3

Ts and Train Tracks

Parallel and Perpendicular Lines

MATERIALS

Patty paper
Straightedges

Lesson Overview

Students investigate segments on a coordinate grid and use patty paper to create parallel and perpendicular segments. They then construct parallel lines off the coordinate plane and graph parallel lines on the coordinate plane. Students identify perpendicular lines on the coordinate plane, use a rigid motion transformation to demonstrate that their slopes are negative reciprocals, and extend their understanding of perpendicular lines to include horizontal and vertical lines. They provide an explanation to demonstrate that if two lines are parallel, then their slopes are equal.

Geometry
Coordinate and Transformational Geometry

(2) The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point.

Logical Argument and Constructions

(5) The student uses constructions to validate conjectures about geometric figures. The student is expected to:

(A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools.
(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge.
(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C,4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- The 90° rotation of a line creates a line perpendicular to the original line.
- Perpendicular lines have slopes that are negative reciprocals of each other.
- The translation of a line creates an identical line or a line parallel to the original line.
- Parallel lines have equal slopes.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: All Aboard the Clue Train!

Students investigate line segments on coordinate grids. They use patty paper to create segments parallel and perpendicular to the original segments; they describe how to create each segment and relate the slope of the resulting segment with the slope of the original. Finally, students conjecture about the slopes of parallel and perpendicular lines.

Develop

Activity 3.1: Constructing Parallel Lines

Students construct a line parallel to a given line three different ways: constructing two lines perpendicular to the same line, constructing two congruent alternate interior angles, and using translations. Before using the two congruent alternate interior angles strategy, they analyze a worked example that demonstrates how to duplicate an angle using a compass and straightedge.

Day 2

Activity 3.2: Slopes of Perpendicular Lines

Students formalize what they recognized about the slopes of perpendicular lines in the Getting Started. They identify perpendicular lines on the coordinate plane and use a rigid motion transformation to demonstrate the negative reciprocal relationships of their slopes.

Activity 3.3: Horizontal and Vertical Lines

Students extend their understanding of parallel and perpendicular lines to include horizontal and vertical lines. They reason why the slopes of horizontal lines are zero and the slopes are vertical lines are undefined, relate their slopes to parallelism and perpendicularity, and write equations for lines parallel or perpendicular to horizontal or vertical lines through a given point.

Day 3

Activity 3.4: Writing Equations of Parallel and Perpendicular Lines

Students write the equation of a line parallel or perpendicular to a given line that passes through a given point.

Demonstrate

Talk the Talk: Parallels the Lesson

Students provide an explanation to demonstrate the statement, "If two lines are parallel, then their slopes are equal."

Facilitation Notes

In this activity, students investigate line segments on coordinate grids. They use patty paper to create segments parallel and perpendicular to the original segments; they describe how to create each segment and relate the slope of the resulting segment with the slope of the original. Finally, students conjecture about the slopes of parallel and perpendicular lines.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- The orientation of the grid. There are no axes on the grid, and students may rotate the grid and get a different slope than their classmates. While they can come to the same conclusion at either orientation, you may want to remind students to hold the grid so that the title is at the top of the page.
- Whether students recognize the difference between translating a segment and rotating a segment. To create parallel segments, they should be translating, or sliding, their patty paper. To create perpendicular segments, they should be rotating, or spinning, their patty paper.
- Where they align their perpendicular segment. Although there is a segment perpendicular to any point along $\overline{AB}$, it is easier to verify that a segment is perpendicular using the grid when the segments intersect at an endpoint. Extending $\overline{AB}$ may make this easier to visualize and verify the relationship.

Questions to ask

- How did you move the patty paper to create a parallel segment? A perpendicular segment?
- How is the movement needed to create a perpendicular segment different from the movement needed to create a parallel segment?
- How can you use the grid to determine whether two segments are parallel?
- How can you use the grid to determine whether two segments are perpendicular?
- What is the sign of the slope of $\overline{AB}$?
- What is the sign of the slope of the segment parallel to $\overline{AB}$? Perpendicular to $\overline{AB}$?
- Did everyone in your class create the same parallel segment? How do the slopes compare?
- Did everyone in your class create the same perpendicular segment? How do the slopes compare?

Have students work with a partner or in a group to complete Questions 4 through 7. Share responses as a class.

Misconception

Because the grid isn't numbered, students may not consider the direction of the line when identifying the slopes of parallel and perpendicular lines. Remind students that the slope of a line indicates both steepness and direction.

Questions to ask

- How is the movement of the patty paper different to create a segment perpendicular to $\overline{CD}$ versus $\overline{AB}$?
- Are the signs of the slopes the same for $\overline{CD}$ and the segment parallel to $\overline{CD}$?
- Are the signs of the slopes the same for $\overline{CD}$ and the segment perpendicular to $\overline{CD}$?
- How does your conjecture compare with your classmates'?
- How can you show that your conjecture is true in more cases?

Summary

Translations can create parallel lines with the same slope, while rotations of 90° create perpendicular lines with negative reciprocal slopes.

DEVELOP

Activity 3.1
Constructing Parallel Lines

Facilitation Notes

In this activity, students construct a line parallel to a given line three different ways: constructing two lines perpendicular to the same line, constructing two congruent alternate interior angles, and using translations. Before using the two congruent alternate interior angles strategy, they analyze a worked example that demonstrates how to duplicate an angle using a compass and straightedge.

