4

Where Has Polly Gone?

Classifying Shapes on the Coordinate Plane

MATERIALS

None

Lesson Overview

Students use a Venn diagram to sort quadrilaterals and triangles based on shared properties. They are introduced to the Distance Formula and use it to calculate the lengths of sides of triangles and quadrilaterals on the coordinate plane. Students also use the slope formula to determine whether opposite sides of a quadrilateral are parallel and whether consecutive sides of a quadrilateral are perpendicular. They use these skills to classify triangles and quadrilaterals that lie on a coordinate plane, or determine the fourth point of a quadrilateral when given three points. Students are then introduced to the Midpoint Formula and use it to classify secondary figures formed when connecting the midpoints of consecutive sides of quadrilaterals. Finally, students consider translations as a strategy to identify the coordinates that create quadrilaterals with parallel sides.

Geometry

Coordinate and Transformational Geometry

(2) The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines.

Similarity, Proof, and Trigonometry

(9) The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:

(B) apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean theorem, including Pythagorean triples, to solve problems.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- The Distance Formula states that the distance d between points (x_1, y_1) and (x_2, y_2) on a coordinate plane is given by the equation $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- The Distance Formula can be used to classify triangles and quadrilaterals based on side lengths.
- The slope formula can be used to determine whether opposite sides are parallel or consecutive sides are perpendicular in a quadrilateral on the coordinate plane.
- The Midpoint Formula states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- The use of translations is an efficient strategy when determining endpoints of parallel segments on a coordinate plane.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: You Better Shape Up

Students match polygons to regions of a Venn diagram illustrating the relationship among polygons with different properties. They then interpret the overlapping region of a second Venn diagram. Students determine whether statements about polygons are always true, sometimes true, or never true.

Develop

Activity 4.1: Calculating Distance on the Coordinate Plane

Students classify a quadrilateral on the coordinate plane formed by vertical and horizontal line segments. They analyze a second quadrilateral using slope criteria to classify it as a parallelogram and as a rectangle. Students use one side of a quadrilateral as the hypotenuse of a right triangle, then apply the Pythagorean Theorem to determine the length of the side. The *Distance Formula* is stated and used to classify a quadrilateral as a square.

Day 2

Activity 4.2: Classifying Triangles on the Coordinate Plane

Students use the Distance Formula to classify a triangle as scalene, isosceles, or equilateral, and then apply the slope formula to determine whether the triangle is a right triangle. Students also consider how to use the Pythagorean Theorem to verify the triangle is a right triangle. Next, they use the relationship among the sides of a triangle to determine whether the triangle is acute, right, or obtuse. Students graph a set of three points and apply appropriate theorems to classify the triangle by the length of its sides and by the measures of its angles.

Activity 4.3: Determining an Unknown Point of a Quadrilateral

Students determine the possible locations of a point to create a specific quadrilateral when given three of its vertices. They must consider the properties of the specific quadrilateral, the slope and length of its sides, and the number of possible solutions.

Activity 4.4: Classifying a Quadrilateral on the Coordinate Plane

Students graph four points and identify the type of quadrilateral. They use the Distance Formula, the slope formula, and the characteristics of specific quadrilaterals to inform their decision.

Day 3

Activity 4.5: Classifying a Quadrilateral Formed by Midpoints

Students use the Midpoint Formula to determine the midpoints of each side of a square, form a secondary figure by connecting the midpoints, and classify the secondary figure. They then repeat this entire process beginning with the secondary figure. In a different situation, students sketch a rhombus, connect midpoints to create a secondary figure, classify the secondary figure, and predict the outcome if the process were repeated.

Demonstrate

Talk the Talk: Look, Ma! No Gridlines!

Students determine all possible locations for the fourth point of a quadrilateral that would make the quadrilateral a parallelogram. They then describe how they could use translations to determine the location of the fourth point.

Getting Started: You Better Shape Up

Facilitation Notes

In this activity, students match polygons to regions of a Venn diagram illustrating the relationship among polygons with different properties. They then interpret the overlapping region of a second Venn diagram. Students determine whether statements about polygons are always true, sometimes true, or never true.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- Attempts to assign a single polygon to each letter.
- Overgeneralization of properties. For example, students may erroneously match *trapezoid* to Region A because a trapezoid can be drawn with at least one pair of perpendicular sides. Explain that the property or properties must be true for all figures for it to match a region.

Misconceptions

- A common thought is that trapezoids contain exactly one pair of parallel sides rather than *at least* one pair of parallel sides. The latter definition of a trapezoid is the one that is throughout this series and is stated on the same page of this activity to avoid confusion.
- Students may think that isosceles triangles contain exactly two congruent sides rather than *at least* two congruent sides. The latter definition of an isosceles triangle is the one that is used in this textbook and is stated on the same page of this activity to avoid confusion.

Questions to ask

- Do all trapezoids have at least one pair of perpendicular sides?
- Since all trapezoids do not have at least one pair of perpendicular sides, what letters do not represent trapezoids?
- Do all trapezoids have two pair of parallel sides?
- Since all trapezoids do not have two pair of parallel sides, what additional letters do not represent trapezoids?
- Do all trapezoids have all sides congruent?
- Since all trapezoids do not have all sides congruent, what additional letters do not represent trapezoids?
- What letter must represent all trapezoids?

Differentiation strategy

To extend the activity, ask students to create counterexamples for incorrect responses.

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

Misconceptions

- Students may assume all equiangular polygons are also equilateral polygons. The rectangle is a counterexample. All angles of a rectangle are right angles, therefore equal in measure, however, all four of the sides may not be the same length. When the rectangle is not a square, it is not equilateral.
- Students may assume all equilateral polygons are also equiangular polygons. The rhombus is a counterexample. All sides of a rhombus are congruent, therefore equal in measure, however, all four of the angles may not have the same measure. When the rhombus is not a square, it is not equiangular.

Questions to ask

- What is the definition of a rhombus?
- Do all rhombi have four congruent sides?
- Do all rhombi have four congruent angles?
- What is the definition of a rectangle? a Parallelogram?
- Do all rectangles have two pairs of parallel sides?
- What is the definition of a square?
- Are all rhombi equiangular? Are all squares equiangular?
- Is it possible to draw a right triangle that has sides of different lengths?
- How can the Pythagorean Theorem be used to show a right triangle cannot be an equilateral triangle?
- Is an equilateral triangle also equiangular?

