5

In and Out and All About

Area and Perimeter on the Coordinate Plane

Warm Up

Determine the distance between each set of points. Round your answer to the nearest tenth, if necessary.

1. (2, −3) and (−4, 1)

2. (−4.75, −8.5) and (3.25, 5.5)

3. $\left(\frac{5}{4}, \frac{9}{4}\right)$ and (0, 10)

Learning Goals

- Determine the perimeter and area of rectangles and triangles on the coordinate plane.
- Use transformations to discover efficient strategies to determine the perimeter and area of rectangles and triangles.
- Determine the perimeter and the area of composite figures on a coordinate plane.
- Use the Distance Formula to solve real-world problems involving perimeters of parallelograms, trapezoids, and hexagons.
- Decompose polygons—including trapezoids and hexagons—to solve real-world problems involving area.
- Calculate area under a curve to determine distance in an acceleration model.

Key Term

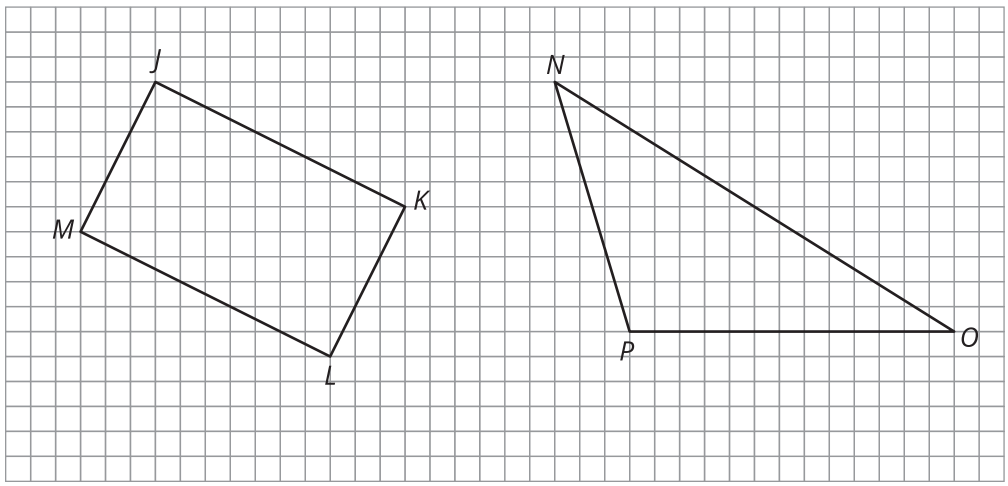
- composite figure

You have used the Distance Formula and the slope formula to classify geometric figures on the coordinate plane. How can you use these same formulas to determine the perimeter and area of polygons on the coordinate plane?

GETTING STARTED

It's Child's Play

A city uses a coordinate grid to map out the locations of two play areas at the park that need to be covered with a rubber surface to prevent injuries. Rectangle *JKLM* represents an area under a swing set and △*NOP* represents an area under a play structure. Each square on the coordinate grid represents one square foot.



1. Describe a way you can use the grid to determine the area of rectangle *JKLM* and △*NOP*.

ACTIVITY
5.1

Perimeter and Area of Figures on the Coordinate Plane

Previously, you classified geometric figures on the coordinate plane by examining the lengths and relationships of their sides. Now, you will determine the perimeter and the area of geometric figures.

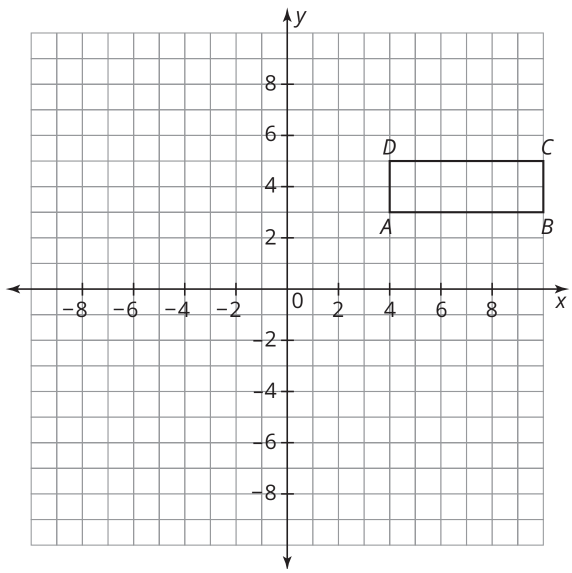
Remember:

The perimeter of a geometric figure is calculated by adding the side lengths.