Ask a student to read the introduction aloud. Discuss as a class.

Questions to ask

- What notation is used to show that the lines are parallel?
- Name a different pair of corresponding angles.
- Identify a pair of congruent angles. How do you know they are congruent?
- What other angle relationships do you recognize?

- If the lines are parallel is that enough information to conclude the pairs of corresponding angles are congruent?
- If pairs of corresponding angles are congruent, is that enough information to conclude the lines are parallel?

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategies

- To scaffold support, suggest they use colored pencils to organize their thinking as they complete the constructions. Have them use the same color for parallel lines and a different color for perpendicular lines.
- To assist all students, remind them to use notation to identify the parallel lines in their figures.

As students work, look for

- The use of other angle pairs noted at the beginning of the activity to recognize that the lines are parallel.
- Extension of $\overline{AE}$ and $\overline{DB}$ to create a diagram with 8 angles similar to the given diagram.
- The use of patty paper or a compass and straightedge.

Questions to ask

- What is the relationship between $\overline{AE}$ and $\overline{AD}$?
- What is the relationship between $\overline{DB}$ and $\overline{AD}$?
- What is the relationship between two lines or line segments perpendicular to the same line?
- Why did you begin by constructing a line perpendicular to line m?
- How did you construct the translated line?
- How many lines can be drawn through point A parallel to line m?

Misconception

Students may be confused by the fact that there is only one line perpendicular to a given line through a given point; however, there is an infinite number of lines perpendicular to a given line.

Differentiation strategy

To extend the activity, ask students to prove why two lines perpendicular to the same line must be parallel to one each other.

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

Differentiation strategies

- To support students who struggle, allow them to construct parallel lines using Gage's method without having to use the starter line q.

- To assist all students,
  - Discuss that Gage is using the fact that congruent alternate interior angles are formed when parallel lines are intersected by a transversal. Discuss Gage's method of using the converse of that theorem.
  - To help them write concise explanations of how they performed constructions, suggest that they label points of intersection in their diagrams. That way, they can refer to locations by the letter of a point rather than a description of the location of the point.
- To extend the activity, have students construct parallel line segments using congruent alternate exterior angles and/or corresponding angles.

As students work, look for

Missing construction arcs. Students may forget to make the first arc to "standardize" the spread of the angle at a fixed distance from the vertex. If so, explain the importance of this step.

Questions to ask

- What angle relationship did Gage use to construct parallel lines?
- What line do you think Gage drew first? Why?
- How could Gage construct both angles to be the same measure?
- Describe Gage's steps to complete his construction.
- How is the method of using perpendicular lines a special case of Gage's method?
- When duplicating an angle, what is the first step?
- How did you decide the initial setting for the compass? Does it make a difference?
- Does the compass setting have to be the same for the arc on the original angle and your construction?
- Where did you place the compass point to create the first arc?
- Why do you think you need to construct that first arc?
- Did you need to change the setting for the compass in the next step? Why?
- Why is the new compass setting so important?
- Will everyone use the same setting? Why not?
- To create the other arc, where do you place the compass point?

Have students work with a partner or in a group to complete Question 6. Share responses as a class.

Questions to ask

- How many units did you translate $y = x$?
- Did you translate $y = x$ horizontally or vertically?

- What information did you use to write the equation of your line?
- Did you use the y-intercept to write the equation of each line?

Differentiation strategy

To extend the activity, discuss the relationship between translations and the y-intercepts of each line.

Summary

Parallel lines can be constructed using translations. Parallel lines have the same slope.

Activity 3.2
Slopes of Perpendicular Lines

Facilitation Notes

In this activity, students formalize what they recognized about the slopes of perpendicular lines in the Getting Started. They identify perpendicular lines on the coordinate plane and use a rigid motion transformation to demonstrate the negative reciprocal relationships of their slopes.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

- The equation of each line written from the graph and a comparison of their equations.
- The equation of the first line written from the graph and the second line written by taking the negative reciprocal of the first line for its slope.
- Failure to recognize that the slopes are negative reciprocals because the fractional slopes are not simplified.

Questions to ask

- How did you determine the equation of the first line? The line perpendicular to it?
- How can knowing the lines are perpendicular help you to identify the slope of one line if you know the slope of the other line?
- How did you determine the y-intercept of each line?
- Is there any relationship between the y-intercepts of perpendicular lines?
- Why do most perpendicular lines have to have one slope that is positive and one slope that is negative?

Analyze the worked example as a class. Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

Misconception

Students may be confused by the $(-b, a)$ label for point D and the positive b-value on the reference triangle. Remind students that b is the horizontal distance from point D to the y-axis, and distance is always a positive value. The coordinate pair represents the location of the point in Quadrant II.

Questions to ask

- What is meant by *negative reciprocals*?
- Provide an example of negative reciprocals.
- What is the product of reciprocals? What is the product of negative reciprocals?
- Why does it make sense to say that the product of the slopes of perpendicular lines is -1?
- Explain why point D has the coordinates $(-b, a)$.
- What is meant by $m_1 = -\frac{1}{m_2}$?
- Explain how the line following $m_1 = -\frac{1}{m_2}$ was determined.
- Does it matter that the negative sign is placed with the b instead of in front of the fraction? Why do you think it is written this way?
- Explain the steps to simplify the complex fraction.
- How does rewriting the complex fraction to get $\frac{b}{a}$ support the explanation?
- What steps did you use to identify the slope of line k?

