2

Hip to Be Square

Constructing a Coordinate Plane

MATERIALS

Compasses
Patty paper
Straightedges

Lesson Overview

Students consider how a coordinate plane can be constructed using squares. They start by completing geometric constructions using patty paper or a compass and a straightedge. They analyze worked examples to construct perpendicular lines, perpendicular bisectors, and duplicated line segments. Students construct a square and then describe how rigid motions can be applied to create a coordinate plane. They then describe rigid motions that can be used to create two-dimensional shapes on a coordinate plane. Students also relate a sequence of translations to the slope of a line.

Geometry

Coordinate and Transformational Geometry

(3) The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity).
The student is expected to:

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

Logical Argument and Constructions

(5) The student uses constructions to validate conjectures about geometric figures.
The student is expected to:

(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge.
(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- When you construct geometric figures, you create exact figures using only a compass and straightedge or patty paper.
- The midpoint of a segment is a point that divides the segment into two congruent segments.

- A segment bisector is a line, line segment, or ray that divides a line segment into two line segments of equal length.
- A perpendicular bisector is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.
- Any point on a perpendicular bisector is equidistant to the endpoints of the original segment it bisects.
- The diagonals of a square are congruent, bisect each other, are perpendicular to one another, and bisect the angles of the square.
- A coordinate plane can be created by constructing a square and applying rigid motion transformations to the square.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Getting Back in Shape

Students distinguish between sketching, drawing, and constructing geometric figures. They draw a right angle and explain their method.

Develop

Activity 2.1: Constructing a Perpendicular Line

Students view the coordinate plane as a geometric figure formed by two perpendicular lines, the *x*-axis and *y*-axis, and a composition of an infinite number of uniform squares, each with the dimension of one unit. They use patty paper to create a line perpendicular to a given segment. Students also analyze worked examples that provide steps to construct a line perpendicular to a given line through a point on the given line and through a point not on the given line. They practice these constructions and then conjecture about the distance from any point on a perpendicular bisector of a line segment to the endpoints of the segment.

Day 2

Activity 2.2: Constructing a Square

Students analyze a worked example that demonstrates the use of construction tools to duplicate a line segment. They practice this construction and then construct a square with a given side length. Finally, they make conjectures about the segments and angles formed by the diagonals of a square and use patty paper to justify their conjectures.

Activity 2.3: Rigid Motions

Students recall the definitions of *rigid motion*, *translation*, *reflection*, and *rotation*. They analyze one strategy to construct a coordinate plane using translations and then describe other methods. Two-dimensional figures are drawn on the coordinate plane, and students describe how the shapes could be formed using transformations of line segments.

Demonstrate

Talk the Talk: Walking on a Thin Line

Students describe a sequence of transformations that can be performed on a given line segment that results in a line. They also determine the slope of the line and write its equation.

Getting Started: Getting Back in Shape

ENGAGE

Facilitation Notes

In this activity, students distinguish between sketching, drawing, and constructing geometric figures. They draw a right angle and explain their method.

Ask a student to read the introduction and definitions aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Misconceptions

- Students may trace the corner of a piece of patty paper and assume it is a right angle. Discuss the issues with that assumption.
- Students may sketch the angle. Remind them that drawing the angle means that they can use tools for precision.

Questions to ask

- Do the lengths of the rays make a difference?
- Do the rays need to be the same length?
- What are some examples of right angles that you see in the classroom?

Summary

Sketches of geometric figures are created without tools. Drawings are created with measuring tools.

Activity 2.1

Constructing a Perpendicular Line

DEVELOP

Facilitation Notes

In this activity, students view the coordinate plane as a geometric figure formed by two perpendicular lines, the *x*-axis and *y*-axis, and as the composition of an infinite number of uniform squares, each with the dimension of one unit. They use patty paper to create a line perpendicular to a given segment. Students also analyze worked examples that provide steps to construct a line perpendicular to a given line through a point on the given line and through a point not on the given line. They practice these constructions and then conjecture about the distance from any point on a perpendicular bisector of a line segment to the endpoints of the segment.

