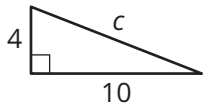
4

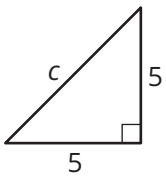
Where Has Polly Gone?

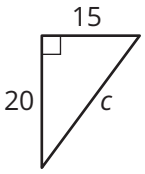
Classifying Shapes on the Coordinate Plane

Warm Up

Determine the length of each hypotenuse. Round your answer to the nearest tenth, if necessary.

1. 

2. 

3. 

Learning Goals

- Use the Pythagorean Theorem to derive the Distance Formula.
- Apply the Distance Formula on the coordinate plane.
- Classify a triangle given the locations of its vertices on a coordinate plane.
- Determine the coordinates of a fourth vertex, given the coordinates of three vertices of a quadrilateral and a description of the quadrilateral.
- Classify a quadrilateral given the locations of its vertices on a coordinate plane.

Key Terms

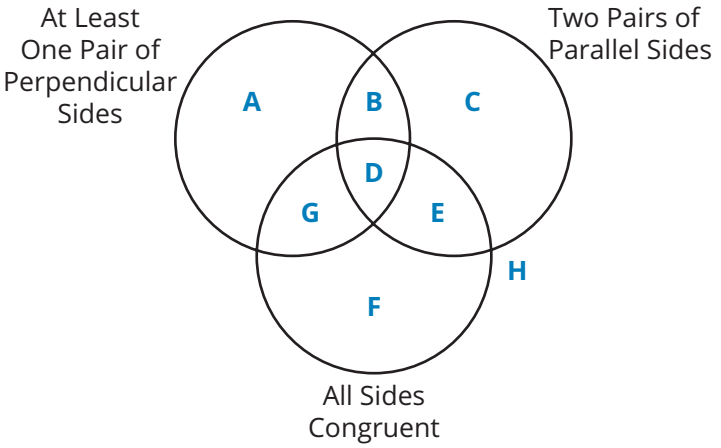
- Distance Formula
- Midpoint Formula

You know the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals. You also know how to determine the length of the hypotenuse of a right triangle. How can you use what you know to classify polygons that lie on a coordinate plane?

You Better Shape Up

Polygons are often classified by properties, such as the lengths of their sides, the relationships between their sides, and the measures of their angles.

The Venn diagram contains three circles each representing a different property. Letters A through H represent any polygon that has the property described by every circle in which it appears.



Remember:

Congruent means to have the same size, shape, and measure.

A letter may be used more than once or not at all!

1. **Match each polygon to one of the lettered regions of the Venn diagram. Write the corresponding letter next to each polygon.**

trapezoid	**square**	**isosceles triangle**
parallelogram	**rhombus**	**equilateral triangle**
rectangle	**right triangle**	**scalene triangle**

2. **Is there a region that cannot be matched to one of the polygons? Explain your reasoning.**

Remember:

A trapezoid has at least one pair of parallel sides.

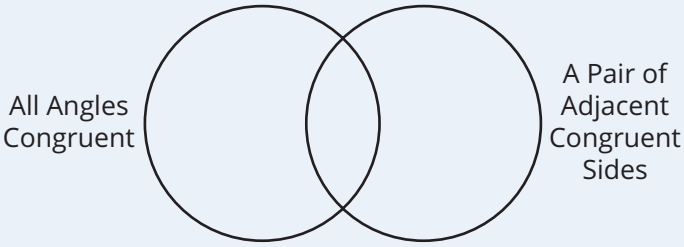
An isosceles triangle has at least two congruent sides.

3. **Use the Venn diagram to compare the properties of each pair of polygons.**

   a. **parallelogram and rhombus**

   b. **rectangle and square**

4. **Marla and Flynn analyze the Venn diagram shown.**



Marla says that the overlapping region describes a rhombus. Flynn says the overlapping region describes a square. Who's correct? Explain your reasoning.

Adjacent sides are sides that share a vertex.