Summary

If two lines are perpendicular, their slopes are negative reciprocals.

Activity 3.3
Horizontal and Vertical Lines

Facilitation Notes

In this activity, students extend their understanding of parallel and perpendicular lines to include horizontal and vertical lines. They reason why the slopes of horizontal lines are zero and the slopes are vertical lines are undefined, relate their slopes to parallelism and perpendicularity, and write equations for lines parallel and perpendicular to horizontal or vertical lines through a given point.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

As students work, look for

- Reasoning using the graphs of the lines. Students may mention that because horizontal lines are flat, their slope is zero.
- Explanations using $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Differentiation strategy

To scaffold support, suggest they select points on the lines and substitute them in the slope formula to make sense of the slopes of horizontal and vertical lines.

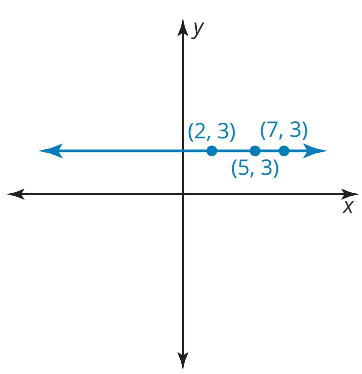
Questions to ask

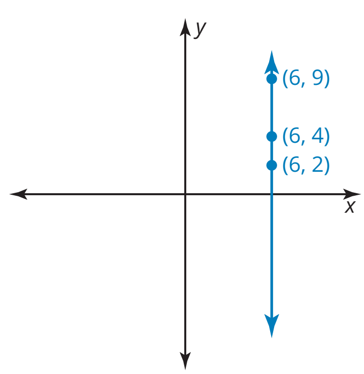
- How would you describe the relationship between the three lines?
- What do you know about the slopes of the three lines?
- Is it possible to divide zero by a number?
- What is zero divided by any number equal to?
- Is it possible to divide a number by zero?
- What is a number divided by zero equal to?
- What is the form in which all vertical lines are written?
- What is the form in which all horizontal lines are written?

Have students work with a partner or in a group to complete Questions 6 through 12. Share responses as a class.

Misconception

Students often confuse the fact that the equation for a horizontal line is $y = a$ and the equation for a vertical line is $x = a$. Suggest a method that students can use to construct the information they need rather than relying on memorization.

<table>
<tr>
<td>Sketch a horizontal line and label a few points.


In all cases, $y = 3$.
Therefore, for horizontal lines, use $y = a$, where a is any real number.</td>
<td>Sketch a vertical line and label a few points.


In all cases, $x = 6$.
Therefore, for vertical lines, use $x = a$, where a is any real number.</td>
</tr>
</table>

Questions to ask

- How does making a sketch with the given information help you to write your equation?
- How do you know whether your equation should begin with $x =$ or $y =$?
- How can you write the equation without using slope-intercept form or point-slope form of a line?
- How do you know what value from the coordinate pair to use in your equation?
- Demonstrate how you can use slope-intercept form or point-slope form to write an equation for a horizontal line.
- Why doesn't using slope-intercept form or point-slope form of a line work for vertical lines?

Summary

Any vertical line is perpendicular to any horizontal line.

Activity 3.4

Writing Equations of Parallel and Perpendicular Lines

Facilitation Notes

In this activity, students write the equation of a line parallel or perpendicular to a given line that passes through a given point.

Have students work with a partner or in a group to complete Questions 1 through 8. Share responses as a class.

Differentiation strategies

- To scaffold support, modify Questions 5 through 8 to provide the same information as Questions 1 through 4.
- To assist all students,
  - Suggest they label their steps with notation such as "$m =$" and "$\perp m =$" for organization of their work.
  - Encourage the use of sketches to make sense of Questions 5 through 8.

As students work, look for

- Errors identifying the perpendicular slope, mainly due to forgetting to take the opposite sign of the original slope.
- Unnecessary steps when solving Questions 5 through 8, such as determining the equation for the line through the two given points, rather than just calculating the slope. If this occurs, discuss the more efficient method with students.
- Use of the slope formula to write the equation of the line.

Questions to ask

- How did you know what slope to use in your equation?
- How did you use the point the line passes through to help write your equation?
- Explain how you used the slope formula to write the equation of the line.
- How can you check that your answer is correct?
- In Questions 5 through 8, how did you determine the slope to use in your equation without being given a slope?
- In Question 8, how was knowing the point of intersection helpful?
- How is the information given in Question 8 similar to the information given in Questions 5 through 7?

Summary

To write the equation of a line perpendicular to a given line, it is necessary to use the negative reciprocal relationship between their slopes. To write an equation of a line parallel to a given line, it is necessary to use the same slope relationship of parallel lines.

Talk the Talk: Parallels the Lesson

DEMONSTRATE

Facilitation Notes

In this activity, students provide an explanation to demonstrate the statement, "If two lines are parallel, then their slopes are equal."

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Differentiation strategy

To scaffold support, suggest they locate two points on line q to determine the hypotenuse of a slope triangle.

As students work, look for

Reference to the worked example in Activity 3.2 when writing their explanation.

Questions to ask

- What is the slope of each line?
- How did you determine the slope?
- Which transformation was used to demonstrate perpendicular lines and their negative reciprocal slopes?
- Is a rotation helpful in this situation? Why not?
- Which transformation does the orientation of the lines on the graph suggest?

