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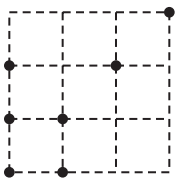
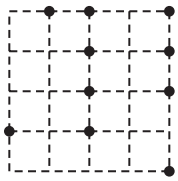
The Squariest Square

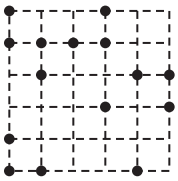
From Informal to Formal Geometric Thinking

Warm Up

Identify and connect the vertices that form a square in each grid.

1. 

2. 

3. 

Learning Goals

- Recall properties of geometric figures.
- Understand that the results from measuring tools can be useful in composing a conjecture; however, they are not an acceptable form of mathematical reasoning to validate a conjecture.
- Make a geometric conjecture and use mathematical reasoning to validate it.

Key Terms

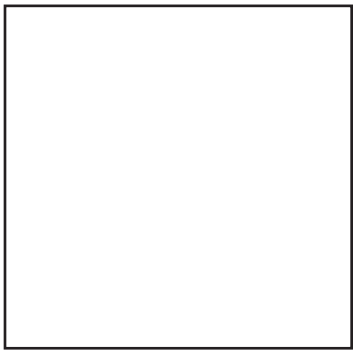
- sketch
- draw
- conjecture
- auxiliary line

You have reasoned about lines and shapes in earlier grades and courses. How can you apply formal geometric reasoning to what you know?

The Perfect Square

Can you sketch a perfect square freehand?

1. **Try to sketch a perfect square, like the one shown, without tracing or using tools.**



2. **Explain how you could decide whether one square is closer to "perfect" than another. Use your criteria to judge your and your classmates' best squares.**

3. **List some properties of squares that you know.**

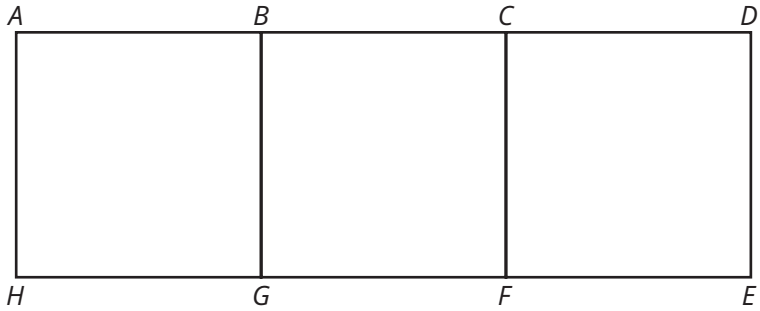
ACTIVITY
1.1

Analyzing a Diagram

In a way, mathematical reasoning is not different from scientific reasoning. In mathematics, you come up with educated guesses and test them to see if they're correct. You can experiment with different patterns and consider arguments about mathematical statements. And, like other scientists, mathematicians gather evidence and become more and more confident about a statement when they obtain more evidence for it.

However, in mathematics, a statement is not true or false until it is proved to be true or false.

Consider the diagram composed of three adjacent squares.



1. Draw $\overline{AG}$, $\overline{AF}$, $\overline{AE}$. Then label $\angle AGH$ using the letter *a*, label $\angle AFG$ using the letter *b*, and label $\angle AEF$ using the letter *c*.

2. Use a protractor to measure $\angle a$, $\angle b$, and $\angle c$. List the angle measures.

3. Compare your measurements with your classmates' measurements. What do you notice?

When you **sketch** a geometric figure, you create the figure without tools. Accuracy is not important. When you **draw** geometric figures, you can use tools such as rulers, protractors, or a coordinate plane to draw exact lengths and areas.

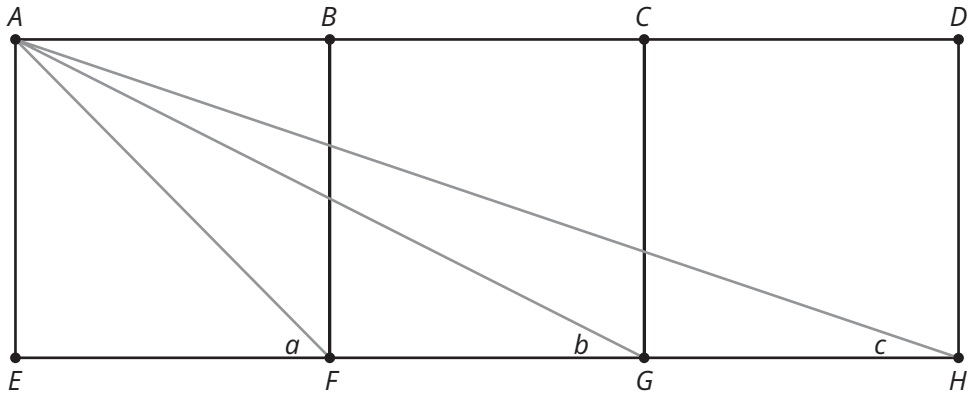
ACTIVITY
1.2 Making Conjectures

A **conjecture** is a mathematical statement that appears to be true, but has not been formally proved.