5. **Determine whether each statement is always true, sometimes true, or never true. Explain your reasoning.**

   a. **A rectangle is a parallelogram.**

   b. **A rhombus is a square.**

   c. **A scalene triangle is a right triangle.**

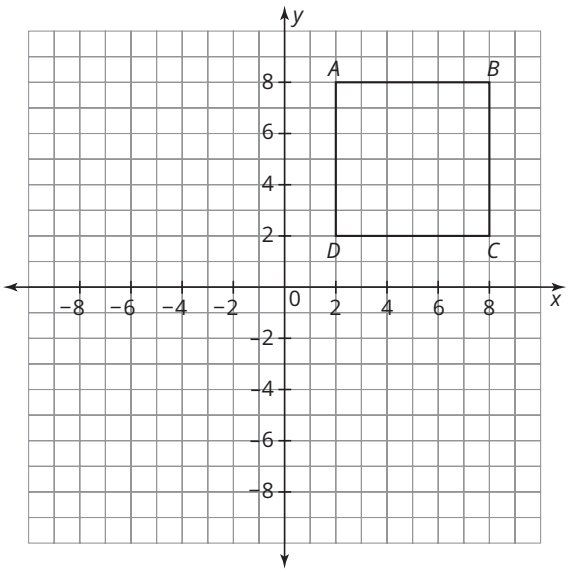
   d. **A parallelogram is a trapezoid.**

   e. **A right triangle is an equilateral triangle.**

ACTIVITY
4.1
Calculating Distance on the Coordinate Plane

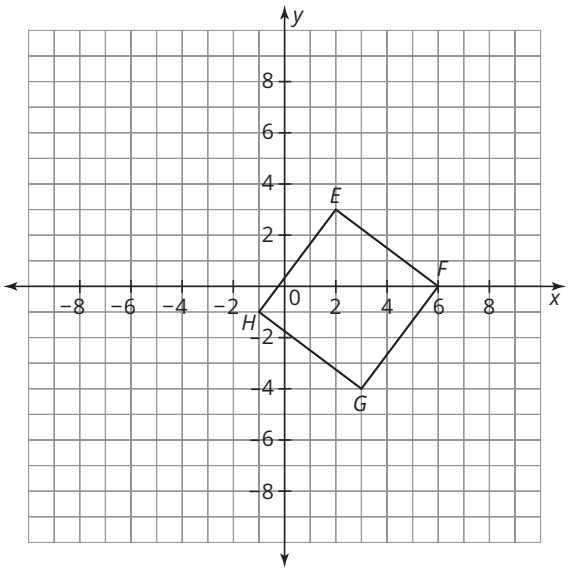
Let's analyze quadrilaterals that lie on a coordinate plane and classify them by their properties.

Consider quadrilateral *ABCD* shown.



1. **Classify the quadrilateral. Justify your reasoning.**

Now consider quadrilateral *EFGH* shown.



2. **Determine whether quadrilateral *EFGH* can be classified as a parallelogram. Justify your reasoning.**

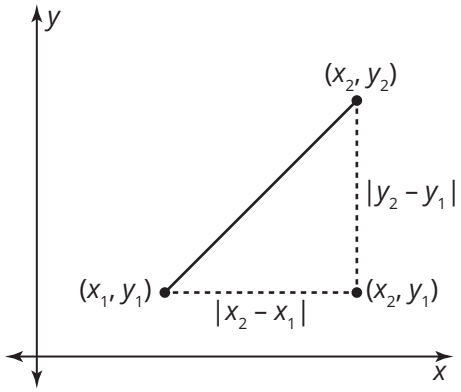
3. **Determine whether quadrilateral *EFGH* can be classified as a rectangle. Justify your reasoning.**

4. What information do you need to classify quadrilateral *EFGH* as a square?

5. On quadrilateral *EFGH*, draw a right triangle *EFR* such that $\overline{EF}$ is the hypotenuse. Use the Pythagorean Theorem to determine the length of $\overline{EF}$.