Differentiation strategies

- To scaffold support, have them use construction tools to do the worked examples for themselves throughout the lesson.
- To assist all students,
  - Allow them to use either patty paper or a compass and straightedge to complete constructions, unless the specific tools to be used are noted in the question.
  - Provide options for placing patty paper constructions in the textbook. Suggest students either staple their patty paper in the book or transfer the solution from their patty paper to the diagram in the textbook. To transfer a solution, trace the solution on the back of the patty paper using a pencil, align the diagram on the patty paper with the diagram in the textbook, and then retrace the answer on the patty paper so that the lead transfers to the diagram in the textbook.

Ask a student to read the introduction. Discuss as a class. Analyze the worked example as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

- To extend the activity, prior to starting the activity, have students draw a line segment on patty paper using a straightedge and then create a line perpendicular to the segment without using any other tools. They can then refer to the worked example to verify their procedure.
- To scaffold support,
  - Discuss the meaning of the term *bisect*. The root word *sect* means to cut, as in dissect. The prefix *bi-* means two, as in a bicycle having two wheels. Therefore, the term *bisect* means to cut into two equal parts.
  - Review the terms *equal* and *congruent*. The term *equal* is used when referring to numbers; for example, $JK = KL$, the measure of $\overline{JK}$ equals the measure of $\overline{KL}$, or $JK = 5$ cm. The term *congruent* is used when referring to figures; for example, $\overline{JK} \cong \overline{KL}$.

As students work, look for

A crease in the student's patty paper that appears perpendicular to the drawn line segment. If the crease does not appear to intersect the line segment at right angles, the line segment was not aligned on top of itself when making the fold. If they are constructing a perpendicular bisector, the endpoints should align when making the fold.

Questions to ask

- How are a midpoint and segment bisector related? How are they different?

- Does a midpoint always lie on the segment bisector? Why?
- Can a line segment have more than one midpoint?
- Does the line segment have to have a horizontal orientation on the patty paper for this to work? Could you have drawn a line segment with a vertical or diagonal orientation?
- When bisecting the line segment, is there more than one way to fold the paper so that the endpoints of the line segment lie on top of each other?
- Did Thomas place the endpoints of the line segment on top of each other before folding the crease in the patty paper? What did he do instead?
- How do you know the two line segments created by the midpoint and segment bisector are congruent?

Ask a student to read the definition following Question 3. Analyze the worked example as a class.

Have students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

As students work, look for

- Placement of the compass on points of the line other than the endpoints.
- Arcs that are less than half the distance between the two endpoints.

Questions to ask

- Explain the steps to create a perpendicular bisector.
- Are $\overline{BC}$ and $\overline{BD}$ both radii of circle B? How does this prove that B is the midpoint of $\overline{CD}$?
- How did you determine the setting of the compass needed to draw the first arc?
- Did you change the setting of the compass before you drew the second arc? Does it matter?
- What is the difference in the first step when constructing a segment bisector and the first step when constructing a perpendicular line through a point on the line?

Have students work with a partner or in a group to analyze the worked example and complete Questions 7 and 8. Share responses as a class.

As students work, look for

Confusion regarding the initial setting of the compass. The setting is not specified because any length for the radius is acceptable.

Questions to ask

- What suggestion would you give to Aaron?
- How do you know when the setting on the compass is greater than half the length of $\overline{RS}$?
- If Aaron's compass setting is equal to exactly half the length of $\overline{RS}$, what would happen? Would the arcs intersect?
- What is the purpose of drawing the first arc?
- Why can't you bisect a line?
- When you construct the perpendicular line, does it bisect the line segment you created? How do you know?
- How would the directions change if point *B* was placed below the line?

Differentiation strategy

To extend this activity, have students complete this construction using patty paper.

Have students work with a partner or in a group to complete Questions 9 through 11. Share responses as a class.

Questions to ask

- How did you determine the setting on the compass to construct the first arc? Does the setting on the compass make a difference in this step?
- How did you determine the setting on the compass to construct the second and third arcs? Does the setting on the compass make a difference in these steps?
- What led you to make your conjecture?
- What reasoning can you use to support your conjecture?
- If you draw segments representing the distance from point *X* to each of the endpoints of the segment, what type of triangles are formed?
- How do you know that the measures of the legs of the right triangles are congruent?
- Without having any measurements provided in the diagram, how do you know the lengths of the hypotenuses of the two right triangles are equal?
- How are the hypotenuse and the distance being investigated related?