- Describe the translation applied to line *q* to create line *p*?
- Did you include slope triangles in your explanation? If so, how?
- What two points on line *q* did you use to create a slope triangle?
- How does knowing translations preserve size and shape support your explanation?

Summary

Translations can be used to demonstrate the statement, “If two lines are parallel, then their slopes are equal.”

3

Ts and Train Tracks

Parallel and Perpendicular Lines

Warm Up

Determine the reciprocal of each value.

1. 3
2. -10
3. $\frac{1}{5}$
4. $-c$
5. $\frac{a}{b}$, $b \neq 0$

Learning Goals

- Construct parallel lines.
- Identify and write the equations of lines perpendicular to given lines.
- Identify and write the equations of parallel lines, including horizontal and vertical lines.

You have constructed line segments, perpendicular lines, squares, and a coordinate plane. How can the coordinate plane be used to justify parallel and perpendicular line relationships?

Warm Up Answers

1. $\frac{1}{3}$
2. $-\frac{1}{10}$
3. 5
4. $-\frac{1}{c}$, $c \neq 0$
5. $\frac{b}{a}$, $a \neq 0$

Answers

1. 2

2a. Sample answer.
I translated the segment 1 unit to the left.

2b. Sample answer.
Both segments lie on lines that are increasing at the same rate. They will never intersect.

2c. 2

3a. Sample answer.
I rotated the segment 90° clockwise.

3b. Sample answer.
They appear to meet at a right angle.
The segment of the perpendicular segment is decreasing at a rate of $\frac{1}{2}$.

3c. $-\frac{1}{2}$

GETTING STARTED

All Aboard the Clue Train!

You have created lines and shapes by translating, reflecting, and rotating squares on the coordinate plane. Let's explore the slopes of line segments constructed using coordinate plane squares.

Let's consider $\overline{AB}$ on Slope Grid A located at the end of the lesson.

1. What is the slope of $\overline{AB}$?

Remember:

The slope of a line indicates both steepness and direction.

2. Consider how to create a segment parallel to $\overline{AB}$.

a. Trace $\overline{AB}$ onto a piece of patty paper. Then move the segment on the patty paper to create a segment parallel to $\overline{AB}$. Describe your movements.

b. How do you know that these segments are parallel?

c. What is the slope of the segment parallel to $\overline{AB}$?

3. Consider how to create a segment perpendicular to $\overline{AB}$.

a. Move the segment on the patty paper to create a segment perpendicular to $\overline{AB}$. Describe your movements.

b. How do you know that these segments are perpendicular?

c. What is the slope of the segment perpendicular to $\overline{AB}$?

Now let's consider $\overline{CD}$ on Slope Grid B located at the end of the lesson.

4. What is the slope of $\overline{CD}$?

5. Use patty paper to create a segment parallel to $\overline{CD}$.

a. Describe how you know that the segments are parallel.

b. Identify the slope of the segment parallel to $\overline{CD}$.

6. Use patty paper to create a segment perpendicular to $\overline{CD}$.

a. Describe how you know that the segments are perpendicular.

b. Identify the slope of the segment perpendicular to $\overline{CD}$.

7. Use your investigation to write a conjecture about the slopes of parallel and perpendicular lines.

Answers

4. $\frac{1}{2}$

5a. Sample answer. Both segments lie on lines that are increasing at the same rate. They will never intersect.

5b. $\frac{1}{2}$

6a. Sample answer. They appear to meet at a right angle. The segment of the perpendicular segment is decreasing at a rate of -2.

6b. -2

7. Sample answer. Parallel lines have the same slope. Perpendicular lines have slopes that are negative reciprocals of each other.

Answer

1. Sample answer. $\overline{DB}$ is a translation of $\overline{AE}$. The angle measures are preserved.

ACTIVITY 3.1

Constructing Parallel Lines

In the previous lesson, you constructed perpendicular lines. You can also construct parallel lines.

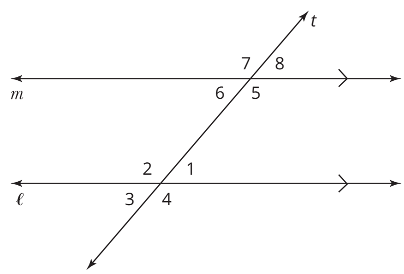
One strategy used to construct parallel lines is translation. The image of a line that has been translated is either the same line or a parallel line.

If line *m* is a translation of line ℓ, then the two lines are parallel. Recall that when parallel lines are cut by a transversal, several pairs of congruent angles are formed.

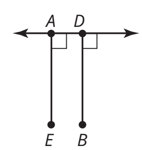
Corresponding angles:
$\angle 8 \cong \angle 1$

Alternate interior angles:
$\angle 1 \cong \angle 6$

Alternate exterior angles:
$\angle 4 \cong \angle 7$

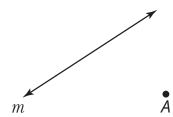


Consider $\overline{AE} \perp$ to $\overleftrightarrow{AD}$ and $\overline{DB} \perp$ to $\overleftrightarrow{AD}$.



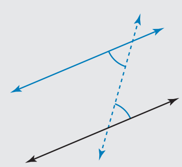
1. Is there enough information to conclude that the two segments are parallel? Explain your reasoning.

