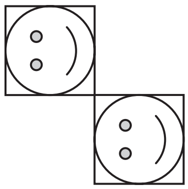
2

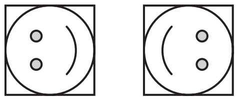
Hip to Be Square

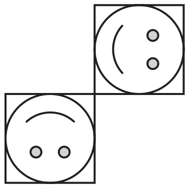
Constructing a Coordinate Plane

Warm Up

Identify whether the transformation shown is a translation, reflection, or rotation. Justify your answer.

1. 

2. 

3. 

Learning Goals

- Construct a perpendicular line segment and bisector using paper folding and using a compass and straightedge.
- Use construction tools to duplicate a line segment and construct a square.
- Use constructions and rigid motions to create a coordinate plane.
- Identify rigid motions that can be used to create shapes on a coordinate plane.
- Identify the coordinates of vertices of shapes on a coordinate plane.

Key Terms

- construct
- compass
- straightedge
- line
- line segment
- point
- midpoint
- segment bisector
- perpendicular bisector
- diagonal
- transformation
- rigid motion

You have recalled and investigated two-dimensional shapes on a plane. How does the coordinate plane help you to analyze geometric objects?

GETTING STARTED

Getting Back in Shape

You may remember that when you were younger you learned to estimate with counting numbers. Accuracy was not important. Then you learned to count and operate with whole numbers and fractions to determine exact amounts. For example, you could calculate that $(2 \times 5) + (2 \times 3)$ is the same as 2×8, or 16. Later, you learned how to reason accurately without the numbers. You could say that $(a \times b) + (a \times c) = a(b + c)$.

Creating and thinking about geometric objects is similar. In the previous lesson, you sketched a square. When you sketch a geometric figure, you create the figure without tools. Accuracy is not important. When you draw geometric figures, you can use tools such as rulers, protractors, and a coordinate plane to draw exact lengths and areas. Finally, when you *construct* geometric figures, you create exact figures without measurements, using paper folding or a compass and a straightedge—and geometric reasoning!

1. Draw a right angle. Explain your method.

A **compass** is a tool used to create arcs and circles.

A **straightedge** is a ruler with no numbers.

In this lesson, you will learn how to **construct** geometric figures. When you construct geometric figures, you create exact figures without measurements, using paper folding or a *compass* and a *straightedge*.

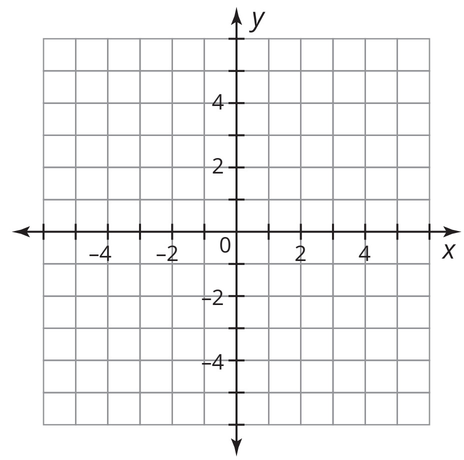
ACTIVITY
2.1

Constructing a Perpendicular Line

You know that a coordinate plane is composed of two intersecting lines—the *x*-axis and *y*-axis.

You can also think of a coordinate plane as being composed of squares. Each square is 1 unit long and 1 unit wide, with the definition of "1 unit" up to the person who constructs the coordinate plane.

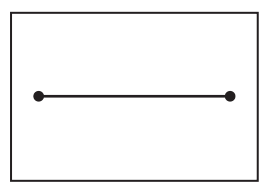
In this lesson, you will consider how a coordinate plane is constructed using squares. To construct a square, you will first need to be able to construct perpendicular lines.



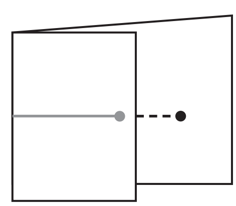
Let's start by experimenting with patty paper to construct a line perpendicular to a given line segment.

Worked Example

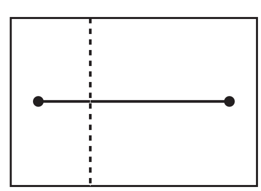
Draw a line segment on a piece of patty paper.



Fold the line segment so that it lies on top of itself.



Open the patty paper. The crease represents a line perpendicular to the given line segment.



A **line** is described as a straight, continuous arrangement of an infinite number of points. A line has an infinite length, but no width.