Summary

Polygons are often classified by properties, such as the lengths of their sides, the relationship between their sides, and the measures of their angles.

Activity 4.1

Calculating Distance on the Coordinate Plane

DEVELOP

Facilitation Notes

In this activity, students classify a quadrilateral on the coordinate plane formed by vertical and horizontal line segments. They analyze a second quadrilateral using slope criteria to classify it as a parallelogram and as a rectangle.

Students use one side of a quadrilateral as the hypotenuse of a right triangle, then apply the Pythagorean Theorem to determine the length of the side. The Distance Formula is stated and used to classify a quadrilateral as a square.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Differentiation strategy

To scaffold support, review relationships between graphic and algebraic representations of lines. Ask students to sketch a graph of the possible relationships between two lines (parallel, perpendicular, intersecting but not perpendicular) on the coordinate plane. Then have students identify two points on each line, calculate the slope, and discuss their conclusions.

As students work, look for

Connections made between graphic and algebraic representations of line segments with respect to their orientation on the coordinate plane. Horizontal lines are associated with zero slopes, and vertical lines are associated with undefined slopes.

Questions to ask

- Are $\overline{AB}$ and $\overline{CD}$ horizontal line segments? How do you know?
- Are $\overline{BC}$ and $\overline{AD}$ vertical line segments? How do you know?
- How can the tick marks on the *x*-axis be used to determine the length of $\overline{AB}$ and $\overline{CD}$?
  How can the tick marks on the *y*-axis be used to determine the length of $\overline{BC}$ and $\overline{AD}$?
- If a quadrilateral has four congruent sides, is that enough information to determine it is a rectangle? What other information is needed?
- If a quadrilateral has four congruent sides, is that enough information to determine it is a square? What other information is needed?
- Are the four angles the same measure? How do you know?
- How is quadrilateral *EFGH* different than quadrilateral *ABCD*?
- What information is needed to classify quadrilateral *EFGH* as a parallelogram?
- How does the slope of each side compare to the slope of the opposite side?

Have students work with a partner or in a group to complete Questions 2 through 5. Share responses as a class.

Questions to ask

- What are the slopes of each side of quadrilateral *EFGH*?
- Are the slopes of each pair of opposite sides equal? What does this imply?
- Are the slopes of each pair of consecutive side opposite reciprocals? What does this imply?
- How are squares and rectangles different?

- Is knowing the side lengths enough information to determine if quadrilateral *EFGH* is a square?
- Where did you locate point *R*?
- Does everyone have the same location for point *R*? Why do you suppose?
- How could you use the coordinates to determine the lengths of $\overline{ER}$ and $\overline{FR}$?

Ask a student to read the information following Question 5 aloud. Discuss the Distance Formula and complete Question 6 as a class.

Differentiation strategies

To scaffold support for all students,

- Suggest they label the axes with x_1, x_2, y_1, and y_2 so they understand how the coordinate pairs for the three vertices were determined.
- Have students trace over the horizontal side of the triangle with a colored pencil, and then use the same color to trace the x_1- and x_2-coordinates of its endpoints. Repeat the process with the y_1- and y_2-coordinates for the vertical side of the triangle. This may help students make sense of the algebraic expressions for the side lengths of the triangle.

Questions to ask

- How is the Distance Formula similar to the Pythagorean Theorem?
- Why does the Distance Formula include subtraction?
- Why are the *y*-coordinates the same for two of the coordinate pairs?
- Why are the *x*-coordinates the same for two of the coordinate pairs?
- Why is the expression $|x_2 - x_1|$ a label for the horizontal segment?
- Why is the expression $|y_2 - y_1|$ a label for the vertical segment?
- Would it be acceptable to use the label $|y_1 - y_2|$ for the vertical segment? Why?

Have students work with a partner or in a group to complete Question 7. Share responses as a class.

Questions to ask

- Why did you need to use the Distance Formula to determine whether quadrilateral *EFGH* is a square?
- What is the length of $\overline{EF}$?
- What are the lengths of the sides of quadrilateral *EFGH*?

Have students work with a partner or in a group to complete Questions 8 and 9. Share responses as a class.

Differentiation strategies

To scaffold support,

- Complete an example as a class using coordinates (2, −3) and (7, 4). Use two methods: graph the pair of ordered pairs on a coordinate plane and use the Pythagorean Theorem; and use the Distance

Formula without any sketch. Discuss similarities, differences, and preferences. Students may prefer to use the diagram and avoid the notation in the Distance Formula.

- Suggest they label the coordinate pairs prior to using the Distance Formula.

$$\overset{x_1,\ y_1}{(2, -3)} \quad \text{and} \quad \overset{x_2,\ y_2}{(7, 4)}$$

- Provide graph paper so they can plot the points and use the Pythagorean Theorem. Then, provide an additional pair of points such as (50, 30) and (90, 105) and demonstrate how to create a sketch rather than use graph paper to support their thinking.
- Graph Question 8, part (*a*) and Question 9 to help make sense of Carlos and Mandy's thinking.

As students work, look for

- Sign errors.
- Errors due to subtraction with an *x*-coordinate and a *y*-coordinate.

Questions to ask

- What substitutions did you complete to calculate the distance?
- How did you deal with two consecutive negative signs?
- What does each difference represent?

Misconception

Each square on the coordinate planes shown in this lesson represents one square unit. Remind students that this might not always be the case and they should check the scale of each graph before performing any calculations.

Summary

The Distance Formula states the distance, d between points (x_1, y_1) and (x_2, y_2) on a coordinate plane is given by the equation $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Activity 4.2

Classifying Triangles on the Coordinate Plane

Facilitation Notes

In this activity, students use the Distance Formula to classify a triangle as scalene, isosceles, or equilateral, and then apply the slope formula to determine whether the triangle is a right triangle. Students also consider how to use the Pythagorean Theorem to verify the triangle is a right triangle. Next, they use the relationship among the sides of a triangle to determine whether the triangle is acute, right, or obtuse. Students graph a set of three points and apply appropriate theorems to classify the triangle by the length of its sides and by the measures of its angles.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Differentiation strategies

To scaffold support, review some labeling conventions.

