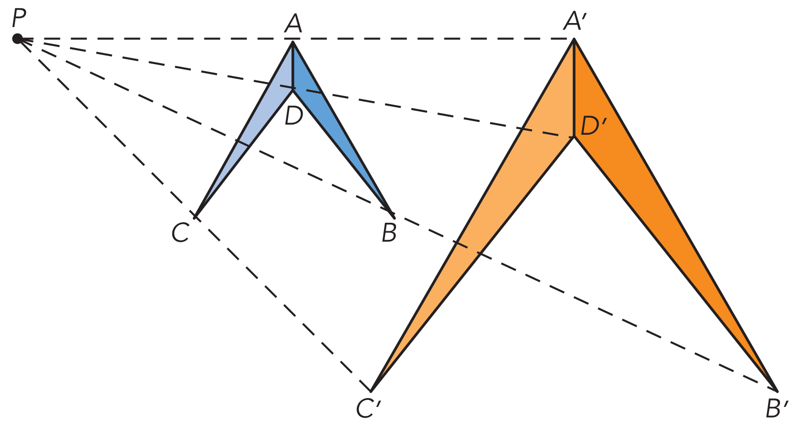
Dilating Figures with a Scale Factor Greater Than 1

In this topic, students will study dilations and similar figures. They will relate dilations to scale factors and scaling up and down.

This image shows the **enlargement** of a logo using point *P* as the center of dilation.

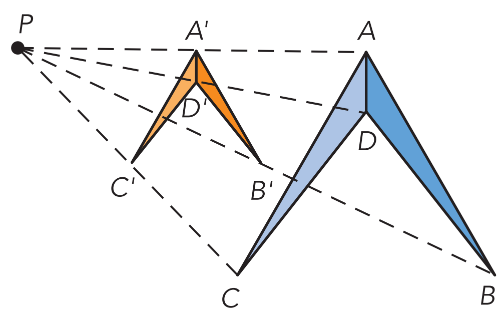


You can express the scale factor as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

When the scale factor is greater than 1, the new figure is an enlargement.

Dilating Figures with a Scale Factor Less Than 1 and Greater Than 0

The image shows a **reduction** of the original logo using point *P* as the center of dilation.



You can express the scale factor as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

When the scale factor is less than 1 and greater than 0, the new figure is a reduction.

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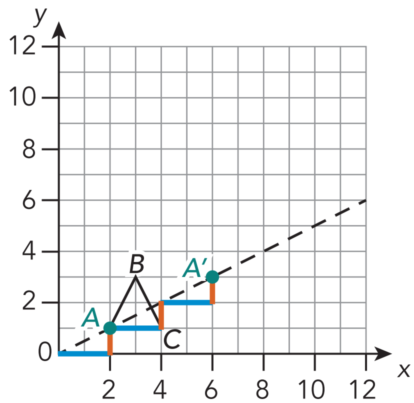
Scaling Up on the Coordinate Plane

Students will know from Topic 1 that a translation moves a point along a line. A sequence of repeated horizontal and/or vertical translations also moves a point along a line. You can use this fact to dilate figures.

For example, to dilate Δ*ABC* by a scale factor of 3 using the origin (0, 0) as the center of dilation, start by dilating Point *A*, located at (2, 1).



Point *A* is 2 units right and 1 unit up from the origin.

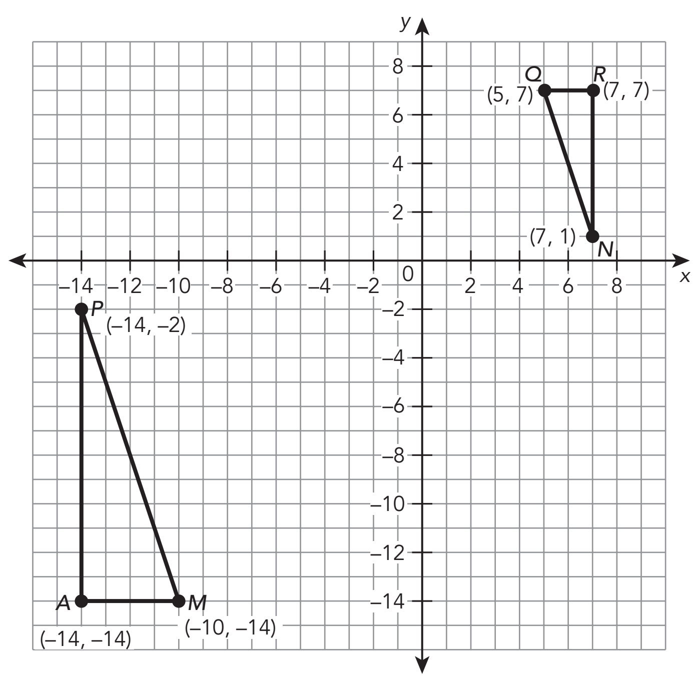
To dilate Point *A* by a scale factor of 3, translate Point *A* by three repeated sequences : **2 units right** and **1 unit up**.