Summary

A segment bisector contains the midpoint of a line segment. A perpendicular bisector is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

Activity 2.2
Constructing a Square

Facilitation Notes

In this activity, students analyze a worked example which demonstrates the use of construction tools to duplicate a line segment. They practice this construction and then construct a square with a given side length. They make conjectures about the segments and angles formed by the diagonals of a square and use patty paper to justify their conjectures.

Discuss the worked example as a class. Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

Confusion regarding the comparison between Jan's and Jackie's methods. They are essentially the same method. Jan extended her arc so that it became a complete circle, whereas Jackie just constructed a small arc of that same circle. Patty paper could be used to show Jackie's arc is on Jan's circle and the extension of Jackie's arc would create Jan's circle.

Questions to ask

- What is a starter line?
- How long is a starter line?
- Does it matter where you locate the initial point on the starter line? Why or why not?
- How is a compass used to measure the length of the segment you are duplicating?
- Where is the compass point placed to cut the arc through the starter line?
- Why is this new segment the exact length of the given segment?
- How is the setting on the compass used to create a congruent segment?
- How many times is the construction done to create a line segment twice the length of the given segment?
- What do Jan's and Jackie's methods have in common? How are they different?
- Did Jan's and Jackie's methods both begin with the construction of a starter line and the location of an initial point on the starter line?
- If Jackie extended the length of her arc, would her arc eventually create a circle similar to Jan's circle?
  - Can a segment be too short to duplicate? Why or why not?
  - How does the orientation of the given segment affect the duplication construction?

Have students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

Differentiation strategy

To scaffold support for constructing the square, suggest they extend $\overline{JK}$ on both sides to construct a perpendicular bisector at point *J* and at point *K*.

Questions to ask

- How did you construct the right angles for your square?
- How did you get the right angles to appear at the vertices instead of at the midpoint of each side?
- How did you get the side lengths to be the same for all four sides?
- How was patty paper used to verify the constructed figure is a square?
- Why isn't it enough to know that the four sides are congruent?
- How did you use patty paper to verify that the angles are right angles?
- Are the diagonals congruent? How do you know?
- Do the diagonals of a square bisect each other? How can you check?
- Are the diagonals of a square perpendicular to each other?
- Do the diagonals of a square bisect the angles at each vertex of the square?
- Do the diagonals of a square form 4 congruent isosceles right triangles?
- How do you know the triangles formed are right? Isosceles? Congruent?

Summary

A line segment can be duplicated using patty paper or construction tools. The diagonals of a square are congruent, bisect each other, are perpendicular to one another, and bisect the angles of the square.

Activity 2.3

Rigid Motions

Facilitation Notes

In this activity, students recall the definitions of *rigid motion*, *translation*, *reflection*, and *rotation*. They analyze one way to construct a coordinate plane, using translations and students are instructed to describe other ways. Two-dimensional figures are drawn on coordinate planes and students describe how the shapes could be formed using transformations of line segments.

Ask a student to read the introduction and definitions aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

Connections made between the properties of a square and the transformations of the square to create a coordinate plane.

Questions to ask

- If Felipe had translated the square only to the right an infinite number of times, how would you describe the results?
- How would you describe the results when a square is translated up an infinite number of times?
- If Felipe had translated the square to the left first instead of the right, then taken the entire row of squares up and down an infinite number of times, would this be considered a different sequence of rigid motions?
- If a square is rotated 90°, how would you describe the resulting image?
- If a square is rotated any multiple of 90°, how would you describe the resulting image?

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

Differentiation strategies

To scaffold support,

- Suggest they use patty paper to transform line segment(s) to solve or verify their response.
- Suggest students label each vertex for easier reference.

Misconception

Students may think they must transform an entire side of a shape. They may transform multiple sides at once for efficiency in parts (a), (b), and (c). They may transform a partial side to complete the reflection in part (d).

Questions to ask

- Did you start with one, two, three, or more line segments? How many?
- How many transformations were performed to produce the shape? Which transformations?
- Can you describe a two-step transformation that would produce the shape?
- Is it possible to describe a three-step transformation that would produce the shape?

- Can you describe a four-step transformation that would produce the shape?
- Did you reflect any segments over an axis? Which axis?
- Did you reflect any segments over a horizontal or vertical line? What is the equation of the line?
- Did you use rotation to produce any shape? Which one? How many degrees rotation?

Summary

Rigid motion transformations can be used to construct a coordinate plane from a single square and to create two-dimensional figures on the coordinate plane.