- *AB* represents a numeric value, the distance between point *A* and point *B*. $\overline{AB}$ represents a figure, the segment between point *A* and point *B*. For example, $AB = 5$, and the slope of $\overline{AB}$ is undefined.
- When labeling a triangle, each vertex is labeled with a capital letter. The side opposite the vertex is labeled with the lower case version of the same letter. Have students use this convention to label the sides of $\triangle ABC$ with *a*, *b*, and *c* to better understand Zach's work.

Questions to ask

- What is the difference between the Distance Formula and the Pythagorean Theorem?
- Why isn't a formula needed to determine the length of $\overline{AB}$?
- Is the Distance Formula or the slope formula used to determine if the triangle is scalene, isosceles, or equilateral? Why?
- Can the Distance Formula be used to determine whether $\triangle ABC$ is a right triangle?
- What are two different methods to determine whether $\triangle ABC$ contains a right angle?
- How is the slope formula used to determine whether a triangle is a right triangle?
- How is the Pythagorean Theorem used to determine whether a triangle is a right triangle?
- Do you prefer to use the slope formula or the Pythagorean Theorem to determine whether a triangle is a right triangle? Why?
- Explain why it might be helpful if the triangle is graphed on the coordinate plane in order to determine if it is a right triangle or not.

Ask a student to read the information following Question 1 aloud. Discuss as a class.

Questions to ask

- In an acute triangle, how many of its angles are acute?
- In an obtuse triangle, how many of its angles are obtuse? What type of angle are each of the other two angles?
- In a right triangle, how many of its angles are right angles? What type of angle are each of the other two angles?
- Explain why when $c^2 < a^2 + b^2$, you know that $\angle C$ is acute.
- Explain why when $c^2 > a^2 + b^2$, you know that $\angle C$ is obtuse.

Differentiation strategy

To scaffold support, remind students that before using any strategy to determine whether a triangle is a right triangle, they should first check that the lengths of the three sides make a triangle by using the Triangle Inequality Theorem. This theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Have students work with a partner or in a group to complete Questions 2 through 4. Share responses as a class.

Questions to ask

- How do you know what value to substitute for *c*?
- What calculations did you make to determine whether $\triangle JKL$ is scalene, isosceles, or equilateral?
- What formula is used to determine whether $\triangle JKL$ contains a right angle?
- How did you know what value to use for *c*?
- How did you determine whether $\triangle JKL$ is an acute or obtuse triangle?

Differentiation strategy

To extend the lesson, discuss an alternative solution method for Question 4, part (b). Expand upon a concept students understand when comparing the measures of the angles and sides of a triangle. The longest side of a triangle is opposite the largest angle, the shortest side of a triangle is opposite the smallest angle, and the side with the medium measurement is opposite the angle with the medium measurement. Discuss the special case when two sides of a triangle are the same length. It follows that if two sides of a triangle are the same length, then the measures of the angles opposite those sides are also equal in measure. This special property of isosceles triangles will be addressed formally at a later time. Reason that if both angles are equal in measure, they cannot both be obtuse. Also, in $\triangle JKL$, the measure of $\angle J$ is less than the measures of the other two angles, so it cannot be obtuse.

Summary

The Distance Formula can be used to classify a triangle according to its side lengths. The slope formula can be used to determine whether a triangle is a right triangle. The Pythagorean Theorem can be used to determine whether a triangle is right, acute, or obtuse.

Activity 4.3
Determining an Unknown Point of a Quadrilateral

Facilitation Notes

In this activity, students determine the possible locations of a point to create a specific quadrilateral when given three of its vertices. They must consider the properties of the specific quadrilateral, the slope and length of its sides, and the number of possible solutions.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To scaffold support for the formulas, allow students to use the graph to determine slope.

As students work, look for

- Use of transformations to locate point D. Point B can be translated left 4 units and up 10 units to locate point A, so point C can be translated left 4 units and up 10 units to locate point D.
- Use of parallel lines.
- Use of perpendicular lines.

Questions to ask

- How do you know $\angle B$ is a right angle?
- What is the slope of $\overline{AB}$? $\overline{BC}$?
- What is the relationship between the slopes of $\overline{AB}$ and $\overline{BC}$? $\overline{BC}$ and $\overline{CD}$? $\overline{CD}$ and $\overline{AB}$?
- What should be the slope of $\overline{CD}$? $\overline{DA}$?
- How do you know all the sides are the same length?
- Can point D have more than one possible location? Why or why not?

Have students work with a partner or in a group to complete Question 3. Share responses as a class.

As students work, look for

- Which sides were kept parallel for the trapezoid.
- Recognition of infinite solutions.
- Limitations of solutions based upon the given points and sides, and the fact that the trapezoid cannot have vertex E at (5, 3).

Questions to ask

- How is this question different than the previous question?
- What sides did you keep parallel?
- Are any sides of the trapezoid the same length?
- Are any angles of the trapezoid right angles?
- If $\overline{AE}$ and $\overline{BC}$ are parallel, why is point E limited to being to the right of point A?
- If $\overline{AB}$ and $\overline{CD}$ are parallel, why is point E limited to being above point C?

- Why can't *E* be at (5, 3)?
- What are three possible coordinate pairs for point *E*?

Summary

To determine the coordinates for a quadrilateral, the properties of the specific quadrilateral, the slope and length of its sides, and the number of possible solutions must be considered.

Activity 4.4

Classifying a Quadrilateral on the Coordinate Plane

Facilitation Notes

In this activity, students graph four points and identify the type of quadrilateral. They use the Distance Formula, the slope formula, and the characteristics of specific quadrilaterals to inform their decision.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Misconception

Students may incorrectly identify the slopes as having an opposite reciprocal relationship and think the quadrilateral has perpendicular lines. The slopes are $\frac{4}{3}$ and $-\frac{4}{3}$ and only have an opposite relationship.

Differentiation strategy

To scaffold support, suggest students use the graph to strategize which calculations they need to make. For example, by looking at the graph for Question 3, it is obvious that opposite sides $\overline{BC}$ and $\overline{AD}$ are not parallel because they are going in opposite directions.

Questions to ask

- What quadrilaterals have all sides congruent?
- Is knowing the length and slope of each line segment forming the sides of a quadrilateral enough information to classify the quadrilateral? Why or why not?
- How can the slope of the sides be used to determine information about the angles formed at each vertex?
- Are there perpendicular relationships between any two line segments? How do you know?
- Do opposite sides of the quadrilateral have the same slope? What does this tell you about the quadrilateral?
- Is quadrilateral *ABCD* a parallelogram? A rectangle? A rhombus? A square?