A **line segment** is a part of a line between two points on the line, called the endpoints. A distance along a line is the length of a line segment connecting two points on the line.

A **point** is described simply as a location. A point in geometry has no size or shape, but it is often represented using a dot. In a diagram, a point can be labeled using a capital letter.

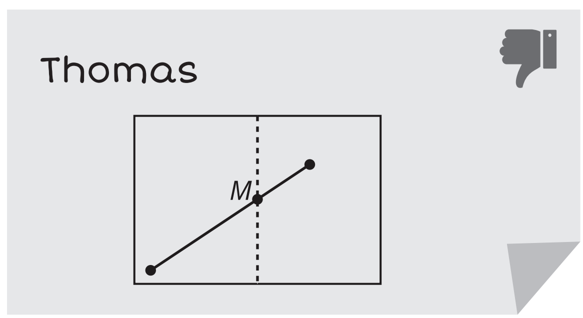
1. **Consider $\overline{AB}$.**

   a. **Use patty paper to construct 2 different perpendicular lines through $\overline{AB}$.**

A **midpoint** of a line segment is the point that divides the line segment into two congruent segments.

   b. **How can you fold the patty paper so that the perpendicular crease intersects the midpoint of $\overline{AB}$?**

2. **Thomas determined the midpoint of a line segment incorrectly. Explain what he did.**



Think about:

How can you be sure that you correctly bisected the line segment?

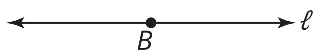
3. **Use patty paper to bisect $\overline{ST}$.**

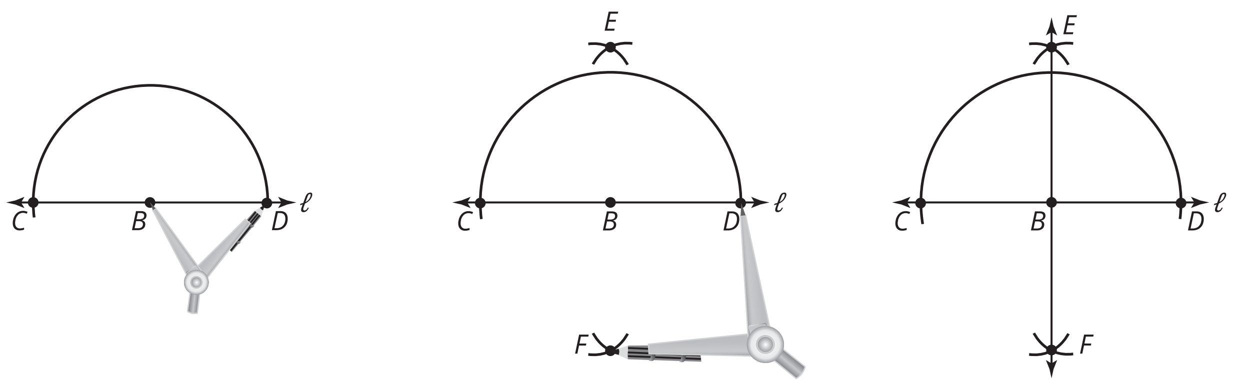
A **segment bisector** is a line, line segment, or ray that divides a line segment into two line segments of equal length. The basic geometric construction used to locate a midpoint of a line segment is called bisecting a line segment.

You can also construct a perpendicular line through a point on a line using a compass and straightedge. To do this construction, you make use of the fact that all the radii of a circle have an equal length.

Worked Example

You can construct a line perpendicular to line ℓ through point B.





Construct an Arc

Use B as the center and construct an arc. Label the intersection points C and D.

→

Construct Other Arcs

Open the compass larger than the radius. Use C and D as centers and construct arcs above and below the line. Label the intersection points E and F.

→

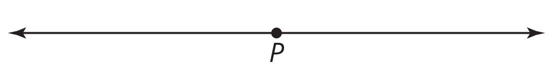
Construct a Line

Use a straightedge to connect points E and F. Line EF is perpendicular to $\overleftrightarrow{CD}$.