You used the Pythagorean Theorem to calculate the distance between two points on the coordinate plane. Your method can be written as the *Distance Formula*. The **Distance Formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the distance d between (x_1, y_1) and (x_2, y_2) is calculated using the formula given.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



The absolute value symbols are used because the difference represents a distance.

6. When you use the Distance Formula, does it matter which point you identify as (x_1, y_1) and which point you identify as (x_2, y_2)? Explain your reasoning.

7. Can quadrilateral *EFGH* be classified as a square? Justify your reasoning.

8. Use the Distance Formula to calculate the distance between each pair of points. Round your answer to the nearest tenth, if necessary. Show all your work.

a. (1, 2) and (3, 7)

b. (−6, 4) and (2, −8)

c. (−5, 2) and (−6, 10)

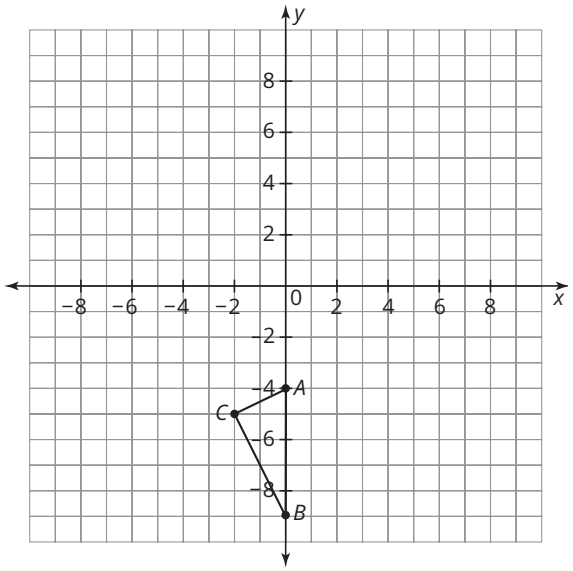
9. Calculate the distance between the points (−1, −2) and (−3, −7). Notice the similarity between this problem and Question 8, part (a).

Carlos says that the solution must be the negative of the solution of part (a). Mandy disagrees and says that the solution will be the same as the solution of part (a). Who is correct? Explain your reasoning and state the correct solution.

ACTIVITY
4.2
Classifying Triangles on the Coordinate Plane

Let's analyze triangles that lie on a coordinate plane and classify them by their properties.

Consider $\triangle ABC$.



1. Classify $\triangle ABC$.

a. Consider the sides of $\triangle ABC$ to describe it as scalene, isosceles, or equilateral. Explain your reasoning.

b. Consider the slope of each side to determine whether $\triangle ABC$ is a right triangle. Justify your conclusion.

Ask yourself:

How can you determine the lengths of the sides of this triangle?

c. Zach used the Pythagorean Theorem to determine whether $\triangle ABC$ was a right triangle.

Zach

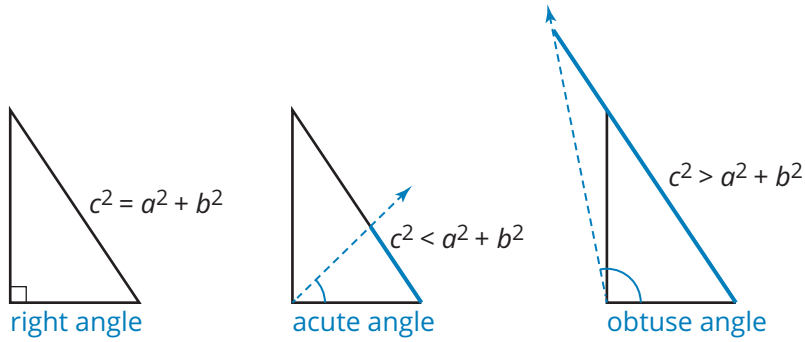
$$a^2 + b^2 = c^2$$
$$(\sqrt{5})^2 + (\sqrt{20})^2 = 5^2$$
$$5 + 20 = 25$$
$$25 = 25$$

Describe why Zach's reasoning is correct.