- In Question 3, what characteristics helped you identify the quadrilateral?
- Is it necessary to calculate the length of all 4 sides? Why or why not?
- Is it necessary to calculate the slopes of all 4 sides? Why or why not?
- How does the graph help you determine what calculations are necessary?

Differentiation strategy

To extend the activity, ask students to create sets of coordinates that could be used to describe each of the following figures: a quadrilateral with no parallel sides, a trapezoid with one pair of parallel sides, an isosceles trapezoid, a parallelogram, a rhombus, a rectangle, and a square. As an extra challenge, do not allow the use of any horizontal and vertical lines.

Summary

A graph is a resource that informs what calculations need to be made to identify a quadrilateral by the lengths of its sides, the measures of its angles, and the relationships between its sides.

Activity 4.5

Classifying a Quadrilateral Formed by Midpoints

Facilitation Notes

In this activity, students use the Midpoint Formula to determine the midpoints of each side of a square, form a secondary figure by connecting the midpoints, and classify the secondary figure. They then repeat this entire process beginning with the secondary figure. In a different situation, students sketch a rhombus, connect midpoints to create a secondary figure, classify the secondary figure, and predict the outcome if the process were repeated.

Ask a student to read the introduction aloud. Discuss as a class.

Questions to ask

- Explain how to calculate the midpoint in your own words.
- How is the formula for the midpoint related to the formula for the mean? For the median?

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To scaffold support, provide the coordinate pairs (2, 9), (8, 1), (2, −7) and (−4, 1) for the rhombus.

Questions to ask

- Are the angles of the secondary figure right angles? How do you know?
- Are the four sides of the secondary figure the same length? How do you know?
- If the four sides are the same length and the angles are right angles, is the figure a rhombus? A rectangle? A square?
- After several more iterations of the process, will the new figure formed always be an image of itself? Why?
- Why is a rectangle, rather than a square, formed by connecting the consecutive midpoints of the sides of a rhombus?
- Why is a rhombus formed by connecting the consecutive midpoints of the sides of a rectangle?

Differentiation strategy

To extend the activity, ask students to repeat the activity with a parallelogram and trapezoid. Analyze the results based on whether the original figure was a square, rhombus, parallelogram, or trapezoid.

Summary

The Midpoint Formula states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

DEMONSTRATE

Talk the Talk: Look, Ma! No Gridlines!

Facilitation Notes

In this activity, students determine all possible locations for the fourth point of a quadrilateral that would make the quadrilateral a parallelogram. They then describe how they could use translations to determine the location of the fourth point.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What are the properties of a parallelogram?
- Did everyone locate the same fourth point?
- Is there more than one location for the fourth point?

- Did you use the length of the segments to locate the fourth point? If so, how?
- Can you use the slope of the segments to locate the fourth point? If so, how?
- How can translation be used to locate the fourth point?
- What method do you prefer to use when solving problems like this?

Differentiation strategies

To extend the activity,

- Provide students with the coordinates (2, 2), (7, 8), and (13, 8), without labeling or connecting the points. Ask students to graph the points and describe all possible locations for the coordinates of a fourth point such that the quadrilateral formed by connecting the points is a parallelogram. This will open the problem up to allow two solutions, either point (−4, 2) or point (8, 2), as the fourth point.
- Discuss how the process of translating a point vertically and horizontally is related to determining the slope of a line and creating a segment parallel to a given segment.

Summary

The use of translations is an effective strategy when determining endpoints of parallel segments on a coordinate plane.

NOTES

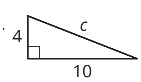
4

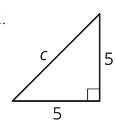
Where Has Polly Gone?

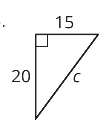
Classifying Shapes on the Coordinate Plane

Warm Up

Determine the length of each hypotenuse. Round your answer to the nearest tenth, if necessary.

1.


2.


3.


Learning Goals

- Use the Pythagorean Theorem to derive the Distance Formula.
- Apply the Distance Formula on the coordinate plane.
- Classify a triangle given the locations of its vertices on a coordinate plane.
- Determine the coordinates of a fourth vertex, given the coordinates of three vertices of a quadrilateral and a description of the quadrilateral.
- Classify a quadrilateral given the locations of its vertices on a coordinate plane.

Key Terms

- Distance Formula
- Midpoint Formula

You know the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals. You also know how to determine the length of the hypotenuse of a right triangle. How can you use what you know to classify polygons that lie on a coordinate plane?

Warm Up Answers

1. 10.8
2. 7.1
3. 25

Answers

1. A: right triangle
   B: rectangle
   C: parallelogram
   D: square
   E: rhombus
   F: equilateral triangle
   H: trapezoid, isosceles triangle, scalene triangle
2. Region G cannot be matched to a polygon in the set. The only polygon that has all sides congruent and at least one pair of perpendicular sides is a square, but it also has two pairs of parallel sides, so it belongs in Region D.

3a. A parallelogram and a rhombus both have two pairs of parallel sides, but a rhombus also has all sides congruent.

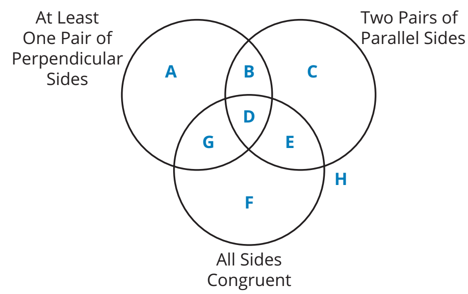
3b. A rectangle and a square both have two pairs of parallel sides and at least one pair of perpendicular sides, but a square also has all sides congruent.

GETTING STARTED

You Better Shape Up

Polygons are often classified by properties, such as the lengths of their sides, the relationships between their sides, and the measures of their angles.

The Venn diagram contains three circles each representing a different property. Letters A through H represent any polygon that has the property described by every circle in which it appears.



Remember:

Congruent means to have the same size, shape, and measure.

A letter may be used more than once or not at all!

1. Match each polygon to one of the lettered regions of the Venn diagram. Write the corresponding letter next to each polygon.

trapezoid	**square**	**isosceles triangle**
parallelogram	**rhombus**	**equilateral triangle**
rectangle	**right triangle**	**scalene triangle**

2. Is there a region that cannot be matched to one of the polygons? Explain your reasoning.

Remember:

A trapezoid has at least one pair of parallel sides.

