3

Ts and Train Tracks

Parallel and Perpendicular Lines

Warm Up

Determine the reciprocal of each value.

1. 3
2. -10
3. $\frac{1}{5}$
4. $-c$
5. $\frac{a}{b}$, $b \neq 0$

Learning Goals

- Construct parallel lines.
- Identify and write the equations of lines perpendicular to given lines.
- Identify and write the equations of parallel lines, including horizontal and vertical lines.

You have constructed line segments, perpendicular lines, squares, and a coordinate plane. How can the coordinate plane be used to justify parallel and perpendicular line relationships?

All Aboard the Clue Train!

You have created lines and shapes by translating, reflecting, and rotating squares on the coordinate plane. Let's explore the slopes of line segments constructed using coordinate plane squares.

Let's consider $\overline{AB}$ on Slope Grid A located at the end of the lesson.

1. What is the slope of $\overline{AB}$?

Remember:

The slope of a line indicates both steepness and direction.

2. Consider how to create a segment parallel to $\overline{AB}$.

a. Trace $\overline{AB}$ onto a piece of patty paper. Then move the segment on the patty paper to create a segment parallel to $\overline{AB}$. Describe your movements.

b. How do you know that these segments are parallel?

c. What is the slope of the segment parallel to $\overline{AB}$?

3. Consider how to create a segment perpendicular to $\overline{AB}$.

a. Move the segment on the patty paper to create a segment perpendicular to $\overline{AB}$. Describe your movements.

b. How do you know that these segments are perpendicular?

c. What is the slope of the segment perpendicular to $\overline{AB}$?

Now let's consider $\overline{CD}$ on Slope Grid B located at the end of the lesson.

4. What is the slope of $\overline{CD}$?

5. Use patty paper to create a segment parallel to $\overline{CD}$.

a. Describe how you know that the segments are parallel.

b. Identify the slope of the segment parallel to $\overline{CD}$.

6. Use patty paper to create a segment perpendicular to $\overline{CD}$.

a. Describe how you know that the segments are perpendicular.

b. Identify the slope of the segment perpendicular to $\overline{CD}$.

7. Use your investigation to write a conjecture about the slopes of parallel and perpendicular lines.

ACTIVITY 3.1 Constructing Parallel Lines

In the previous lesson, you constructed perpendicular lines. You can also construct parallel lines.

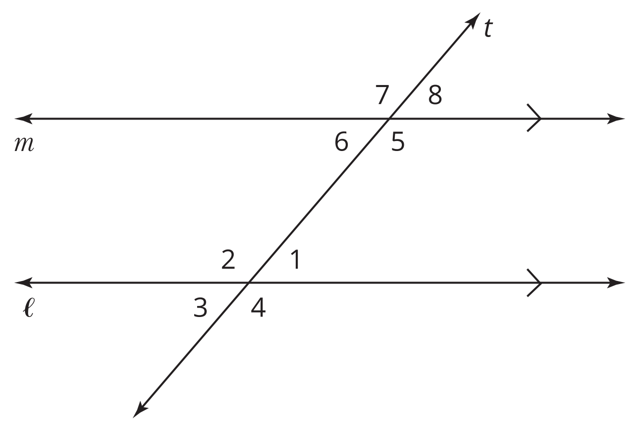
One strategy used to construct parallel lines is translation. The image of a line that has been translated is either the same line or a parallel line.

If line *m* is a translation of line ℓ, then the two lines are parallel. Recall that when parallel lines are cut by a transversal, several pairs of congruent angles are formed.

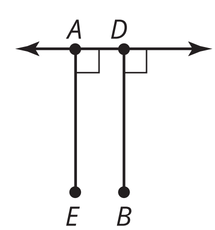
Corresponding angles:
$\angle 8 \cong \angle 1$

Alternate interior angles:
$\angle 1 \cong \angle 6$

Alternate exterior angles:
$\angle 4 \cong \angle 7$

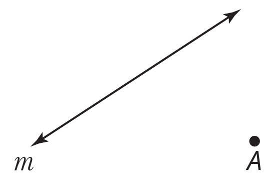


Consider $\overline{AE} \perp$ to $\overleftrightarrow{AD}$ and $\overline{DB} \perp$ to $\overleftrightarrow{AD}$.