Remember:

Any set of three positive integers a, b, and c that satisfies the equation $a^2 + b^2 = c^2$ is a Pythagorean triple. For example, the integers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.

You can use the relationship among the sides of a triangle to determine whether the triangle is acute or obtuse. Given a, b, and c are the sides of a triangle with c as the longest side, when $c^2 < a^2 + b^2$, the triangle is acute, and when $c^2 > a^2 + b^2$, the triangle is obtuse.



2. Determine whether each set of side lengths creates an acute, right, or obtuse triangle. Explain your reasoning.

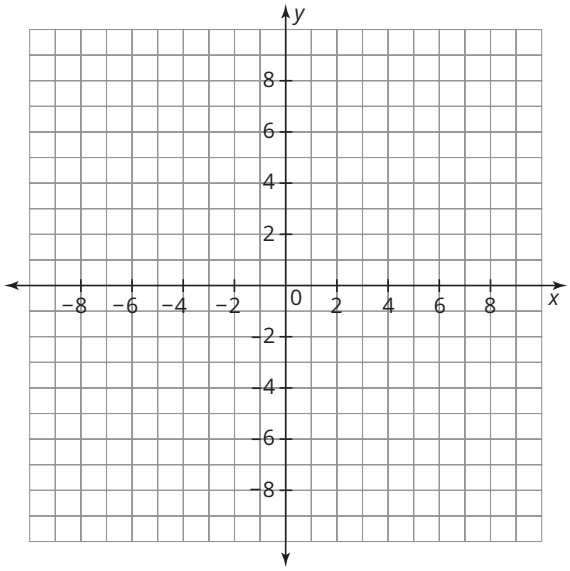
a. 42 cm, 36 cm, 15 cm

b. 10 cm, 6 cm, 8 cm

c. 18.5 m, 11 m, 15 m

d. 4 ft, $\sqrt{65}$ ft, 7 ft

3. Graph △*JKL* using points *J* (−2, 4), *K* (8, 4), and *L* (6, −2).



Think about:

Are you using a straightedge to draw the triangle?

4. Classify △*JKL*.

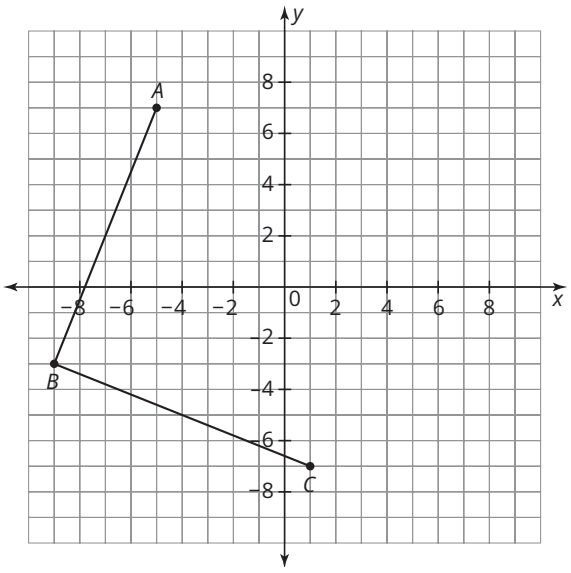
a. Consider the sides of △*JKL*. Describe the triangle as scalene, isosceles, or equilateral. Explain your reasoning.

b. Consider the angles of △*JKL*. Describe the triangle as acute, obtuse, or right. Explain your reasoning.

ACTIVITY
4.3
Determining an Unknown Point of a Quadrilateral

You have classified quadrilaterals by their sides and angles. You can use this information to compose quadrilaterals on a coordinate plane.

Analyze the given points *A*, *B*, and *C*. Suppose you want to plot point *D* such that quadrilateral *ABCD* is a square.



1. Consider the properties of a square.

a. How does knowing that a square has two pairs of parallel sides help to determine the unknown location?

b. How does knowing that a square has four right angles help to determine the unknown location?