An isosceles triangle has at least two congruent sides.

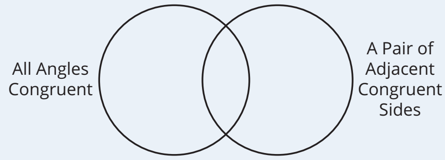
3. Use the Venn diagram to compare the properties of each pair of polygons.

a. parallelogram and rhombus

b. rectangle and square

2 • TOPIC 1: Using a Rectangular Coordinate System

4. **Marla and Flynn analyze the Venn diagram shown.**



Marla says that the overlapping region describes a rhombus. Flynn says the overlapping region describes a square. Who's correct? Explain your reasoning.

Remember:

Adjacent sides are sides that share a vertex.

5. **Determine whether each statement is always true, sometimes true, or never true. Explain your reasoning.**

   a. **A rectangle is a parallelogram.**

   b. **A rhombus is a square.**

   c. **A scalene triangle is a right triangle.**

   d. **A parallelogram is a trapezoid.**

   e. **A right triangle is an equilateral triangle.**

Answers

4. Flynn is correct. Both a rhombus and a square have a pair of adjacent congruent sides, but only a square has all angles congruent as well, since it has four right angles.

5a. always true;
A parallelogram is a quadrilateral with two pairs of opposite sides that are parallel. All rectangles have that same property.

5b. sometimes true;
A rhombus has two pairs of opposite sides that are parallel and four congruent sides, but it does not have to have four congruent angles as a square does.

5c. sometimes true;
A scalene triangle has three different side lengths, but one of its angles may or may not be a right angle.

5d. always true;
A trapezoid is a quadrilateral with at least one pair of parallel sides. All parallelograms have that same property.

5e. never true;
An equilateral triangle has congruent angles, which each measure 60°, so it can never be a right triangle.

Answers

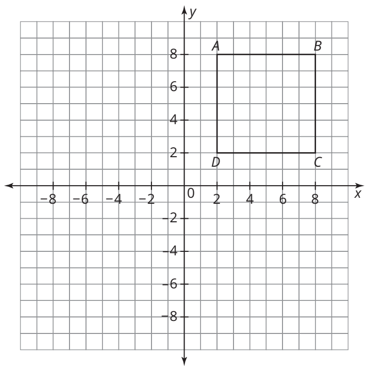
1. Quadrilateral *ABCD* is a square. Because its sides are horizontal and vertical line segments, I know they are perpendicular, so the figure has four right angles. I can use the number line on the axes to determine the distance between the vertices and conclude that the side lengths are all congruent.
2. Quadrilateral *EFGH* is a parallelogram. The slope of side *EF* is $-\frac{3}{4}$, the slope of side *FG* is $\frac{4}{3}$, the slope of side *GH* is $-\frac{3}{4}$, and the slope of side *HE* is $\frac{4}{3}$. The opposite sides of the quadrilateral have the same slope so they are parallel to each other.
3. Quadrilateral *EFGH* is a rectangle. I can use the slopes I calculated in the previous question. Since the slopes of sides *EF* and *GH* are opposite reciprocals of the slopes of sides *FG* and *HE*, the sides are perpendicular to each other. Therefore, the quadrilateral has four right angles.

ACTIVITY **4.1**

Calculating Distance on the Coordinate Plane

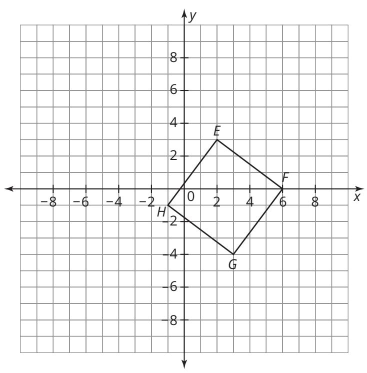
Let's analyze quadrilaterals that lie on a coordinate plane and classify them by their properties.

Consider quadrilateral *ABCD* shown.



1. Classify the quadrilateral. Justify your reasoning.

Now consider quadrilateral *EFGH* shown.



2. Determine whether quadrilateral *EFGH* can be classified as a parallelogram. Justify your reasoning.

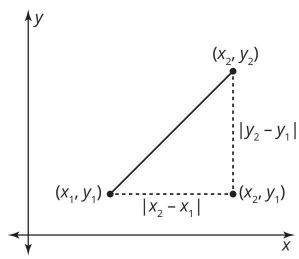
3. Determine whether quadrilateral *EFGH* can be classified as a rectangle. Justify your reasoning.

4. What information do you need to classify quadrilateral *EFGH* as a square?

5. On quadrilateral *EFGH*, draw a right triangle *EFR* such that $\overline{EF}$ is the hypotenuse. Use the Pythagorean Theorem to determine the length of $\overline{EF}$.

You used the Pythagorean Theorem to calculate the distance between two points on the coordinate plane. Your method can be written as the *Distance Formula*. The **Distance Formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the distance d between (x_1, y_1) and (x_2, y_2) is calculated using the formula given.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



The absolute value symbols are used because the difference represents a distance.

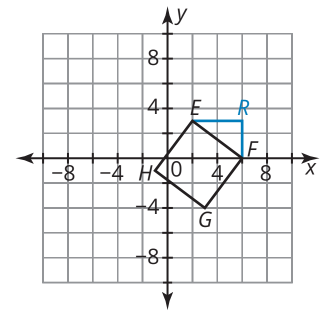
6. When you use the Distance Formula, does it matter which point you identify as (x_1, y_1) and which point you identify as (x_2, y_2)? Explain your reasoning.

7. Can quadrilateral *EFGH* be classified as a square? Justify your reasoning.

Answers

4. I need to know whether the side lengths are congruent.

5.



$ER^2 + FR^2 = EF^2$

$4^2 + 3^2 = EF^2$

$16 + 9 = EF^2$

$\sqrt{25} = EF$

$EF = 5$

6. No. Once I square the differences, the values are the same so order does not matter. For example, $(3 - 5)^2 = (-2)^2 = 4$ and $(5 - 3)^2 = 2^2 = 4$.