The dashed line helps you see that Point *A′* is a dilation of Point *A* by a factor of 3.

Proving Similarity Through Transformations

Finally, students will use dilations to map from a figure to a similar figure, eventually identifying a sequence of transformations that map from a figure to a similar figure.

For example, Δ*MAP* is similar to Δ*QRN*. The ratio of corresponding sides is equal to 2, or $\frac{1}{2}$. A possible sequence of transformations to map Δ*QRN* onto Δ*MAP* is a rotation of 180° about the origin and a dilation by a scale factor of 2. Images created from the same pre-image are always similar figures.



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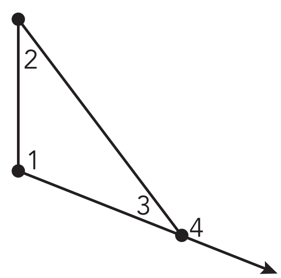
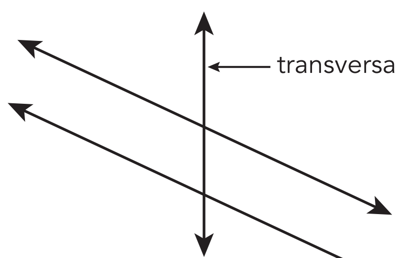
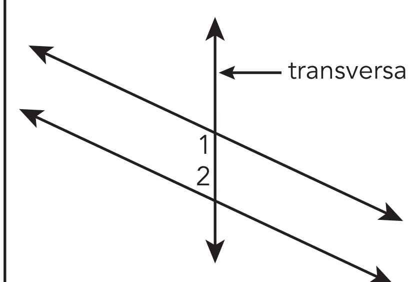
Topic 3: Line and Angle Relationships

Key Terms		
• Triangle Sum Theorem • exterior angle of a polygon • remote interior angles of a triangle	• Exterior Angle Theorem • transversal • alternate interior angles • alternate exterior angles	• same-side interior angles • same-side exterior angles • Angle-Angle (AA) Similarity Theorem
The **remote interior angles of a triangle** are the two angles that are non-adjacent to the specified exterior angle.  Angles 1 and 2 are remote interior angles of a triangle with respect to $\angle 4$.	A **transversal** is a line that intersects two or more lines at distinct points. 	**Same-side interior angles** form when a transversal intersects two other lines. These angle pairs are on the same side of the transversal and are between the other two lines.  Angles 1 and 2 are same-side interior angles.

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In this topic, students explore important triangle relationships and use what they know about transformations, congruence, and similarity to establish additional geometric facts. They investigate relationships of angles formed when parallel lines are cut by a transversal.

TEXAS MATH SOLUTION

Triangle Sum Theorem

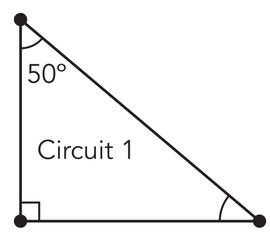
The *Triangle Sum Theorem* states the relationship between the three angles inside a triangle.

THEOREM

TRIANGLE SUM THEOREM

The sum of the measures of the interior angles of a triangle is 180°.

Trevor organizes a bike race called the Tri-Cities Criterium. Criteriums consist of several laps around a closed circuit. He designs a triangular circuit.



Use the Triangle Sum Theorem to determine the measure of the third angle in the triangular circuit.

$$x + 90° + 50° = 180°$$

$$x + 140° = 180°$$

$$x = 40°$$

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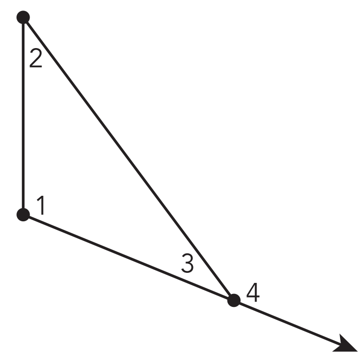
Exterior Angle Theorem

An **exterior angle of a polygon** is an angle between a side of a polygon and the extension of its adjacent side. You can extend a ray from one side of the polygon to form an exterior angle.