1. Is there enough information to conclude that the two segments are parallel? Explain your reasoning.

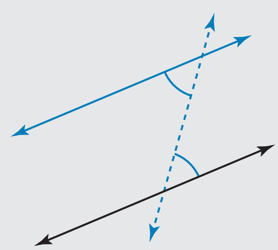
2. **Use your reasoning from Question 1 to construct a line parallel to line *m* through point *A*. Describe your process.**



3. **Explain why Gage is correct.**

Gage

You can duplicate any angle measure, not just right angles, to construct parallel lines.



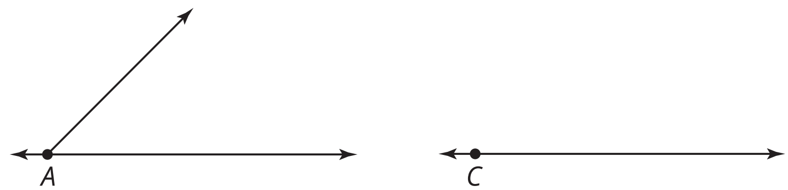
To use Gage's method, you need to know how to use construction tools to duplicate an angle.

Worked Example

Construct a Starter Line

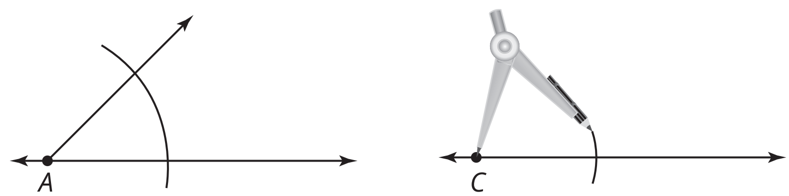
Use a straightedge to construct a starter line. Label point *C* on the new line.

Given:



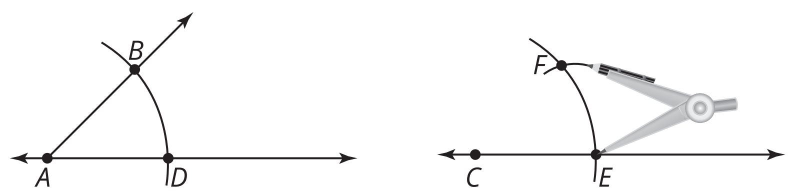
Construct an Arc

Construct an arc with center *A*. Using the same radius, construct an arc with center *C*.



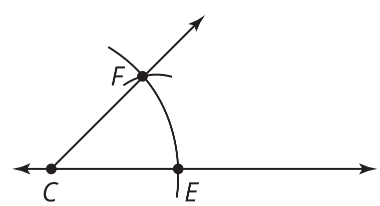
Construct Another Arc

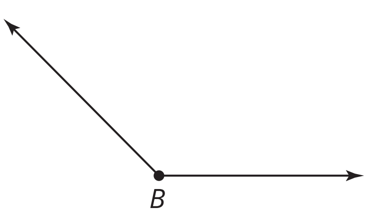
Label points *B*, *D*, and *E*. Construct an arc with a radius the length of *BD* and center *E*. Label the intersection *F*.



Construct a Ray

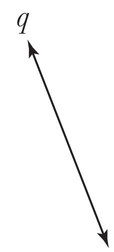
Construct ray *CF*.
$\angle BAD \cong \angle FCE$





4. Duplicate angle *B*. Verify with patty paper.

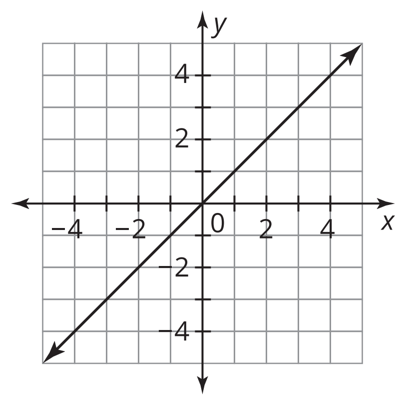
5. Now that you know how to duplicate an angle, use Gage's method to construct a line parallel to line *q*.