2. Use your reasoning from Question 1 to construct a line parallel to line *m* through point *A*. Describe your process.



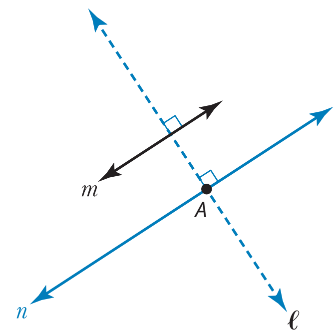
3. Explain why Gage is correct.

> **Gage**
> You can duplicate any angle measure, not just right angles, to construct parallel lines.



Answers

2. Check students' constructions.
   I constructed a line perpendicular to line *m*, labeled line *ℓ*. Then I can translate line *m* along line *ℓ* to point *A* to create line *n*. Line *n* is parallel to line *m*.



3. Gage reasoned that if alternate interior angles are congruent when two lines are cut by a transversal, then the lines are parallel.

Answer

4. Check students' diagrams.

To use Gage's method, you need to know how to use construction tools to duplicate an angle.

Worked Example

Construct a Starter Line

Use a straightedge to construct a starter line. Label point *C* on the new line.

Construct an Arc

Construct an arc with center *A*. Using the same radius, construct an arc with center *C*.

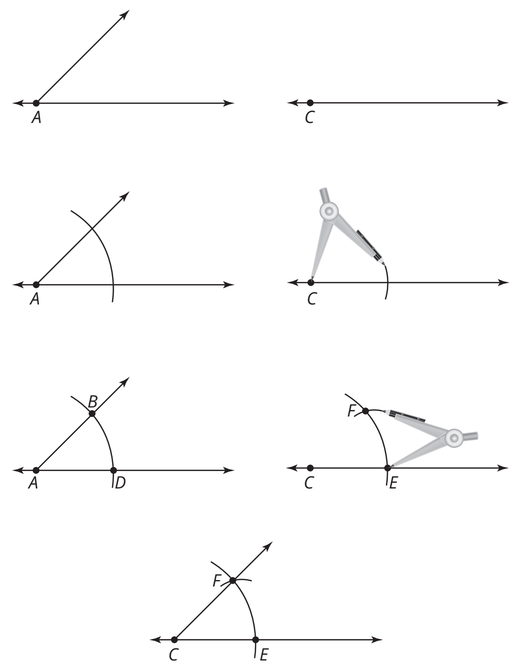
Construct Another Arc

Label points *B*, *D*, and *E*. Construct an arc with a radius the length of *BD* and center *E*. Label the intersection *F*.

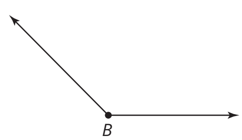
Construct a Ray

Construct ray *CF*.
$\angle BAD \cong \angle FCE$

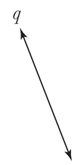
Given:



4. Duplicate angle *B*. Verify with patty paper.



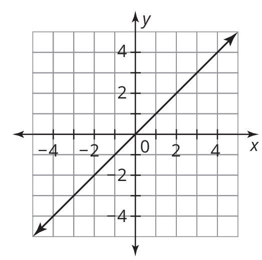
5. Now that you know how to duplicate an angle, use Gage's method to construct a line parallel to line q.



6. Consider the line $y = x$ on the coordinate plane shown.

a. Translate the line $y = x$ to create a parallel line.

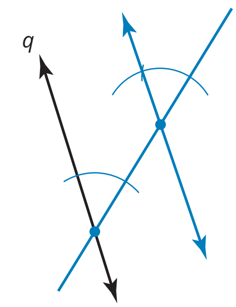
b. Write the equation of your line.



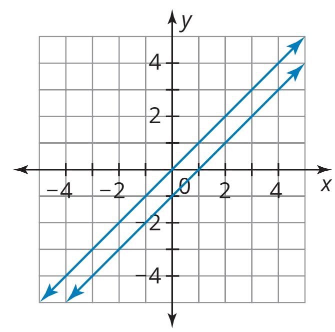
c. Describe the slopes of parallel lines on the coordinate plane.

Answers

5. Check students' constructions.



6a. Sample answer.



6b. Sample answer. $y = x - 1$

6c. The slopes are equal values.

Answers

1. Yes. Each line is rotated 90°, so the angle formed by the intersection of the lines measures 90°, which is a right angle. When two lines form right angles, they are perpendicular.
2. $A: y = x$

   $A': y = -x$

   $B: y = \frac{1}{3}x + 5$

   $B': y = -3x + 5$

   $C: y = -\frac{2}{3}x + 9$

   $C': y = \frac{3}{2}x - 4$

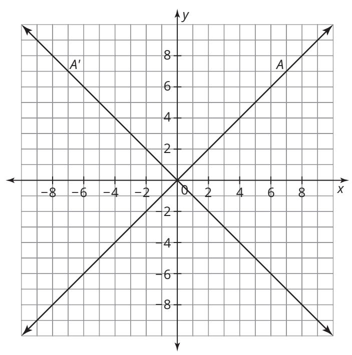
   The slopes of each pair of perpendicular lines are reciprocals of each other with opposite signs.