Remember:

The formula for area of a rectangle is $A = bh$, where A represents the area, b represents the base, and h represents the height.

1. Consider rectangle *ABCD*.



a. Determine the perimeter of rectangle *ABCD*.

b. Determine the area of rectangle *ABCD*.

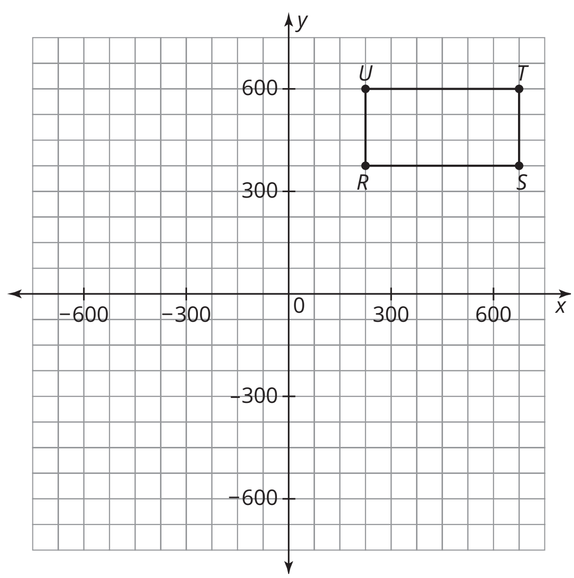
2. Horace says that he determined the area of rectangle *ABCD* by determining the product *CD*(*CB*). Bernice says that Horace is incorrect because he needs to use the base of the rectangle and that the base is $\overline{AB}$, not $\overline{CD}$. Horace responded by saying that $\overline{CD}$ is one of the bases. Who's correct? Explain your reasoning.

When a rectangle is graphed along gridlines, you can determine the perimeter and area by simply counting units or square units on the coordinate plane. This is true if all coordinates are integers. If they are fractions or decimals, it presents a challenge.

Analyze rectangle *RSTU* on the coordinate plane shown.

Think about:

Notice the intervals along the axes.



3. Calculate the perimeter and area of rectangle *RSTU*.

4. How would doubling the height of the rectangle affect the area?

5. How would doubling the length of the base of the rectangle affect the area?

Shantelle used another strategy to determine the perimeter and area of rectangle *RSTU* from the previous activity.

Shantelle

If I translate rectangle *RSTU* to have at least one point of image *R′S′T′U′* on the origin, it is easier to calculate the perimeter and area of rectangle *RSTU* because one of the points will have coordinates (0, 0).

6. How do you know a translation of rectangle *RSTU* will have the same area and perimeter as the pre-image *RSTU*? Explain your reasoning.

7. Explain why Shantelle's rationale is correct.

8. Translate rectangle *RSTU* so that point *R* is located at the origin.

a. List the coordinates of rectangle *R′S′T′U′*.

b. Determine the perimeter and area of *R′S′T′U′*. What do you notice?

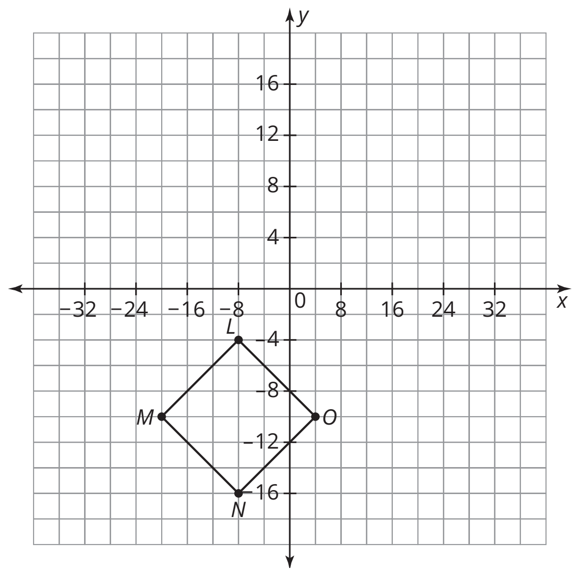
When the sides of a rectangle do not lie on the gridlines of the coordinate plane, you can use the Distance Formula to determine the lengths of the sides.

Think about:

Is the quadrilateral a square, a rectangle, or a rhombus?

9. Consider quadrilateral *LMNO*.

a. Determine the perimeter and area of quadrilateral *LMNO*. Round your answer to the nearest hundredth, if necessary.