2. Determine the location of point *D*. Plot and label point *D* on the coordinate plane.

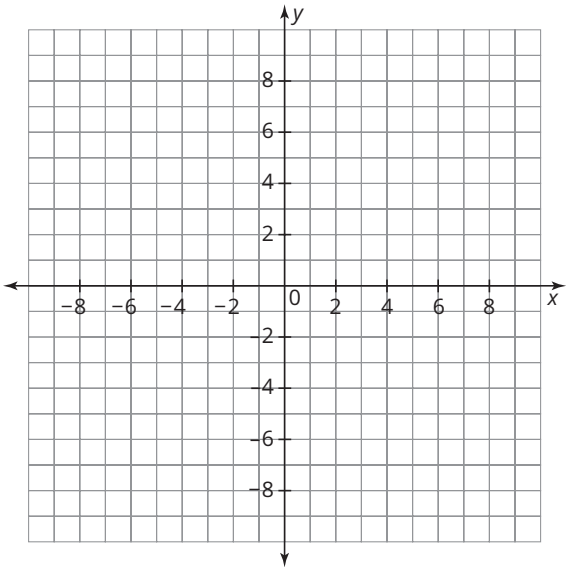
3. **Use the same locations for points *A*, *B*, and *C* to identify the location of a new point *E*, such that quadrilateral *ABCE* is a trapezoid with only one pair of parallel sides.**

   a. **Identify information that is helpful to locate point *E*. Explain your reasoning.**

   b. **Describe the possible locations of point *E* such that quadrilateral *ABCE* is a trapezoid with only one pair of parallel sides.**

ACTIVITY
4.4
Classifying a Quadrilateral on the Coordinate Plane

In this activity, you will classify quadrilaterals by examining the lengths and relationships of their sides.



1. Graph quadrilateral *ABCD* using points *A* (−5, 6), *B* (−8, 2), *C* (−5, −2), and *D* (−2, 2).

2. Consider the sides of quadrilateral *ABCD*.

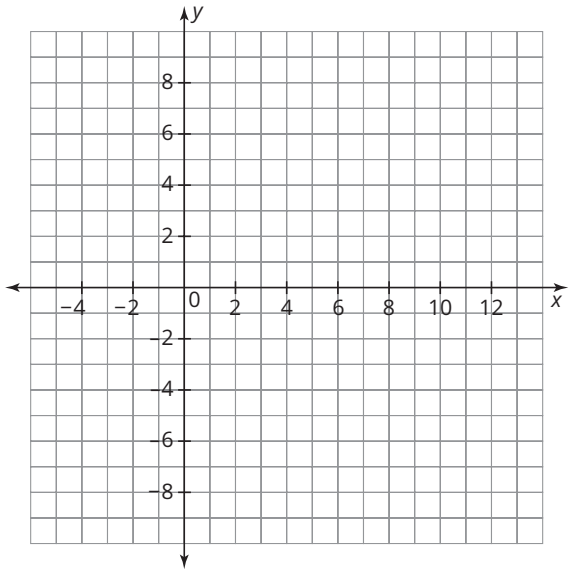
a. Determine each side length of quadrilateral *ABCD*. Can you classify quadrilateral *ABCD* from its side lengths? If so, identify the type of figure. If not, explain why not.

Think about:

What is the difference between a square and a rhombus?

b. Determine the slope of each line segment in the quadrilateral. Describe the relationship between the slopes. Can you identify the figure? If so, identify the type of figure. If not, explain why not.

3. Graph quadrilateral *ABCD* using points *A* (8, 8), *B* (3, −7), *C* (10, −6), and *D* (13, 3). Classify this quadrilateral as a trapezoid, a rhombus, a rectangle, a square, or none of these. Explain your reasoning.



Think about:

Which types of figures can you eliminate as you determine information about the figure?