6. Consider the line $y = x$ on the coordinate plane shown.

a. Translate the line $y = x$ to create a parallel line.

b. Write the equation of your line.



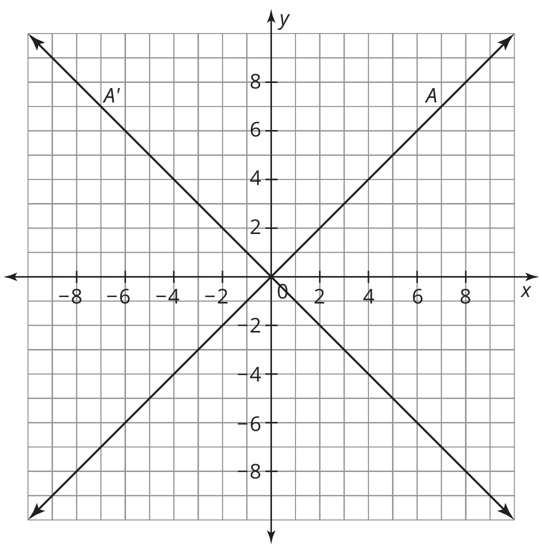
c. Describe the slopes of parallel lines on the coordinate plane.

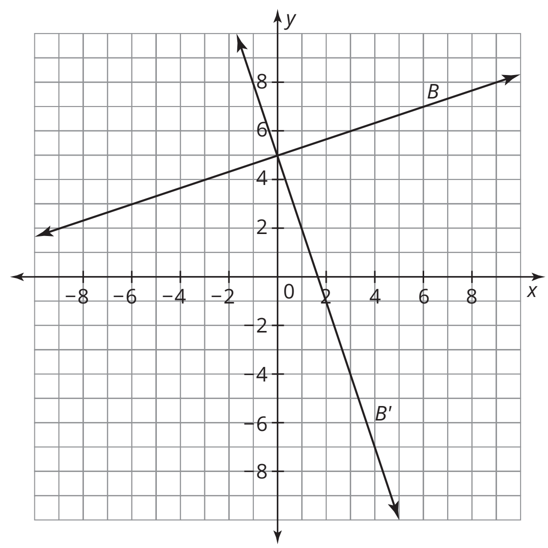
ACTIVITY
3.2

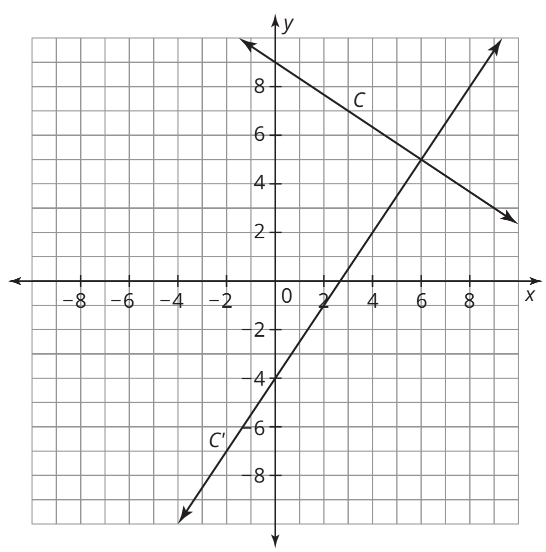
Slopes of Perpendicular Lines

Recall that perpendicular lines or line segments form a right angle at the point of intersection.

Consider the three graphs shown. Each shows a line and its rotation 90° about a point, which is also the point of intersection.







Remember:

The reciprocal of a number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. Because the product of a number and its reciprocal is one, reciprocal numbers are also known as multiplicative inverses.