7. Quadrilateral *EFGH* is a square.
   In Question 3, I determined it was a rectangle. Also, all four sides are congruent.
   I already determined that side *EF* had a length of 5 units in Question 5.

$FG = \sqrt{(6 - 3)^2 + [0 - (-4)]^2}$

$= \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

$GH = \sqrt{[3 - (-1)]^2 + [-4 - (-1)]^2}$

$= \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

$HE = \sqrt{(-1 - 2)^2 + (-1 - 3)^2}$

$= \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5$

Answers

8a. $d = \sqrt{(3-1)^2 + (7-2)^2}$

$d = \sqrt{2^2 + 5^2}$

$d = \sqrt{29}$

$d \approx 5.4$

8b. $d = \sqrt{[2-(-6)]^2 + (-8-4)^2}$

$d = \sqrt{8^2 + (-12)^2}$

$d = \sqrt{208}$

$d \approx 14.4$

8c. $d = \sqrt{[-6-(-5)]^2 + (10-2)^2}$

$d = \sqrt{(-1)^2 + 8^2}$

$d = \sqrt{65}$

$d \approx 8.1$

9. Mandy is correct. Because you are squaring the differences, the solution will be positive. The distance between those points is also approximately 5.4 units.

8. Use the Distance Formula to calculate the distance between each pair of points. Round your answer to the nearest tenth, if necessary. Show all your work.

a. (1, 2) and (3, 7)

b. (−6, 4) and (2, −8)

c. (−5, 2) and (−6, 10)

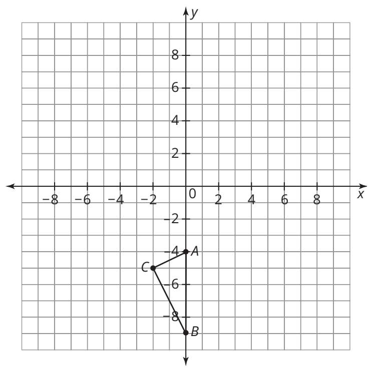
9. Calculate the distance between the points (−1, −2) and (−3, −7). Notice the similarity between this problem and Question 8, part (a).

Carlos says that the solution must be the negative of the solution of part (a). Mandy disagrees and says that the solution will be the same as the solution of part (a). Who is correct? Explain your reasoning and state the correct solution.

ACTIVITY 4.2 Classifying Triangles on the Coordinate Plane

Let's analyze triangles that lie on a coordinate plane and classify them by their properties.

Consider $\triangle ABC$.



1. Classify $\triangle ABC$.

a. Consider the sides of $\triangle ABC$ to describe it as scalene, isosceles, or equilateral. Explain your reasoning.

Ask yourself:

How can you determine the lengths of the sides of this triangle?

b. Consider the slope of each side to determine whether $\triangle ABC$ is a right triangle. Justify your conclusion.

Answers

1a. Triangle ABC is a scalene triangle.

$AB = 5$

$BC = \sqrt{20} \approx 4.47$

$AC = \sqrt{5} \approx 2.24$

1b. Triangle ABC is a right triangle.

Slope of $\overline{AB}$ is undefined.

Slope of $\overline{AC} = \frac{1}{2}$

Slope of $\overline{BC} = -2$

The slopes of the segments that form angle C are opposite reciprocals of each other, so they must be perpendicular, which means they form a right angle.

Answers

1c. The Pythagorean Theorem only works for right triangles. Since the side lengths of triangle *ABC* satisfied the Pythagorean Theorem, Zach proved that triangle *ABC* is a right triangle.

2a. $42^2 = 1{,}764$
$36^2 + 15^2 = 1{,}521$
$1{,}764 > 1521$

The triangle is obtuse.

2b. I know 6, 8, 10 is a Pythagorean triple. So, $6^2 + 8^2 = 10^2$.

The triangle is a right triangle.

2c. $18.5^2 = 342.25$
$11^2 + 15^2 = 346$
$342.25 < 346$

The triangle is acute.

2d. $(\sqrt{65})^2 = 65$
$4^2 + 7^2 = 65$
$65 = 65$

The triangle is a right triangle.

c. Zach used the Pythagorean Theorem to determine whether $\triangle ABC$ was a right triangle.

Zach

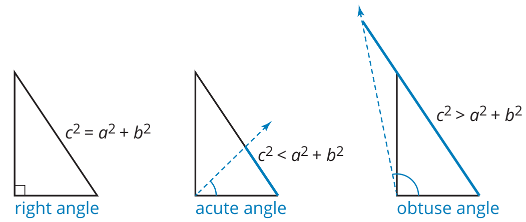
$$a^2 + b^2 = c^2$$
$$(\sqrt{5})^2 + (\sqrt{20})^2 = 5^2$$
$$5 + 20 = 25$$
$$25 = 25$$

Describe why Zach's reasoning is correct.

Remember:

Any set of three positive integers *a*, *b*, and *c* that satisfies the equation $a^2 + b^2 = c^2$ is a Pythagorean triple. For example, the integers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.

You can use the relationship among the sides of a triangle to determine whether the triangle is acute or obtuse. Given *a*, *b*, and *c* are the sides of a triangle with c as the longest side, when $c^2 < a^2 + b^2$, the triangle is acute, and when $c^2 > a^2 + b^2$, the triangle is obtuse.



2. Determine whether each set of side lengths creates an acute, right, or obtuse triangle. Explain your reasoning.

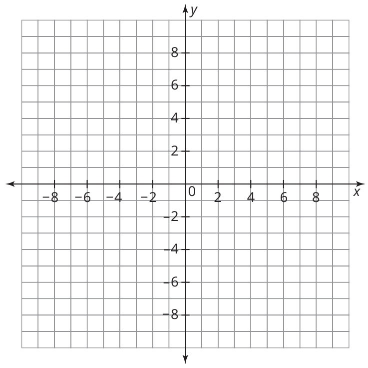
a. 42 cm, 36 cm, 15 cm

b. 10 cm, 6 cm, 8 cm

c. 18.5 m, 11 m, 15 m

d. 4 ft, $\sqrt{65}$ ft, 7 ft

3. Graph △*JKL* using points *J* (−2, 4), *K* (8, 4), and *L* (6, −2).



Think about:

Are you using a straightedge to draw the triangle?