4. Explain why $\overleftrightarrow{EF}$ is a bisector of $\overline{CD}$.

A **perpendicular bisector** is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

5. Construct a line perpendicular to the given line through point *P*.

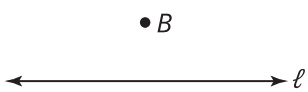


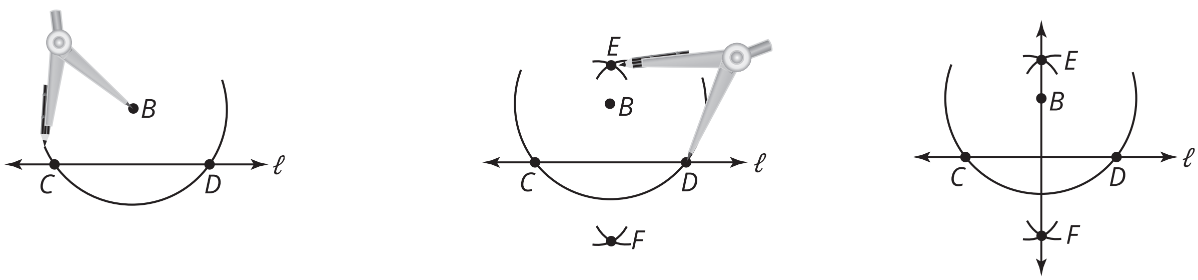
6. How are constructing a segment bisector and constructing a perpendicular line through a point on a line different?

You can also construct a perpendicular line through a point that is not on a line.

Worked Example

You can construct a line perpendicular to line ℓ through point *B*.





Construct an Arc

Use *B* as the center and construct an arc. Label the intersection points *C* and *D*.

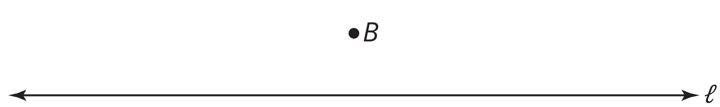
Construct Other Arcs

Open the compass larger than the radius. Use *C* and *D* as centers and construct arcs above and below the line. Label the intersection points *E* and *F*.

Construct a Line

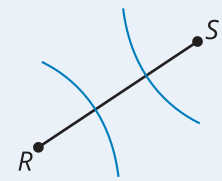
Use a straightedge to connect points *E* and *F*. Line *EF* is perpendicular to $\overleftrightarrow{CD}$ and passes through point *B*.

7. Construct a line perpendicular to line ℓ through point *B*.



8. **Aaron is constructing the perpendicular bisector of $\overline{RS}$. His work is shown.**

Aaron says that because the arcs do not intersect, this line segment does not have a midpoint. Kate disagrees and tells him he drew his arcs incorrectly and that he must redraw his arcs to determine the midpoint. Who is correct? Explain your reasoning.

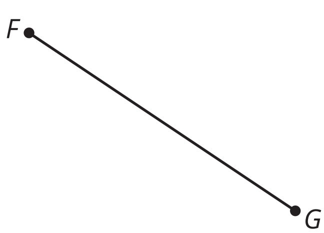


9. **Use construction tools to locate the perpendicular bisector of each given line segment. Label each midpoint as *M*.**

a.

b.

c.



10. Choose a point on the perpendicular bisector of one of the line segments in Question 9 and label it *X*. Measure the distances from point *X* to each of the segment's endpoints. Choose another point on the perpendicular bisector and label it *Y*. Measure the distances from point *Y* to each of the segment's endpoints. What do you notice?

11. Make a conjecture about the distance from any point on a perpendicular bisector to the endpoints of the original segment.

ACTIVITY

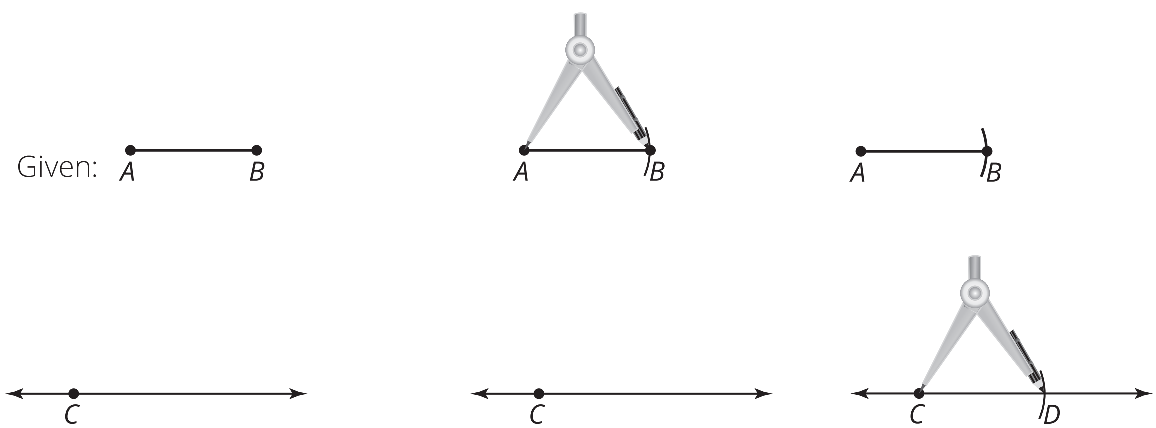
2.2 Constructing a Square

Along with constructing perpendicular lines, you will need to be able to duplicate line segments in order to construct a square.