1. **Are the lines in each graph perpendicular? Explain your reasoning.**

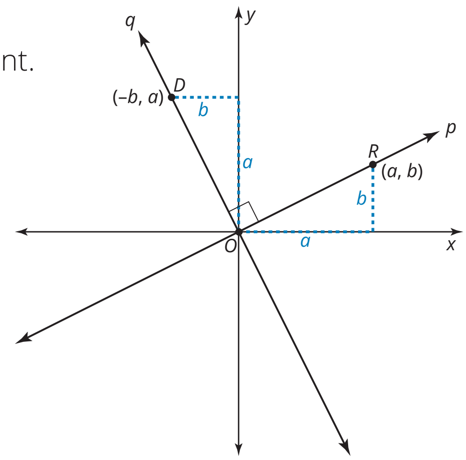
2. **Write the equation for each line and its transformation. What do you notice?**

It appears that if two lines are perpendicular, then their slopes are negative reciprocals. Let's investigate.

Worked Example

If two lines are perpendicular, then their slopes are negative reciprocals.

The graph shown can be used to analyze the validity of this statement.



Assumption: $p \perp q$

Let m_1 = slope of line p and let m_2 = slope of line q.

Point R lies on line p.

Conclusion: $m_1 = -\frac{1}{m_2}$

Perform a 90° counterclockwise rotation of point R using point O as the center of rotation. Since p and q are perpendicular, the image (point D) will lie on line q due to a 90° rotation.

Since this rotation maps the positive x-axis to the positive y-axis, and the positive y-axis to the negative x-axis, then the coordinates of R (a, b) are transformed into the coordinates of D $(-b, a)$. Graphically, you can follow the movement of lengths a and b under the rotation.

Using the graph, you can identify the slope of line p as $m_1 = \frac{b}{a}$, and the slope of line q as $m_2 = \frac{a}{-b}$.

Using these slopes, you can demonstrate that $m_1 = -\frac{1}{m_2}$.

$$\begin{aligned} \frac{b}{a} &= -\frac{1}{\frac{a}{-b}} \\ &= -1 \cdot \frac{-b}{a} \\ &= \frac{b}{a} \end{aligned}$$

The slope of line q is the negative reciprocal of the slope of line p.

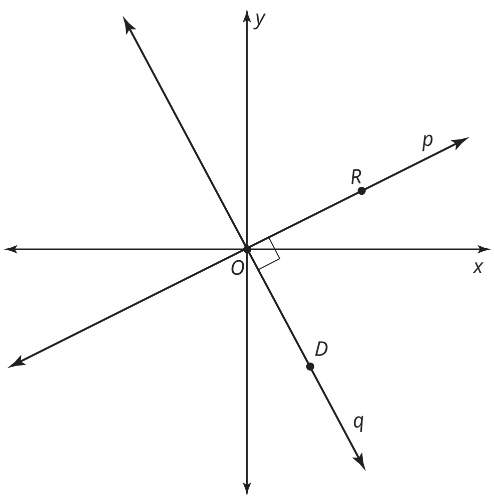
The product of the slopes of perpendicular lines is −1.

Remember:

The symbol ⊥ means *is perpendicular to.*

There is often more than one way to prove a theorem. Suppose that point *R* is rotated 90° clockwise using point *O* as the center of rotation.

3. Rewrite the assumption and conclusion using the clockwise rotation of point *R*.



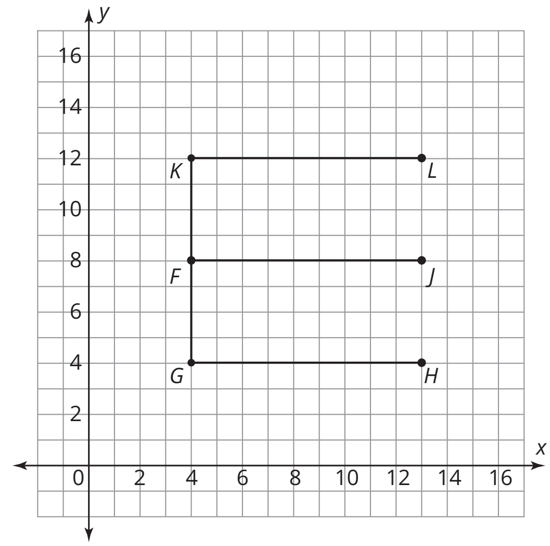
4. Line *j* and line *k* are perpendicular. Given each slope of line *j*, determine the slope of line *k*.

a. $m = \frac{2}{3}$ **b.** $m = -\frac{4}{5}$ **c.** $m = -3$

ACTIVITY **3.3**

Horizontal and Vertical Lines

Consider the graph shown.