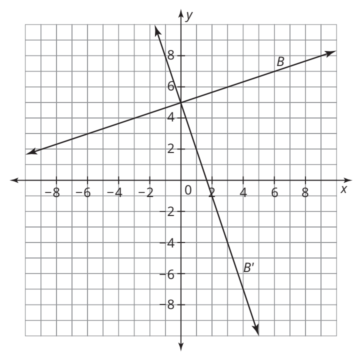
ACTIVITY **3.2**

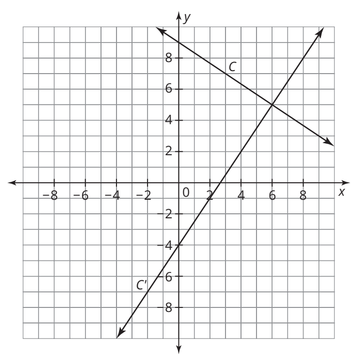
Slopes of Perpendicular Lines

Recall that perpendicular lines or line segments form a right angle at the point of intersection.

Consider the three graphs shown. Each shows a line and its rotation 90° about a point, which is also the point of intersection.







Remember:

The reciprocal of a number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. Because the product of a number and its reciprocal is one, reciprocal numbers are also known as multiplicative inverses.

1. Are the lines in each graph perpendicular? Explain your reasoning.

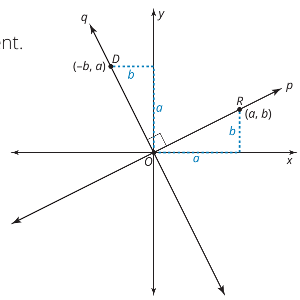
2. Write the equation for each line and its transformation. What do you notice?

It appears that if two lines are perpendicular, then their slopes are negative reciprocals. Let's investigate.

Worked Example

If two lines are perpendicular, then their slopes are negative reciprocals.

The graph shown can be used to analyze the validity of this statement.



Assumption: $p \perp q$

Let m_1 = slope of line p and let m_2 = slope of line q.

Point R lies on line p.

Conclusion: $m_1 = -\frac{1}{m_2}$

Perform a 90° counterclockwise rotation of point R using point O as the center of rotation. Since p and q are perpendicular, the image (point D) will lie on line q due to a 90° rotation.

Since this rotation maps the positive x-axis to the positive y-axis, and the positive y-axis to the negative x-axis, then the coordinates of R (a, b) are transformed into the coordinates of D $(-b, a)$. Graphically, you can follow the movement of lengths a and b under the rotation.

Using the graph, you can identify the slope of line p as $m_1 = \frac{b}{a}$, and the slope of line q as $m_2 = \frac{a}{-b}$.

Using these slopes, you can demonstrate that $m_1 = -\frac{1}{m_2}$.

$$\begin{aligned} \frac{b}{a} &= -\frac{1}{\frac{a}{-b}} \\ &= -1 \cdot \frac{-b}{a} \\ &= \frac{b}{a} \end{aligned}$$

The slope of line q is the negative reciprocal of the slope of line p.

The product of the slopes of perpendicular lines is −1.

Remember:

The symbol ⊥ means *is perpendicular to.*

Answers

3. Since the rotation of point R clockwise maps onto line q as $D(-b, a)$, the slope of line p is $m_1 = \frac{b}{a}$ and the slope of line q is $m_2 = \frac{-a}{b}$.

   Using these slopes, I can demonstrate that $m_1 = -\frac{1}{m_2}$.

   $$\frac{b}{a} = \frac{-1}{\frac{-a}{b}}$$
   $$= -1 \cdot \frac{b}{-a}$$
   $$= \frac{b}{a}$$

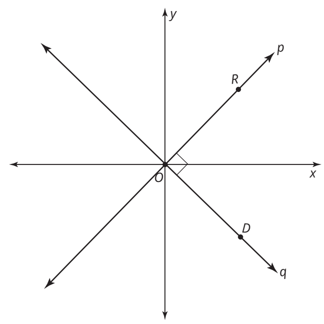
4a. For line k, $m = -\frac{3}{2}$.

4b. For line k, $m = \frac{5}{4}$.

4c. For line k, $m = \frac{1}{3}$.

There is often more than one way to prove a theorem. Suppose that point R is rotated 90° clockwise using point O as the center of rotation.

3. Rewrite the assumption and conclusion using the clockwise rotation of point *R*.

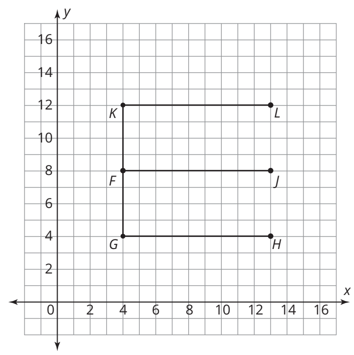


4. Line *j* and line *k* are perpendicular. Given each slope of line *j*, determine the slope of line *k*.

a. $m = \frac{2}{3}$ **b.** $m = -\frac{4}{5}$ **c.** $m = -3$

ACTIVITY
3.3
Horizontal and Vertical Lines

Consider the graph shown.



1. Use a straightedge to extend $\overline{GK}$ to create line *p*, extend $\overline{GH}$ to create line *q*, extend $\overline{FJ}$ to create line *r*, and extend $\overline{KL}$ to create line *s*.

2. Consider the three horizontal lines you drew for Question 1. For any horizontal line, if *x* increases by one unit, by how many units does *y* change?

3. Describe the slope of any horizontal line. Explain your reasoning.

4. Consider the vertical line you drew in Question 1. Suppose that *y* increases by one unit. By how many units does *x* change?