In the previous activity, you may have noticed that the sum of the measures of $\angle a$, $\angle b$, and $\angle c$ is close to or equal to 90°. Jayda made a *conjecture* about the sum of the angle measures.

> **Jayda**
> The size of the squares doesn't matter. Given any three adjacent and congruent squares, if the diagonals are drawn in the same way, the sum of the angle measures will always be 90°.

Let's consider a diagram of three differently-sized adjacent and congruent squares. The same lines are drawn and triangles formed.



1. Without measuring, do you think the size of the squares will affect the sum of the measures of $\angle a$, $\angle b$, and $\angle c$? Explain your thinking.

2. **Use a protractor to test your prediction on these differently-sized squares. Record your results.**

Do your results support Jayda's conjecture?

There are many different ways to verify that the sum of the angle measures could be 90°. Experimenting with different methods and visualizing are important tools that mathematicians use to approach problems in effective ways and gain confidence in their conclusions.

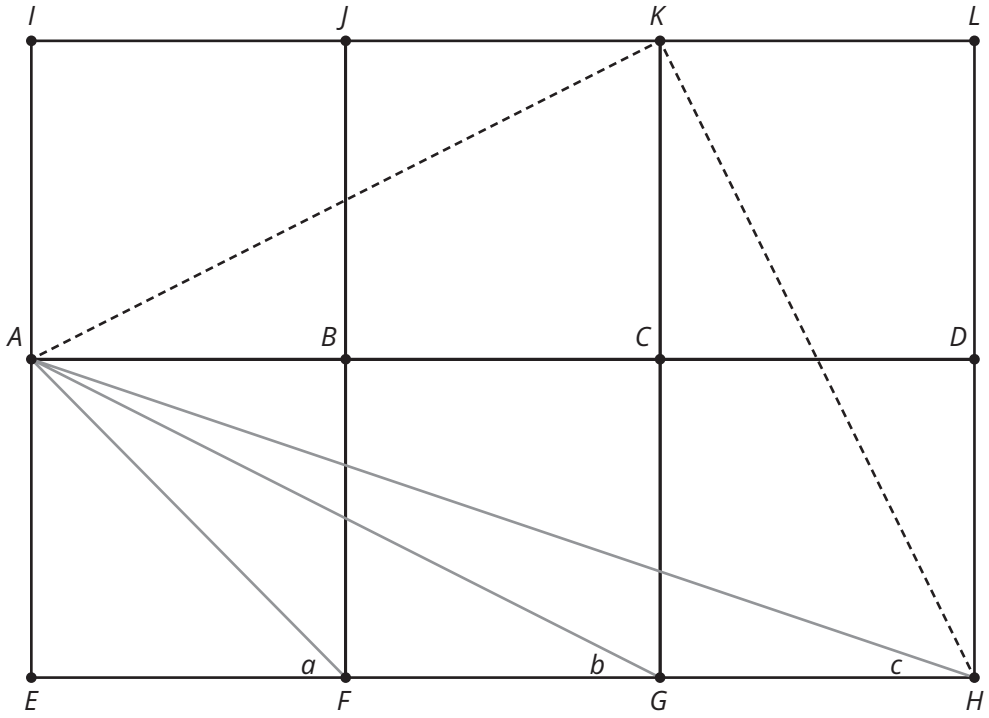
3. **Copy each of the angles *a*, *b*, and *c* from the diagram onto a different piece of patty paper. How can you manipulate the three angles to show that their sum is 90°?**

ACTIVITY
1.3 Drawing Auxiliary Lines

An **auxiliary line** is a line or line segment added to a diagram to help in solving or proving a concept.

Making arguments about statements in mathematics may seem like a rigid process at times, but it can also involve a lot of creativity. You can, for example, draw extra lines, called *auxiliary lines*, and perform rigid motions like translations, reflections, and rotations when you are reasoning geometrically.

Let's consider a new diagram that is the result of translating the original three squares up and drawing two auxiliary line segments.



1. What other angle measures or side lengths can you determine using these added figures? List all the concepts and facts you use.

TALK the TALK

Proving Yourself

In this course, you will move from making conjectures and creating informal arguments to proving, for good, that certain mathematical statements must be true. You will learn to use properties and definitions to prove or disprove many conjectures.

1. **Write a paragraph describing the differences that you think exist between the informal geometric reasoning you have used in the past and the formal thinking used to prove conjectures. Use examples to illustrate your answer.**

NOTES