DEMONSTRATE

Talk the Talk: Walking on a Thin Line

Facilitation Notes

In this activity, students describe a sequence of transformations that can be performed on a line segment located in the square on the coordinate plane that results in a given line. They also determine the slope of the line and write its equation.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategy

To scaffold support for all learners, discuss the relationship between translations performed on a single point and the positive or negative slope value of the line segment formed by these transformations.
In a classroom discussion, pose questions similar to:

- If a point on the coordinate plane is translated up one unit and to the right one unit, is the slope of the line segment formed by connecting the two points a positive or negative value?
- If a point on the coordinate plane is translated up one unit and to the left one unit, is the slope of the line segment formed by connecting the two points a positive or negative value?
- If a point on the coordinate plane is translated down one unit and to the right one unit, is the slope of the line segment formed by connecting the two points a positive or negative value?
- If a point on the coordinate plane is translated down one unit and to the left one unit, is the slope of the line segment formed by connecting the two points a positive or negative value?

Questions to ask

- Did you start with a line segment constructed in one square or more than one square?
- Does it matter which line segment you use to perform the first translation?
- What are the endpoints of the line segment you used first?
- Did you translate the initial line segment up or down? To the left or right?
- If you translate the initial line segment up first, what effect does it have on the horizontal translation?
- If you translate the initial line segment down first, is the second translation to the left or to the right?
- What is the definition of *slope*?
- How do you determine the slope of the line on the coordinate plane?
- What is the slope of the line on the coordinate plane?
- Is the slope of the line 1 or -1? How do you know?
- What information do you need to determine the equation of the line?
- Does the graph of the line pass through the origin?
- What are the coordinates of the y-intercept?
- How can the equation of a line be written using the slope and y-intercept?

Summary

A sequence of translations can be related to the slope of a line.

NOTES

Warm Up Answers

1. translation
2. reflection
3. rotation or reflection

2

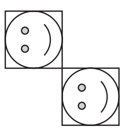
Hip to Be Square

Constructing a Coordinate Plane

Warm Up

Identify whether the transformation shown is a translation, reflection, or rotation. Justify your answer.

1.



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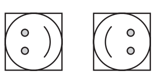

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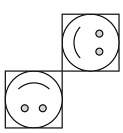


Learning Goals

- Construct a perpendicular line segment and bisector using paper folding and using a compass and straightedge.
- Use construction tools to duplicate a line segment and construct a square.
- Use constructions and rigid motions to create a coordinate plane.
- Identify rigid motions that can be used to create shapes on a coordinate plane.
- Identify the coordinates of vertices of shapes on a coordinate plane.

Key Terms

- construct
- compass
- straightedge
- line
- line segment
- point
- midpoint
- segment bisector
- perpendicular bisector
- diagonal
- transformation
- rigid motion

You have recalled and investigated two-dimensional shapes on a plane. How does the coordinate plane help you to analyze geometric objects?

Answer

1 Check students' drawings.

GETTING STARTED

Getting Back in Shape

You may remember that when you were younger you learned to estimate with counting numbers. Accuracy was not important. Then you learned to count and operate with whole numbers and fractions to determine exact amounts. For example, you could calculate that $(2 \times 5) + (2 \times 3)$ is the same as 2×8, or 16. Later, you learned how to reason accurately without the numbers. You could say that $(a \times b) + (a \times c) = a(b + c)$.

Creating and thinking about geometric objects is similar. In the previous lesson, you sketched a square. When you sketch a geometric figure, you create the figure without tools. Accuracy is not important. When you draw geometric figures, you can use tools such as rulers, protractors, and a coordinate plane to draw exact lengths and areas. Finally, when you *construct* geometric figures, you create exact figures without measurements, using paper folding or a compass and a straightedge—and geometric reasoning!

1. Draw a right angle. Explain your method.

A **compass** is a tool used to create arcs and circles.

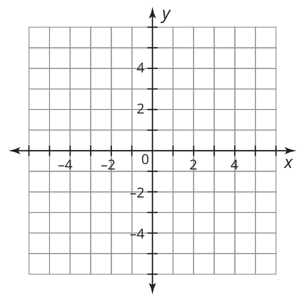
A **straightedge** is a ruler with no numbers.