5. Describe the slope of any vertical line. Explain your reasoning.

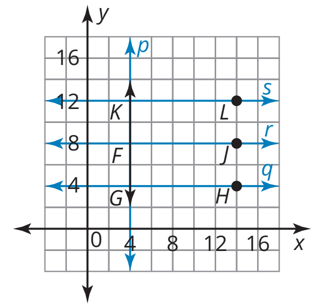
LESSON 3: Ts and Train Tracks • 11

ELL Tip

Determine whether students are familiar with the term *extend*. If not, state the two definitions of *extend* as *to make longer or wider*, and *to hold something out toward someone*. Discuss real-life examples of the term *extend*, such as *extending* a roadway, *extending* a deadline, *extending* the range of acceptable answers on a test, and *extending* a hand for someone to shake. Ensure students' understanding of the context of *extend* in Question 1, as *to make the given line segment longer.*

Answers

1.



2. The value of *y* does not change.
3. The slope of any horizontal line is zero because as *x* increases, the value of *y* stays the same, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{x_2 - x_1} = 0$.
4. The value of *x* does not change.
5. The slope of any vertical line is undefined because as *y* increases, the value of *x* stays the same. $x_2 - x_1 = 0$, so $m = \frac{y_2 - y_1}{x_2 - x_1}$ is undefined.

Answers

6a. always true;
The slopes of all vertical lines are undefined, and therefore the same. So, all vertical lines are parallel.

6b. always true;
The slopes of all horizontal lines are zero, and therefore the same. So, all horizontal lines are parallel.

7. Any vertical line and any horizontal line are perpendicular to each other.

8. Horizontal line: $y = -1$
Vertical line: $x = 2$

9. $y = 0$

10. $x = 5$

11. $y = 2$

12. $x = 7$

6. Determine whether each of the given statements is always true, sometimes true, or never true. Explain your reasoning.

a. All vertical lines are parallel.

b. All horizontal lines are parallel.

7. Describe the relationship between any vertical line and any horizontal line.

8. Write an equation for a horizontal line and an equation for a vertical line that passes through the point (2, −1).

9. Write an equation for a line that is perpendicular to the line given by $x = 5$ and passes through the point (1, 0).

10. Write an equation for a line that is perpendicular to the line given by $y = -2$ and passes through the point (5, 6).

11. Write an equation for a line that is parallel to the line $y = 4$ and passes through the point (−1, 2).

12. Write an equation for a line that is parallel to the line $x = -3$ and passes through the point (7, 4).

12 • TOPIC 1: Using a Rectangular Coordinate System

ACTIVITY 3.4

Writing Equations of Parallel and Perpendicular Lines

You can write the equation of a parallel or perpendicular line using what you know about the slope of that line and any point on that line.

1. Write the equation of the line perpendicular to $y = 2x + 1$ that passes through the point (6, 2).

2. Write the equation of a line that is parallel to $y = -3x - 1$ and passes through the point (−1, 5).

3. Write the equation of the line parallel to $y = \frac{1}{2}x$ that passes through the point (14, 2).

4. Write the equation of the line perpendicular to $y = -\frac{3}{4}x$ that passes through the point (3, −8).

5. Write the equation of the line that passes through the point (6, 2) and is perpendicular to a line that passes through the points (−5, 3) and (−1, −9).

6. Write the equation of a line that passes through the point (−2, 7) and is perpendicular to a line that passes through the points (−6, 1) and (0, 4).

7. Write the equation of a line that passes through the point (4, −6) and is parallel to a line that passes through the points (−2, 3.5) and (4, 5).

8. A pair of perpendicular lines intersect at the point (5, 9). Write the equation of the line that is perpendicular to the line that also passes through the point (−4, 4).

Remember:

You can use point-slope form to write an equation for any line if you know its slope and one point on that line.

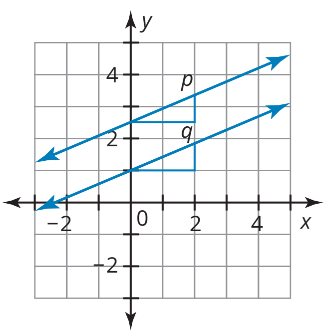
LESSON 3: Ts and Train Tracks • 13

Answers

1. $y = -\frac{1}{2}x + 5$
2. $y = -3x + 2$
3. $y = \frac{1}{2}x - 5$
4. $y = \frac{4}{3}x - 12$
5. $y = \frac{1}{3}x$
6. $y = -2x + 3$
7. $y = \frac{1}{4}x - 7$
8. $y = -\frac{9}{5}x + 18$

Answer

1. Sample answer.
 A translation of line q produces line p.
 A right triangle can be constructed on line q as shown to represent the slope of line q, m_2.
 When line q is translated to produce line p, this triangle is also translated. Since translations preserve size, shape, and orientation, line p must have the same slope as line q ($m_1 = m_2$), because the translated right triangle is congruent to the original.



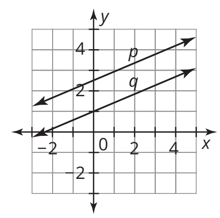
NOTES

TALK the TALK

Parallels the Lesson

Previously, you analyzed a worked example that demonstrates, "If two lines are perpendicular, then their slopes are negative reciprocals of each other."

1. Using similar reasoning, write an explanation that justifies, "If two lines are parallel, then their slopes are equal." Include a sketch.