In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are interior angles of the triangle, and $\angle 4$ is an exterior angle of the triangle.



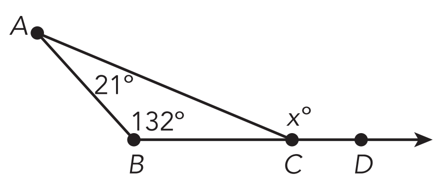
THEOREM

EXTERIOR ANGLE THEOREM

The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

If $\angle 1$ and $\angle 2$ are remote interior angles and $\angle 4$ is an exterior angle, then:

$\angle 1 + \angle 2 = \angle 4$.

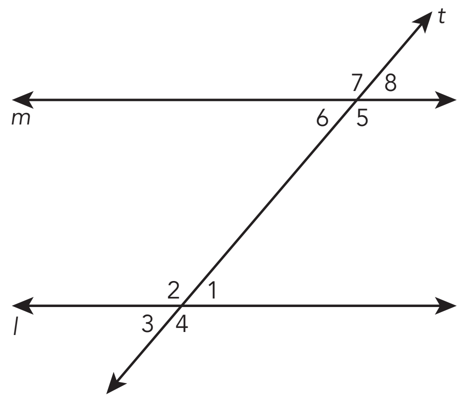


$$x° = 21° + 132°$$

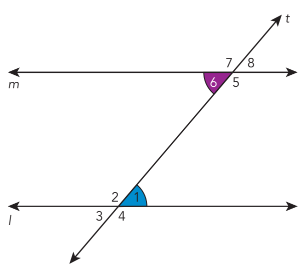
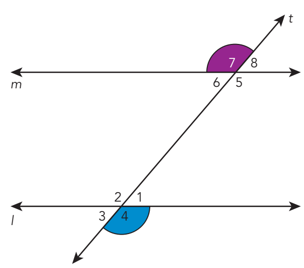
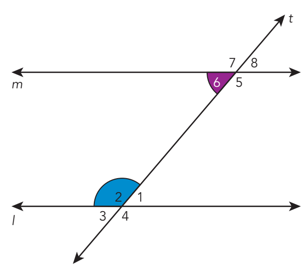
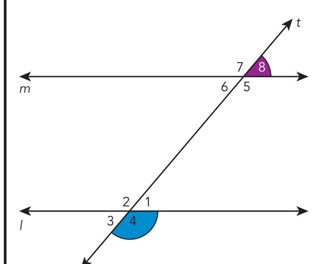
$$x° = 153°$$

Angle Relationships

A transversal is a line that intersects, or crosses, two or more lines. When the two lines intersected by a transversal are parallel, special relationships between the angle measurements form. In this diagram, two parallel lines, *m* and *l*, are intersected by a transversal, *t*.



Corresponding angles have the same relative positions in geometric figures. An example of corresponding angles are ∠2 and ∠7.

Alternate interior angles	Alternate exterior angles	Same-side interior angles	Same-side exterior angles
			
Alternate interior angles are on opposite sides of the transversal and are between the two other lines. An example of alternate interior angles are ∠1 and ∠6.	**Alternate exterior angles** are on opposite sides of the transversal and are outside the other two lines. An example of alternate exterior angles are ∠4 and ∠7.	**Same-side interior angles** are on the same side of the transversal and are between the other two lines. An example of same-side interior angles are ∠2 and ∠6.	**Same-side exterior angles** are on the same side of the transversal and are outside the other two lines. An example of same-side exterior angles are ∠4 and ∠8.

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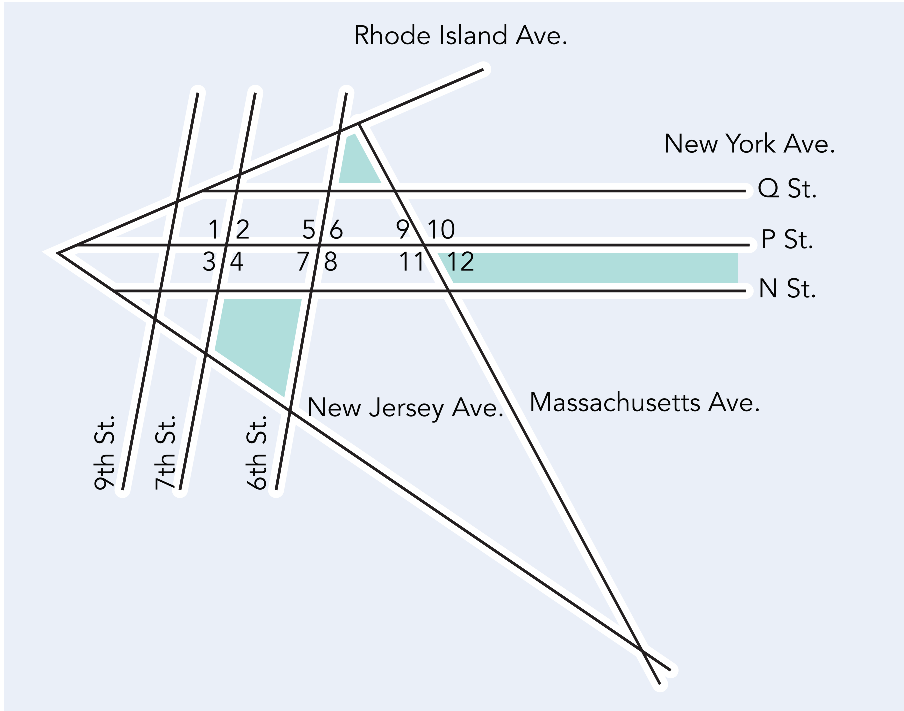