In this lesson, you will learn how to **construct** geometric figures. When you construct geometric figures, you create exact figures without measurements, using paper folding or a *compass* and a *straightedge.*

ACTIVITY **2.1**

Constructing a Perpendicular Line

You know that a coordinate plane is composed of two intersecting lines—the *x*-axis and *y*-axis.



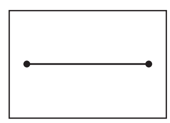
You can also think of a coordinate plane as being composed of squares. Each square is 1 unit long and 1 unit wide, with the definition of "1 unit" up to the person who constructs the coordinate plane.

In this lesson, you will consider how a coordinate plane is constructed using squares. To construct a square, you will first need to be able to construct perpendicular lines.

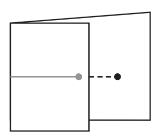
Let's start by experimenting with patty paper to construct a line perpendicular to a given line segment.

Worked Example

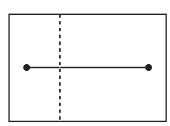
Draw a line segment on a piece of patty paper.



Fold the line segment so that it lies on top of itself.



Open the patty paper. The crease represents a line perpendicular to the given line segment.



A **line** is described as a straight, continuous arrangement of an infinite number of points. A line has an infinite length, but no width.

A **line segment** is a part of a line between two points on the line, called the endpoints. A distance along a line is the length of a line segment connecting two points on the line.

A **point** is described simply as a location. A point in geometry has no size or shape, but it is often represented using a dot. In a diagram, a point can be labeled using a capital letter.

Answer

1a. Check students' patty paper.

1b. Fold the paper so that the endpoints of the line segment lie on top of each other.

2. Thomas folded the patty paper by aligning the corners of the patty paper. He should have folded the patty paper so that endpoints of the line segment lie on top of one another.

3. Check students' patty paper folds.

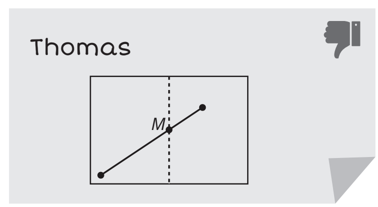
1. Consider $\overline{AB}$.

a. Use patty paper to construct 2 different perpendicular lines through $\overline{AB}$.

A **midpoint** of a line segment is the point that divides the line segment into two congruent segments.

b. How can you fold the patty paper so that the perpendicular crease intersects the midpoint of $\overline{AB}$?

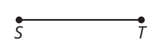
2. Thomas determined the midpoint of a line segment incorrectly. Explain what he did.



Think about:

How can you be sure that you correctly bisected the line segment?

3. Use patty paper to bisect $\overline{ST}$.

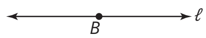


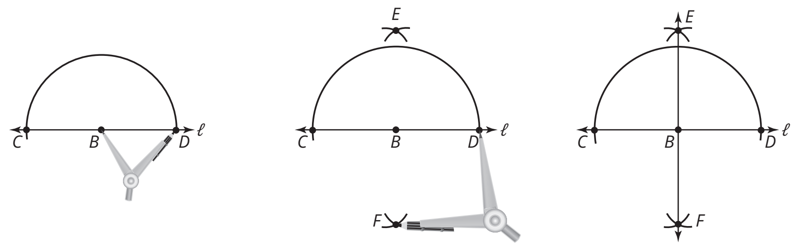
A **segment bisector** is a line, line segment, or ray that divides a line segment into two line segments of equal length. The basic geometric construction used to locate a midpoint of a line segment is called bisecting a line segment.

You can also construct a perpendicular line through a point on a line using a compass and straightedge. To do this construction, you make use of the fact that all the radii of a circle have an equal length.

Worked Example

You can construct a line perpendicular to line ℓ through point *B*.





Construct an Arc

Use *B* as the center and construct an arc. Label the intersection points *C* and *D*.

→

Construct Other Arcs

Open the compass larger than the radius. Use *C* and *D* as centers and construct arcs above and below the line. Label the intersection points *E* and *F*.

→

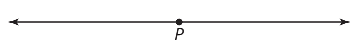
Construct a Line

Use a straightedge to connect points *E* and *F*. Line *EF* is perpendicular to $\overleftrightarrow{CD}$.