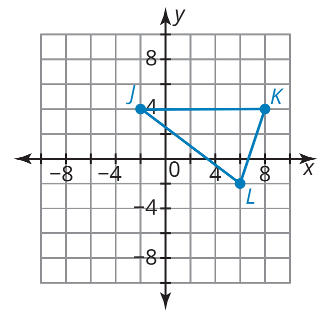
4. Classify △*JKL*.

a. Consider the sides of △*JKL*. Describe the triangle as scalene, isosceles, or equilateral. Explain your reasoning.

b. Consider the angles of △*JKL*. Describe the triangle as acute, obtuse, or right. Explain your reasoning.

Answers

3.



4a. Triangle *JKL* is isosceles because sides $\overline{JK}$ and $\overline{JL}$ are the same length.

$JK = 10$

$KL = \sqrt{40} \approx 6.32$

$JL = 10$

4b. Triangle *JKL* is an acute triangle.

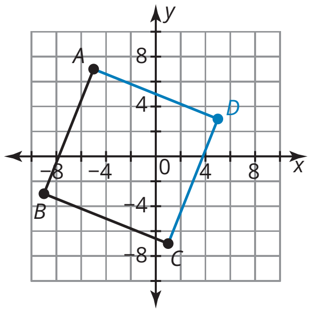
The longest sides are $\overline{JK}$ and $\overline{JL}$. Because $10^2 < 10^2 + (\sqrt{40})^2$, the triangle is acute.

Answers

1a. Because each pair of opposite sides of a square are parallel, the opposite sides have the same slope. I can use the slope of $\overline{AB}$ to draw side $\overline{DC}$ and the slope of $\overline{BC}$ to draw side $\overline{AD}$. Their intersection determines the vertex, *D*.

1b. Because consecutive sides of a square are perpendicular, consecutive sides have slopes that are opposite reciprocals. I can use the slope of $\overline{AB}$ to draw consecutive side $\overline{AD}$, and the slope of $\overline{BC}$ to draw consecutive side $\overline{CD}$. Their intersection determines the vertex, *D*.

2. The coordinates of point *D* are (5, 3).

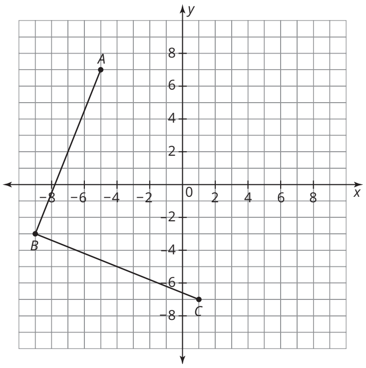


ACTIVITY **4.3**

Determining an Unknown Point of a Quadrilateral

You have classified quadrilaterals by their sides and angles. You can use this information to compose quadrilaterals on a coordinate plane.

Analyze the given points *A*, *B*, and *C*. Suppose you want to plot point *D* such that quadrilateral *ABCD* is a square.



1. Consider the properties of a square.

a. How does knowing that a square has two pairs of parallel sides help to determine the unknown location?

b. How does knowing that a square has four right angles help to determine the unknown location?

2. Determine the location of point *D*. Plot and label point *D* on the coordinate plane.

3. **Use the same locations for points *A*, *B*, and *C* to identify the location of a new point *E*, such that quadrilateral *ABCE* is a trapezoid with only one pair of parallel sides.**

   a. **Identify information that is helpful to locate point *E*. Explain your reasoning.**

   b. **Describe the possible locations of point *E* such that quadrilateral *ABCE* is a trapezoid with only one pair of parallel sides.**

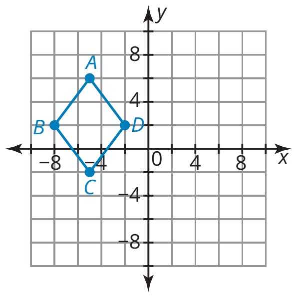
Answers

3a. I need to decide what two sides of quadrilateral *ABCD* from Question 2 I will keep parallel, and whether I will extend or shorten a remaining side so that the other two sides are not parallel.

3b. Point *E* can lie on $\overleftrightarrow{AD}$ anywhere to the right of point *A* except at point (5, 3). Point *E* can lie on $\overleftrightarrow{CD}$, anywhere above point *C* except at (5, 3).

Answers

1.



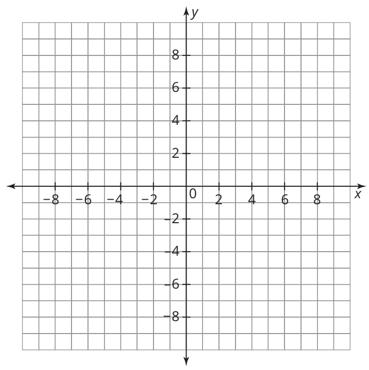
2a. All sides are 5 units in length.
The quadrilateral is a rhombus because all four sides are the same length. It might also be a square, but I would have to know the measure of the angles.

2b. Slope of $\overline{AB} = \frac{4}{3}$
Slope of $\overline{BC} = -\frac{4}{3}$
Slope of $\overline{CD} = \frac{4}{3}$
Slope of $\overline{AD} = -\frac{4}{3}$
Opposite sides have the same slope.
None of the slopes have an opposite reciprocal relationship.
Quadrilateral *ABCD* is a rhombus and not a square.

ACTIVITY **4.4**

Classifying a Quadrilateral on the Coordinate Plane

In this activity, you will classify quadrilaterals by examining the lengths and relationships of their sides.



1. Graph quadrilateral *ABCD* using points *A* (−5, 6), *B* (−8, 2), *C* (−5, −2), and *D* (−2, 2).

2. Consider the sides of quadrilateral *ABCD*.

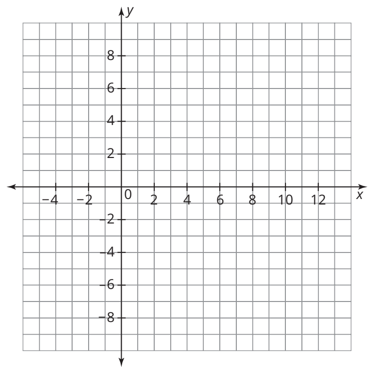
a. Determine each side length of quadrilateral *ABCD*. Can you classify quadrilateral *ABCD* from its side lengths? If so, identify the type of figure. If not, explain why not.

Think about:

What is the difference between a square and a rhombus?

b. Determine the slope of each line segment in the quadrilateral. Describe the relationship between the slopes. Can you identify the figure? If so, identify the type of figure. If not, explain why not.