1. Use a straightedge to extend $\overline{GK}$ to create line *p*, extend $\overline{GH}$ to create line *q*, extend $\overline{FJ}$ to create line *r*, and extend $\overline{KL}$ to create line *s*.

2. Consider the three horizontal lines you drew for Question 1. For any horizontal line, if *x* increases by one unit, by how many units does *y* change?

3. Describe the slope of any horizontal line. Explain your reasoning.

4. Consider the vertical line you drew in Question 1. Suppose that *y* increases by one unit. By how many units does *x* change?

5. Describe the slope of any vertical line. Explain your reasoning.

6. **Determine whether each of the given statements is always true, sometimes true, or never true. Explain your reasoning.**

   a. **All vertical lines are parallel.**

   b. **All horizontal lines are parallel.**

7. **Describe the relationship between any vertical line and any horizontal line.**

8. **Write an equation for a horizontal line and an equation for a vertical line that passes through the point (2, −1).**

9. **Write an equation for a line that is perpendicular to the line given by $x = 5$ and passes through the point (1, 0).**

10. **Write an equation for a line that is perpendicular to the line given by $y = -2$ and passes through the point (5, 6).**

11. **Write an equation for a line that is parallel to the line $y = 4$ and passes through the point (−1, 2).**

12. **Write an equation for a line that is parallel to the line $x = -3$ and passes through the point (7, 4).**

ACTIVITY **3.4**

Writing Equations of Parallel and Perpendicular Lines

You can write the equation of a parallel or perpendicular line using what you know about the slope of that line and any point on that line.

1. **Write the equation of the line perpendicular to $y = 2x + 1$ that passes through the point (6, 2).**

2. **Write the equation of a line that is parallel to $y = -3x - 1$ and passes through the point (−1, 5).**

3. **Write the equation of the line parallel to $y = \frac{1}{2}x$ that passes through the point (14, 2).**

4. **Write the equation of the line perpendicular to $y = -\frac{3}{4}x$ that passes through the point (3, −8).**

5. **Write the equation of the line that passes through the point (6, 2) and is perpendicular to a line that passes through the points (−5, 3) and (−1, −9).**

6. **Write the equation of a line that passes through the point (−2, 7) and is perpendicular to a line that passes through the points (−6, 1) and (0, 4).**

7. **Write the equation of a line that passes through the point (4, −6) and is parallel to a line that passes through the points (−2, 3.5) and (4, 5).**

8. **A pair of perpendicular lines intersect at the point (5, 9). Write the equation of the line that is perpendicular to the line that also passes through the point (−4, 4).**

Remember:

You can use point-slope form to write an equation for any line if you know its slope and one point on that line.

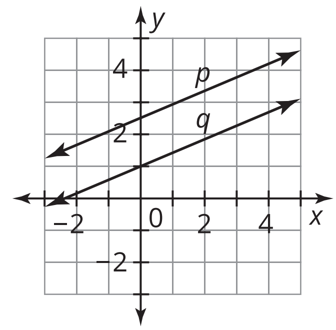
NOTES

TALK the TALK

Parallels the Lesson

Previously, you analyzed a worked example that demonstrates, "If two lines are perpendicular, then their slopes are negative reciprocals of each other."

1. Using similar reasoning, write an explanation that justifies, "If two lines are parallel, then their slopes are equal." Include a sketch.