4. Explain why $\overleftrightarrow{EF}$ is a bisector of $\overline{CD}$.

A **perpendicular bisector** is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

5. Construct a line perpendicular to the given line through point *P*.

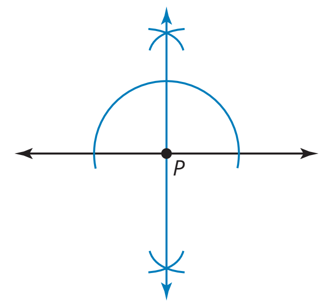


6. How are constructing a segment bisector and constructing a perpendicular line through a point on a line different?

Answers

4. Segments *BD* and *BC* are radii of the same circle, Circle *B*, so they are congruent.

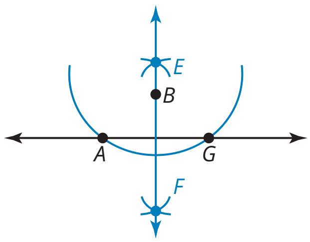
5. Check students' constructions.



6. When constructing a segment bisector, the endpoints of the segment are used to make arcs. When constructing a perpendicular line through a point on the line, there are no endpoints. I must start by placing my compass on the given point and creating two points equidistant from the given point. Then, I can make arcs from those two points I created.

Answer

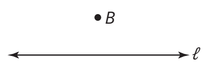
7. Check students' constructions.

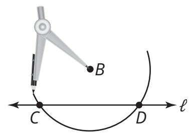


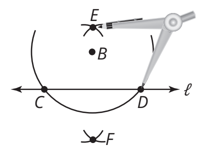
You can also construct a perpendicular line through a point that is not on a line.

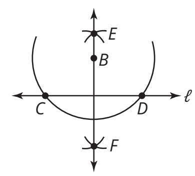
Worked Example

You can construct a line perpendicular to line ℓ through point *B*.









Construct an Arc

Use *B* as the center and construct an arc. Label the intersection points *C* and *D*.

→

Construct Other Arcs

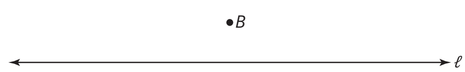
Open the compass larger than the radius. Use *C* and *D* as centers and construct arcs above and below the line. Label the intersection points *E* and *F*.

→

Construct a Line

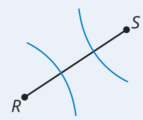
Use a straightedge to connect points *E* and *F*. Line *EF* is perpendicular to $\overleftrightarrow{CD}$ and passes through point *B*.

7. Construct a line perpendicular to line ℓ through point *B*.



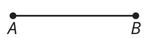
8. **Aaron is constructing the perpendicular bisector of $\overline{RS}$. His work is shown.**

Aaron says that because the arcs do not intersect, this line segment does not have a midpoint. Kate disagrees and tells him he drew his arcs incorrectly and that he must redraw his arcs to determine the midpoint. Who is correct? Explain your reasoning.



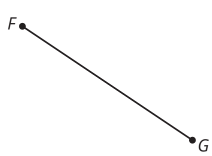
9. **Use construction tools to locate the perpendicular bisector of each given line segment. Label each midpoint as *M*.**

a.



b.

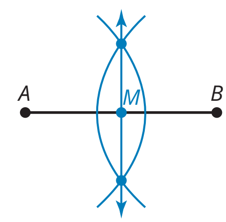
c.



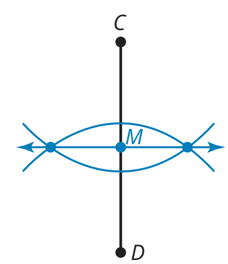
Answers

8. Kate is correct. All line segments have midpoints. If the arcs do not intersect, Aaron should set his compass so that it is open more than half the distance between the two endpoints and redraw the arcs.

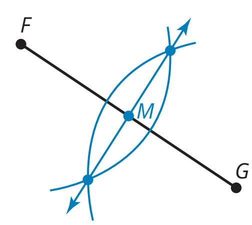
9a. Check students' constructions.



9b. Check students' constructions.



9c. Check students' constructions.



Answers

10. The distances from a point on the perpendicular bisector to each of the segment's endpoints are equal.
11. Any point on a perpendicular bisector is equidistant to the endpoints of the original segment.

10. Choose a point on the perpendicular bisector of one of the line segments in Question 9 and label it *X*. Measure the distances from point *X* to each of the segment's endpoints. Choose another point on the perpendicular bisector and label it *Y*. Measure the distances from point *Y* to each of the segment's endpoints. What do you notice?