3. **Graph quadrilateral *ABCD* using points *A* (8, 8), *B* (3, −7), *C* (10, −6), and *D* (13, 3). Classify this quadrilateral as a trapezoid, a rhombus, a rectangle, a square, or none of these. Explain your reasoning.**

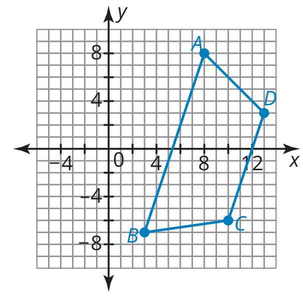


Think about:

Which types of figures can you eliminate as you determine information about the figure?

Answer

3.



Quadrilateral *ABCD* is a trapezoid. It has one pair of parallel sides. The slope of $\overline{AB}$ and the slope of $\overline{CD}$ are 3.

Answers

1a. Midpoint of $\overline{AB}$ is $\left(\frac{5}{2}, \frac{5}{2}\right)$.
Midpoint of $\overline{BC}$ is $\left(\frac{5}{2}, -\frac{5}{2}\right)$.
Midpoint of $\overline{CD}$ is $\left(-\frac{5}{2}, -\frac{5}{2}\right)$.
Midpoint of $\overline{DA}$ is $\left(-\frac{5}{2}, \frac{5}{2}\right)$.

1b. The polygon formed by connecting the consecutive midpoints of each side of a square is also a square. Consecutive sides are perpendicular since they are horizontal and vertical lines. The measures of all 4 sides are 5 units.

1c. The midpoints of the sides of the secondary figure are $\left(-\frac{5}{2}, 0\right)$, $\left(0, \frac{5}{2}\right)$, $\left(\frac{5}{2}, 0\right)$, and $\left(0, -\frac{5}{2}\right)$.
The resulting figure is also a square.

ACTIVITY 4.5

Classifying a Quadrilateral Formed by Midpoints

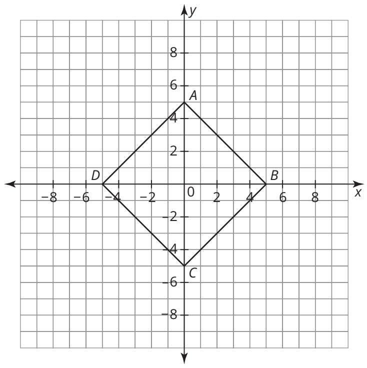
Remember:

A midpoint is the point that is exactly halfway between two given points.

You have used the Distance Formula to determine the distance between two points. To determine the coordinates of a midpoint, you can use the *Midpoint Formula*.

The **Midpoint Formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Use the Midpoint Formula to determine the midpoints of each side of the given figures.



1. Given square *ABCD*.

a. Determine and label the midpoint of each side of the square.

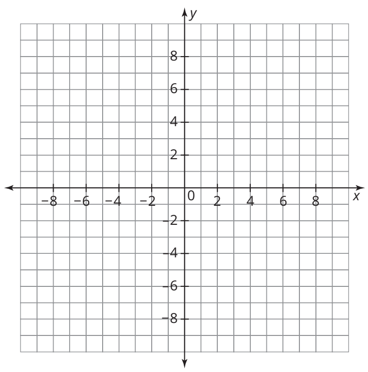
b. Determine the polygon formed by connecting the consecutive midpoints of each side of a square and justify your conclusion.

c. If the same process was repeated one more time by connecting the consecutive midpoints of each side of the polygon determined in part (a), describe the polygon that would result.

14 • TOPIC 1: Using a Rectangular Coordinate System

2. **Sketch any rhombus that is not a square. Label the midpoint of each side of the rhombus.**

   a. **Determine the polygon formed by connecting the consecutive midpoints of each side of a rhombus and justify your conclusion.**

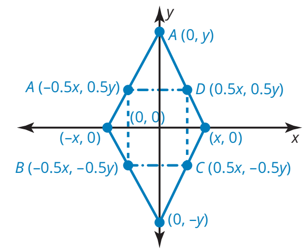


   b. **If the same process was repeated one more time by connecting the consecutive midpoints of each side of the polygon determined in part (a), describe the polygon that would result.**

2b. If the consecutive midpoints of the sides of the rectangle (secondary figure) were connected, the polygon formed would be a rhombus.

Answers

2a. The polygon formed by connecting the consecutive midpoints of each side of a rhombus is a rectangle.



Points A, B, C, and D are midpoints of each side of the given rhombus.

Let $x \neq y$.

$AB = y$

$BC = x$

$CD = y$

$DA = x$

$AB = CD$ *and* $BC = DA$

Slope of $\overline{AB}$: $m = \frac{y}{0}$ (undefined)

Slope of $\overline{BC}$: $m = \frac{0}{-x} = 0$

Slope of $\overline{CD}$: $m = \frac{y}{0}$ (undefined)

Slope of $\overline{DA}$: $m = \frac{0}{x} = 0$

Vertical lines are perpendicular to horizontal lines.

$\overline{AB} \perp \overline{BC}$

$\overline{BC} \perp \overline{CD}$

$\overline{CD} \perp \overline{DA}$

$\overline{DA} \perp \overline{AB}$

$\overline{AB} \parallel \overline{CD}$

$\overline{BC} \parallel \overline{DA}$

ABCD is a rectangle because opposite sides are congruent and all four angles are right angles.

Answers

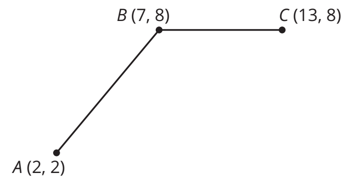
1. Point *D* has the coordinates (8, 2).
2. Sample answers.
   I noticed point *A* was located 5 units to the left and 6 units down from point *B*, so I translated point *D* 5 units to the left and 6 units down from point *C*.
   I noticed point *C* was 6 units to the right of point *B*, so I translated point *D* 6 units to the right of point *A*.

NOTES

Talk the Talk

Look, Ma! No Gridlines!

Consider points *A* (2, 2), *B* (7, 8), and *C* (13, 8).



1. Describe all possible locations for the coordinates of point *D* such that quadrilateral *ABCD* is a parallelogram.

2. Describe how point *D* can be located using a translation.