ACTIVITY
4.5
Classifying a Quadrilateral Formed by Midpoints

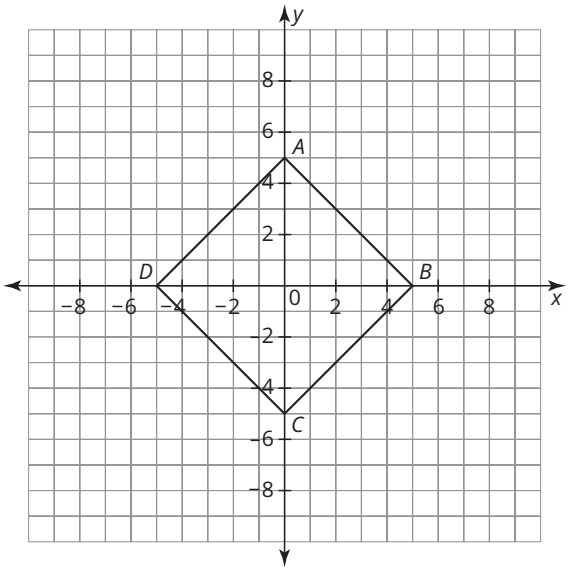
Remember:

A midpoint is the point that is exactly halfway between two given points.

You have used the Distance Formula to determine the distance between two points. To determine the coordinates of a midpoint, you can use the *Midpoint Formula*.

The **Midpoint Formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Use the Midpoint Formula to determine the midpoints of each side of the given figures.



1. Given square *ABCD*.

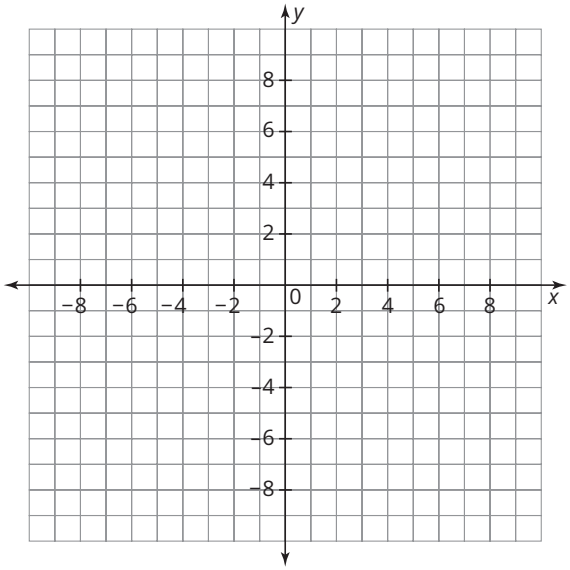
a. Determine and label the midpoint of each side of the square.

b. Determine the polygon formed by connecting the consecutive midpoints of each side of a square and justify your conclusion.

c. If the same process was repeated one more time by connecting the consecutive midpoints of each side of the polygon determined in part (a), describe the polygon that would result.

2. Sketch any rhombus that is not a square. Label the midpoint of each side of the rhombus.

a. Determine the polygon formed by connecting the consecutive midpoints of each side of a rhombus and justify your conclusion.



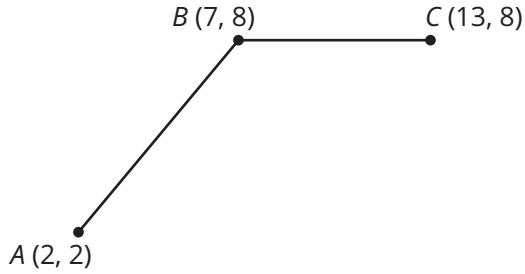
b. If the same process was repeated one more time by connecting the consecutive midpoints of each side of the polygon determined in part (a), describe the polygon that would result.

NOTES

Talk the Talk

Look, Ma! No Gridlines!

Consider points *A* (2, 2), *B* (7, 8), and *C* (13, 8).



1. **Describe all possible locations for the coordinates of point *D* such that quadrilateral *ABCD* is a parallelogram.**

2. **Describe how point *D* can be located using a translation.**