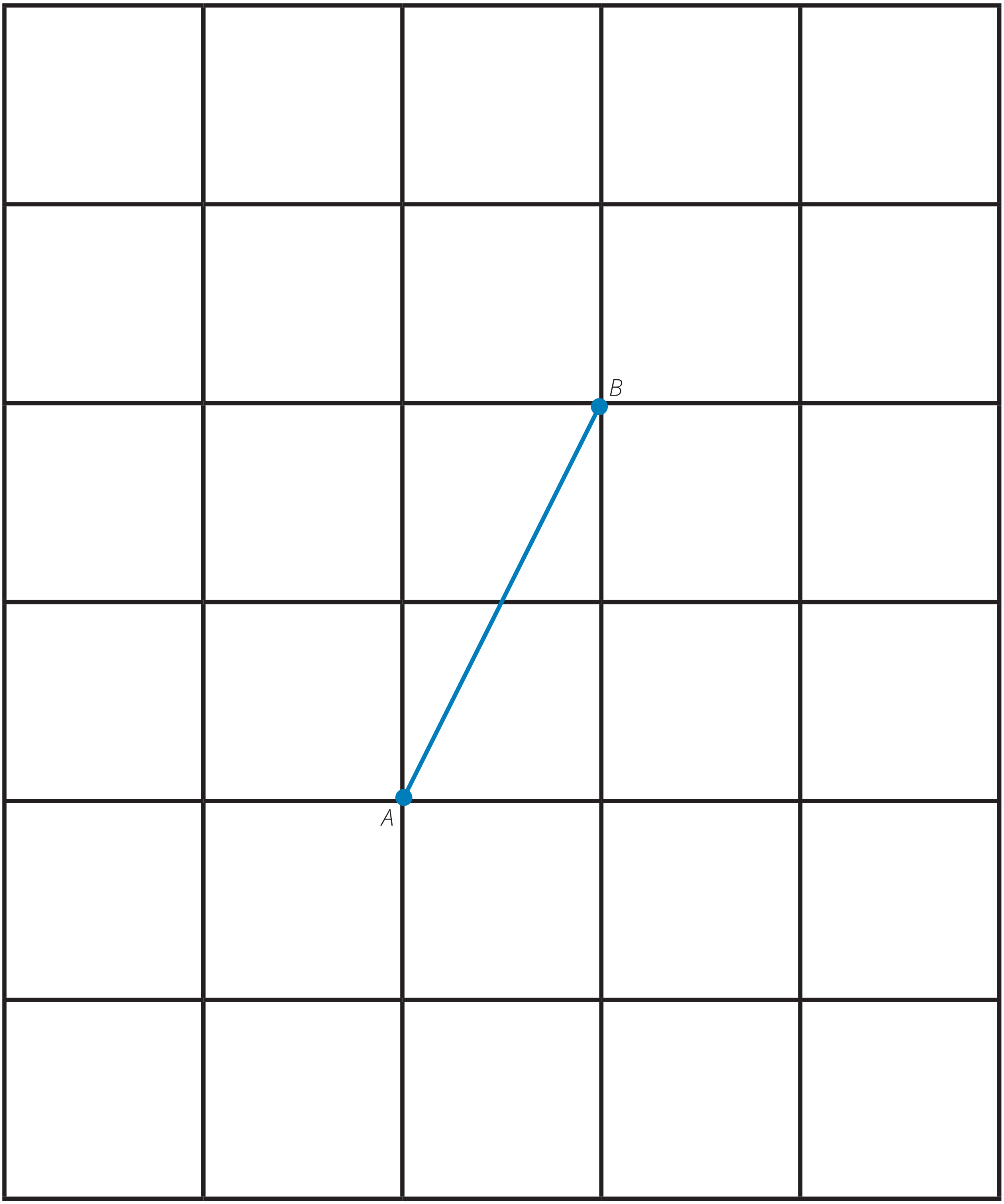


Assumption: $p \parallel q$

Let the slope of p **be** m_1.
Let the slope of q **be** m_2.

Conclusion: $m_1 = m_2$

Slope Grid A



Slope Grid B