Consider the map of Washington, D.C. shown.

Find a pair of alternate interior angles ______________

alternate exterior angles ______________

same-side interior angles ______________

same-side exterior angles ______________

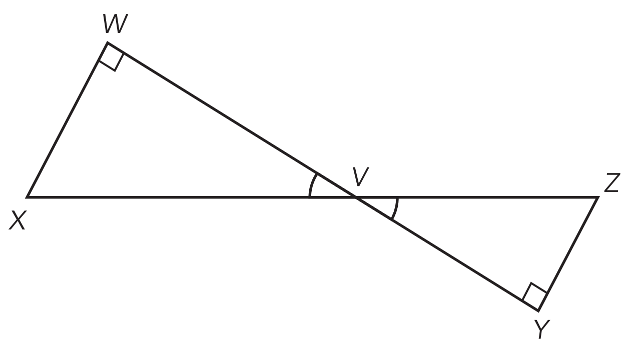
Angle-Angle Similarity Theorem

THEOREM

ANGLE-ANGLE (AA) SIMILARITY THEOREM

When two angles of one triangle are congruent to the corresponding angles of another triangle, the triangles are similar.

In the figure shown, ΔXWV is similar to ΔZYV by the *AA* Similarity Theorem. Because $\angle XWV$ and $\angle ZYV$ are right angles, they are congruent to each other. Because $\angle WVX$ and $\angle YVZ$ are vertical angles, they are congruent to each other. Thus, ΔXWV is similar to ΔZYV.



Your student can use dilations and other transformations, line and angle relationships, measurements, and/or the Angle-Angle Similarity Theorem to demonstrate that two triangles are similar.

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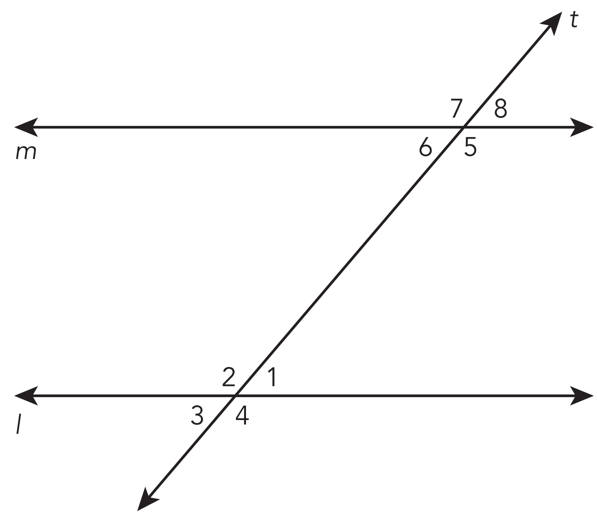
MATH PROCESS STANDARDS **How do the activities in *Line and Angle Relationships* promote student expertise in the math process standards?**	
NOTE: This is an example of the math process standard: *Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.* • I can look closely to identify patterns or structure. Have your student refer to page 2 for more "I can" statements.	 ∠1 and ∠6 are a pair of alternate interior angles, can you name the other pair of alternate interior angles? [*∠2 and ∠5*]

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Discuss important dates throughout this module such as assessments, assignments, or class events with your student. Use the table to record these dates and reference them as your student progresses through the module.

Important Dates	
Date	Reason

Using the link below, visit the Texas Math Solution Support Center for students and caregivers to access additional resources such as:

- Mathematics Glossaries
- Videos
- Topic Materials
- A Letter to Families and Caregivers

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