11. Make a conjecture about the distance from any point on a perpendicular bisector to the endpoints of the original segment.

ACTIVITY **2.2**

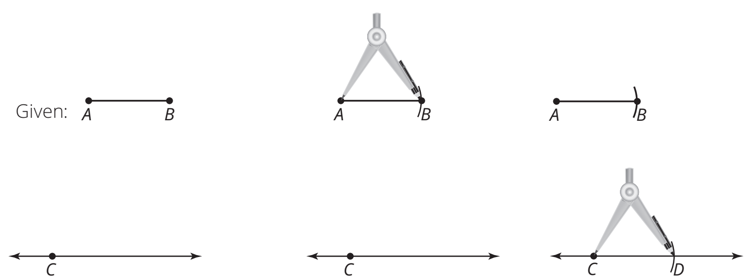
Constructing a Square

Along with constructing perpendicular lines, you will need to be able to duplicate line segments in order to construct a square.

There are different ways to use construction tools to duplicate a line segment.

Worked Example

You can duplicate a line segment by constructing an exact copy of the original line segment.



Construct a Starter Line
Use a straightedge to construct a starter line longer than $\overline{AB}$. Label point C on the line.

→ **Measure Length**
Set your compass at the length AB.

→ **Copy Length**
Place the compass at C. Mark point D on the new segment.

$\overline{CD}$ is a duplicate of $\overline{AB}$.

1. Construct a line segment that is twice the length of $\overline{AB}$.

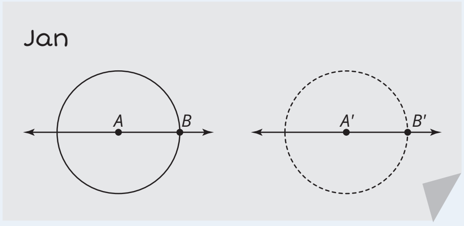
Answer

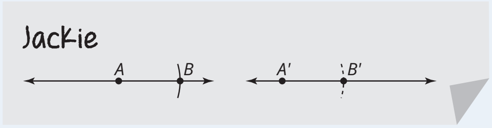
1. 

Answers

2. Both methods are correct. Jackie and Jan are using the same method. The only difference is Jan used the compass to draw complete circles, whereas Jackie drew only the part of the circle, an arc, where it intersects the line segment.
3. Check students' constructions. Any orientation of the line segment is acceptable.

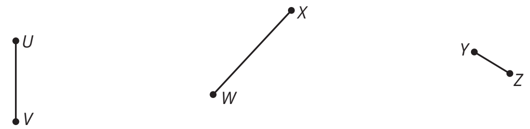
2. Jan and Jackie are duplicating $\overline{AB}$. Their methods are shown.



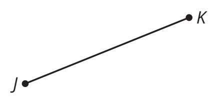


Which method is correct? Explain your reasoning.

3. Use a compass and a straightedge to duplicate each line segment.

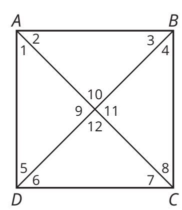


4. Use what you know about duplicating segments and constructing perpendicular lines to construct a square with the same side length as $\overline{JK}$.



5. Use patty paper to verify that the figure you constructed is a square.

6. Draw the *diagonals* of your square and label the angles as shown. What do you notice about the segments and angles? Use patty paper to justify any conjectures.



A **diagonal** is a line segment joining two vertices of a polygon but is not a side of the polygon.

Answer

4. Check students' constructions.
   The figure should have four right angles and four congruent sides.
5. Check students' work.
   Students should be checking that all the angles are right angles and all the sides are congruent.
6. Check students' diagrams.
   Sample answers.
   The diagonals of a square are congruent.
   The diagonals of a square bisect each other.
   The diagonals of a square are perpendicular to each other.
   The diagonals of a square bisect the angles at each vertex of the square.
   The diagonals of a square form four congruent isosceles right triangles.

Answer

1. Sample answer. The square can be translated to the right an infinite number of times and then the single row of squares can be translated up to create the first quadrant of a coordinate plane. This quadrant of squares can then be rotated 90 degrees clockwise (or counterclockwise) 3 times to create the other quadrants of the plane.

