Module 1 Overview
Reasoning with Shapes

"In fact, as one of the fundamental 'implements' of Euclidean geometry, along with the straightedge, the compass is involved and invoked in constructing midpoints of segments, right angles, regular polygons, transformations, and so on. Among other things, what a geometric tool does is implement a definition. The whole development of Euclid's Elements is based on these tools—if Euclid had chosen different ones, the geometry that we do (and teach) may well have looked quite different. The student who remarks with surprise (as many students do), 'I never thought that you needed a circle to make a square!' experiences the lovely realization that the circle (through the compass) leaves its trace on the square." (*Developing Essential Understanding of Geometry*, NCTM, pg. 43)

Why is the Module named Reasoning with Shapes?

In **Reasoning with Shapes**, students take the first steps to transition from the informal reasoning that encompassed much of their geometric studies throughout middle school to the more formal reasoning required in high school geometry. Students begin to reason about the defining characteristics of shapes that they are very familiar with: squares, circles, and quadrilaterals. They consider whether the characteristics that they have investigated and uncovered throughout their elementary and middle school careers are true for any and all cases of that figure. Students reason algebraically—connecting what they know about lines on the coordinate plane to verify simple geometric theorems. They formally define rigid motion transformations: translations, reflections, and rotations. The reasoning done in this module prepares students for the formal proofs that they will write in the next module.

What is the mathematics of Reasoning with Shapes?

Reasoning with Shapes contains three topics: *Using a Rectangular Coordinate System, Rigid Motions on a Plane*, and *Congruence Through Transformations*. Students perform basic constructions' they experiment with geometric transformations on the plane; and they formally define rigid motion transformations.

In the first topic, *Using a Rectangular Coordinate System*, students start with a shape that they are most familiar with: the square. They explore the nature of geometric reasoning as they investigate angles in a diagram composed of three adjacent and congruent squares. Students consider the structure of the coordinate plane and construct a coordinate plane using transformations of squares to do so. They first gain familiarity with the tools of construction and the deductive reasoning required to precisely construct geometric figures. Students then explore geometric figures on a coordinate plane. Using what they know about the translations and rotations of lines, students prove the slope criteria for parallel and perpendicular lines. They recall the properties of quadrilaterals and triangles and verify the properties of shapes with given vertices using the Distance Formula. Students use coordinates to determine the midpoint of a line segment and to recognize the pattern that exists when connecting the midpoints of quadrilaterals. They integrate what they know about lines, distances on the coordinate plane, and the properties of geometric figures to determine the perimeter and area of triangles, rectangles, and quadrilaterals on the coordinate plane.

In *Rigid Motions on a Plane*, students build from their experience in middle school exploring the properties of rotations, reflections, and translations. They know that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. With a transformation machine, students are reminded of how geometric figures move in the plane. Then translations are formally defined in terms of specific distances along parallel lines; reflections are defined in terms of points equal distances across perpendicular lines; and rotations are defined in terms of specific angles of rotations around concentric circles. Students experience each rigid motion transformation and then specify sequences of transformations to map one image onto another to confirm that they are congruent. Next, students determine how transformations affect the coordinates of a figure. Finally, students investigate reflectional and rotational symmetry.

The proofs by construction in *Congruence Through Transformations* utilize the properties of rigid motions developed in the previous topic.

How is Reasoning with Shapes connected to prior learning?

This topic gives students extensive experience in using the formal tools of construction and expands the number of constructions they can make, including copying and bisecting segments and angles, constructing perpendiculars to given lines, and constructing a parallel line to a given line through a point on or off the line. Students build on what they know from middle school to formally define each rigid motion transformation. While much of their experience in middle school was with transforming shapes on a coordinate plane, the structure of the coordinate grid is at first removed, and students transform objects on any plane. This generalization allows them to use what they know but also to generalize about the overarching structure of these transformations. With formal definitions for each rigid motion, students are ready to use formal geometric reasoning to prove the minimum criteria for triangle congruence.

When will students use the knowledge from Reasoning with Shapes in future learning?

Reasoning with Shapes launches that process of more formal reasoning and formulating more formal questions about the geometric objects students will study. In this module, students are learning the basics: tools of construction, geometric reasoning skills, and formal definitions of rigid motions. As students advance in geometry, they will continue to use constructions and the deductive reasoning that constructions require. Students will use constructions to compose and decompose figures, learning the structure of geometric relationships. They will use formal reasoning to prove geometric relationships, adding two-column, flowchart, and paragraph proofs to their repertoire of proof tools. In many of these proofs, students will use triangle congruence to prove angle and side relationships, and these additional proofs will become part of a large array of geometric theorems that they know to be true in all cases and can use to further reason geometrically.