There are different ways to use construction tools to duplicate a line segment.

Worked Example

You can duplicate a line segment by constructing an exact copy of the original line segment.



Construct a Starter Line
Use a straightedge to construct a starter line longer than $\overline{AB}$. Label point C on the line.

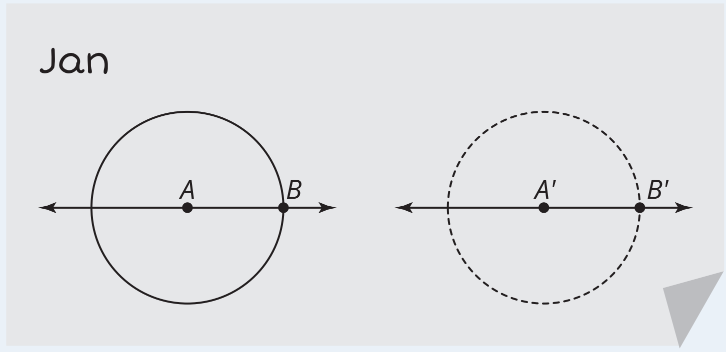
Measure Length
Set your compass at the length AB.

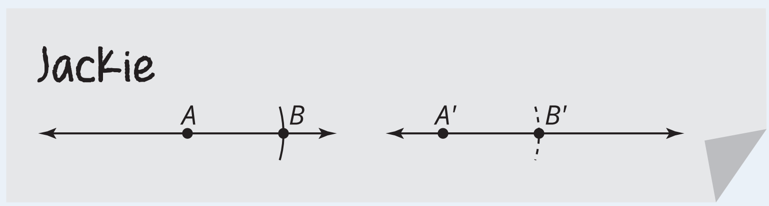
Copy Length
Place the compass at C. Mark point D on the new segment.

$\overline{CD}$ is a duplicate of $\overline{AB}$.

1. Construct a line segment that is twice the length of $\overline{AB}$.

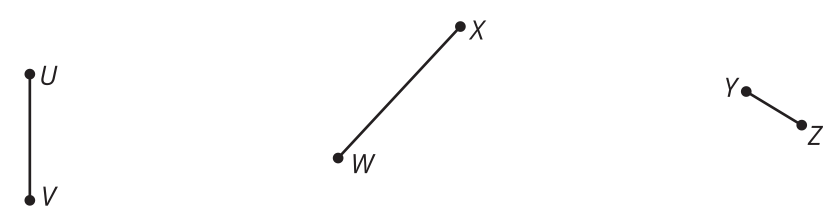
2. **Jan and Jackie are duplicating $\overline{AB}$. Their methods are shown.**



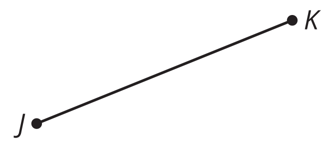


Which method is correct? Explain your reasoning.

3. **Use a compass and a straightedge to duplicate each line segment.**

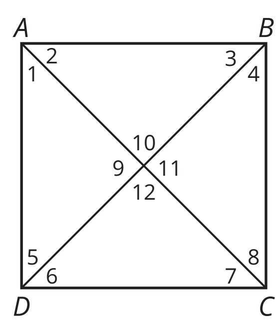


4. **Use what you know about duplicating segments and constructing perpendicular lines to construct a square with the same side length as $\overline{JK}$.**



5. **Use patty paper to verify that the figure you constructed is a square.**

6. **Draw the *diagonals* of your square and label the angles as shown. What do you notice about the segments and angles? Use patty paper to justify any conjectures.**



A **diagonal** is a line segment joining two vertices of a polygon but is not a side of the polygon.

ACTIVITY
2.3

Rigid Motions

To complete the construction of a coordinate plane, you can perform rigid motions of a constructed square.

Recall that a **transformation** is the mapping, or movement, of the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of the figure.

Felipe used translations to create a coordinate plane.