ACTIVITY **2.3**

Rigid Motions

To complete the construction of a coordinate plane, you can perform rigid motions of a constructed square.

Recall that a **transformation** is the mapping, or movement, of the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of the figure.

Felipe used translations to create a coordinate plane.

Remember:

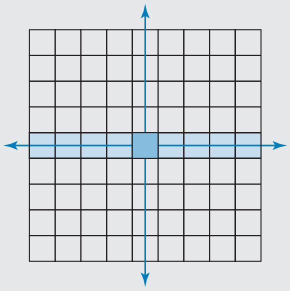
A translation "slides" a figure up, down, left, or right.

A reflection "flips" a figure across a line.

A rotation "spins" a figure about a point.

Felipe

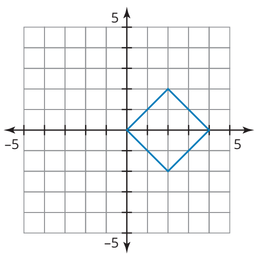
I can translate a square to the right and to the left an infinite number of times. Then, I can translate that entire row of squares up and down an infinite number of times to create a coordinate plane.



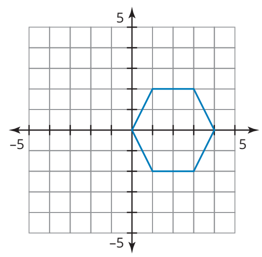
1. **What other sequences of rigid motions of a square can you use to create a coordinate plane? Show your work and explain your reasoning.**

2. The figures shown were each constructed using rigid motions, starting with line segments constructed in one or more squares. Describe a sequence of transformations of a figure that could produce the resulting shape. Then give the coordinates of the vertices of the shape.

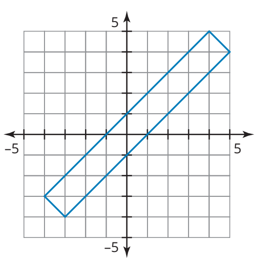
a.



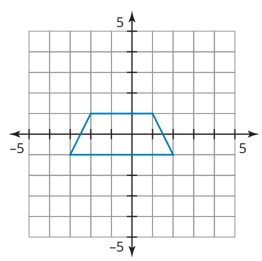
b.



c.



d.



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Answers

2a. Sample answer. Start with the line segment with endpoints at the origin and (2, 2). Rotate the line segment 90° counterclockwise about the point at (2, 2). Then reflect the two line segments together across the *x*-axis.

2b. Sample answer. Start with the figure formed by two line segments: one with endpoints at the origin and (1, 2) and one with endpoints at (1, 2) and (2, 2). Reflect this figure across the vertical line $x = 2$ to create an isosceles trapezoid shape in the first quadrant. Then reflect the entire shape across the *x*-axis.

2c. Sample answer. Start with the figure formed by two line segments: one with endpoints at (−4, −3) and (4, 5) and one with endpoints at (4, 5) and (5, 4). Rotate this figure 180° about the point at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

2d. Sample answer. Start with the figure formed by three line segments: one with endpoints at (2, −1) and (1, 1), one with endpoints at (1, 1) and (−0.5, 1), and one with endpoints at (2, −1) and (−0.5, −1). Reflect this figure across the vertical line $x = -0.5$.

Answers

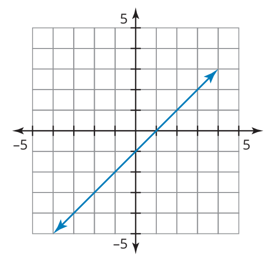
1. Sample answer. Translate the line segment with endpoints at (0, −1) and (1, 0) up 1 and right 1 repeatedly, and translate it down 1 and left 1 repeatedly.
2. The slope of the line is 1. The slope is the ratio of translations up and to the right $\frac{+1}{+1} = 1$ or translations down and to the left $\frac{-1}{-1} = 1$.
3. $y = x - 1$

NOTES

TALK the TALK

Walking on a Thin Line

Consider the line shown on the coordinate plane.



1. Suppose the line was constructed using rigid motions, starting with any line segment constructed in one square. Describe one possible sequence of translations that could produce the line.

2. How is the sequence of translations related to the slope of the line?

3. What is the equation for the line?

Remember:

The equation for a line can be written in the form $y = mx + b$, where m represents the slope of the line, and b represents the y-intercept.

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