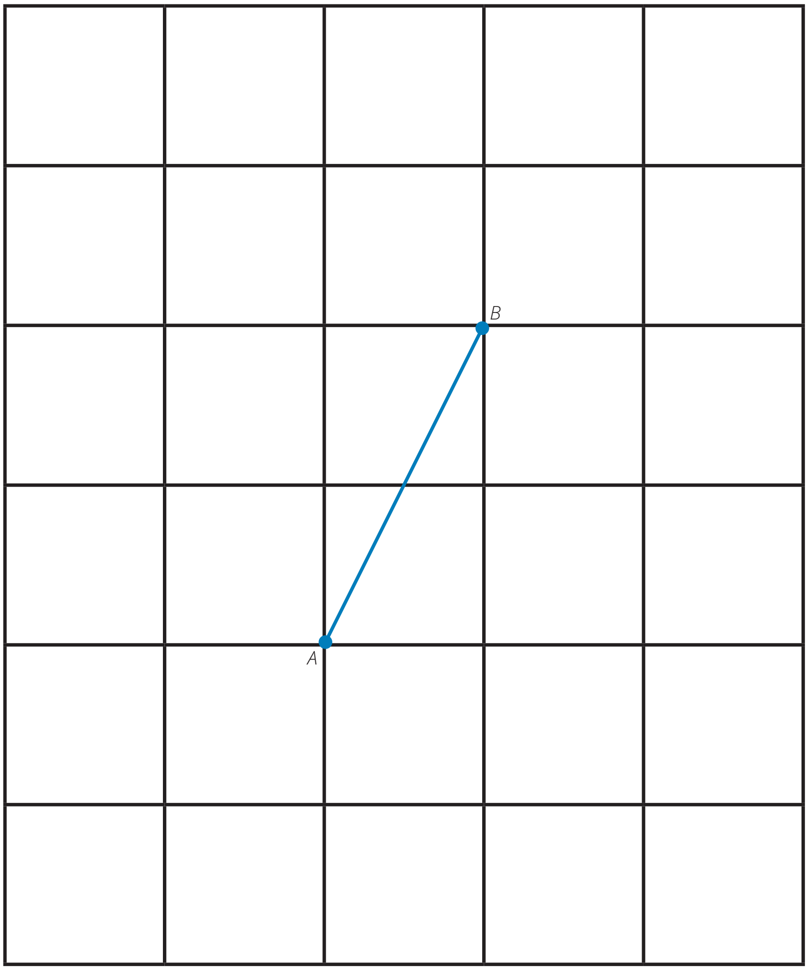


Assumption: $p \parallel q$

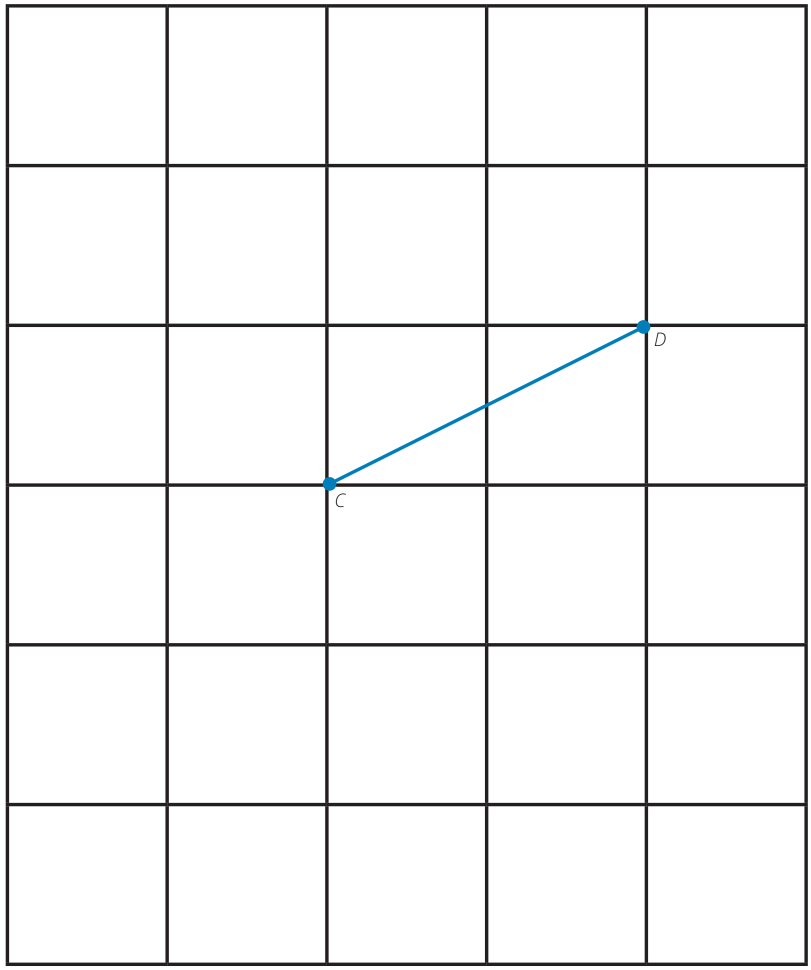
Let the slope of p be m_1.
Let the slope of q be m_2.

Conclusion: $m_1 = m_2$

Slope Grid A



Slope Grid B



16 • TOPIC 1: Using a Rectangular Coordinate System