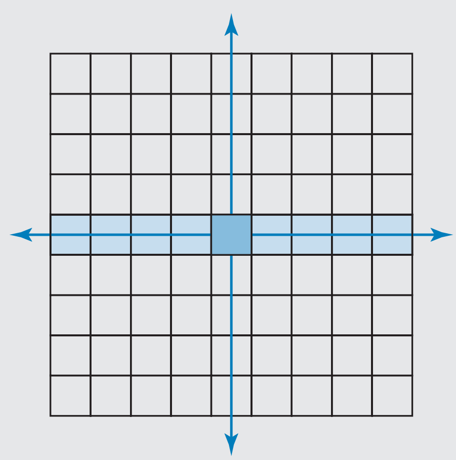
Remember:

A translation "slides" a figure up, down, left, or right.

A reflection "flips" a figure across a line.

A rotation "spins" a figure about a point.

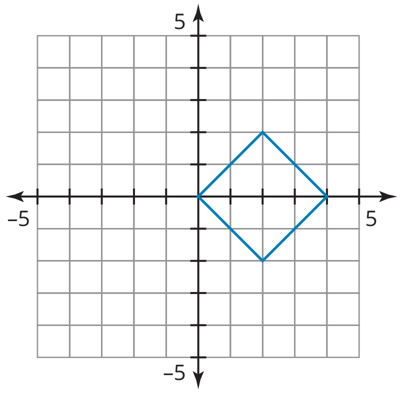
Felipe
I can translate a square to the right and to the left an infinite number of times. Then, I can translate that entire row of squares up and down an infinite number of times to create a coordinate plane.



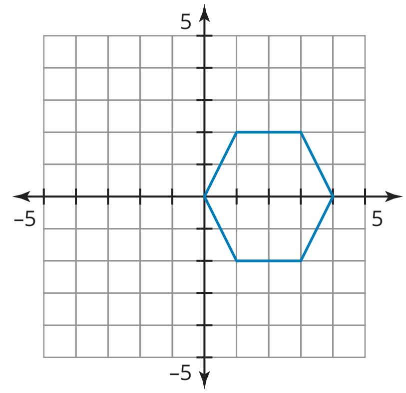
1. What other sequences of rigid motions of a square can you use to create a coordinate plane? Show your work and explain your reasoning.

2. **The figures shown were each constructed using rigid motions, starting with line segments constructed in one or more squares. Describe a sequence of transformations of a figure that could produce the resulting shape. Then give the coordinates of the vertices of the shape.**

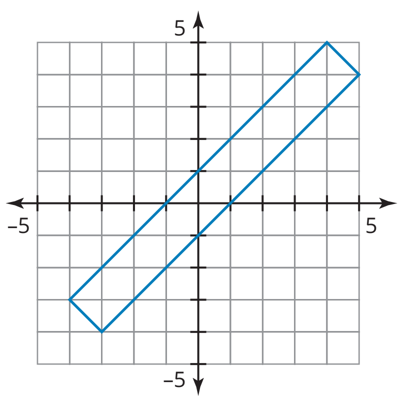
a.



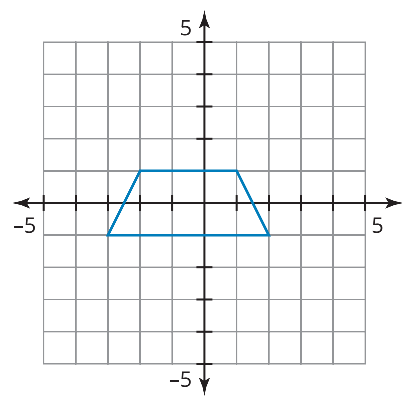
b.



c.



d.

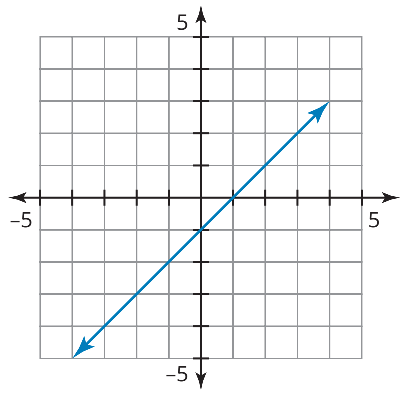


NOTES

TALK the TALK

Walking on a Thin Line

Consider the line shown on the coordinate plane.



1. Suppose the line was constructed using rigid motions, starting with any line segment constructed in one square. Describe one possible sequence of translations that could produce the line.

2. How is the sequence of translations related to the slope of the line?

3. What is the equation for the line?

Remember:

The equation for a line can be written in the form $y = mx + b$, where m represents the slope of the line, and b represents the y-intercept.