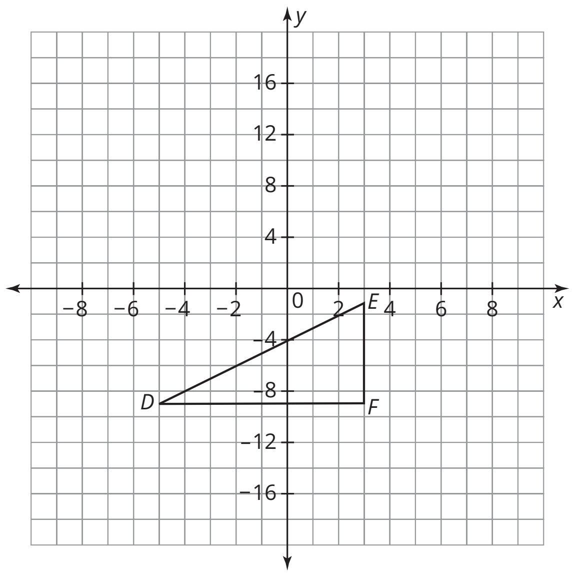


b. Double the side lengths of quadrilateral *LMNO*. How does this affect the area? What are the new coordinates?

c. Describe how you could translate quadrilateral *LMNO* to make the perimeter and area calculations more efficient.

10. Consider $\triangle DEF$ with vertices D (−5, −9), E (3, −1), and F (3, −9).

a. Determine the perimeter and area of $\triangle DEF$. Round your answer to the nearest hundredth, if necessary.

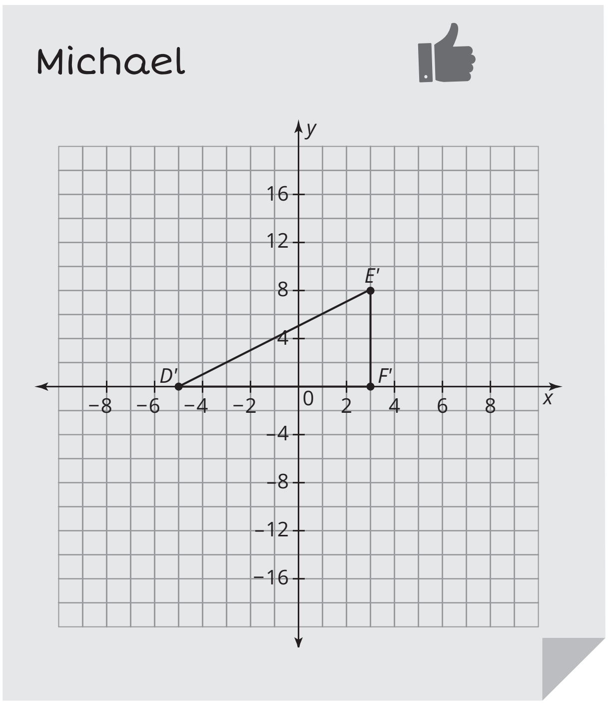


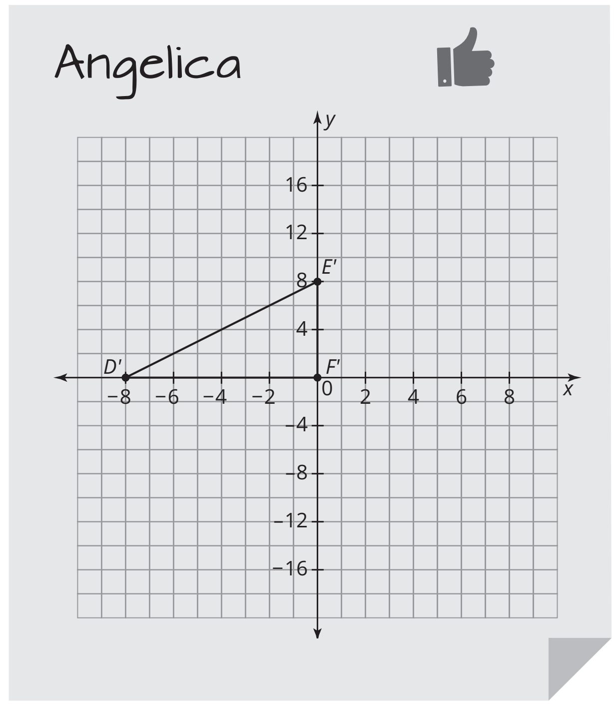
b. Double the height. What are the coordinates of the new triangle? How did this affect the area?

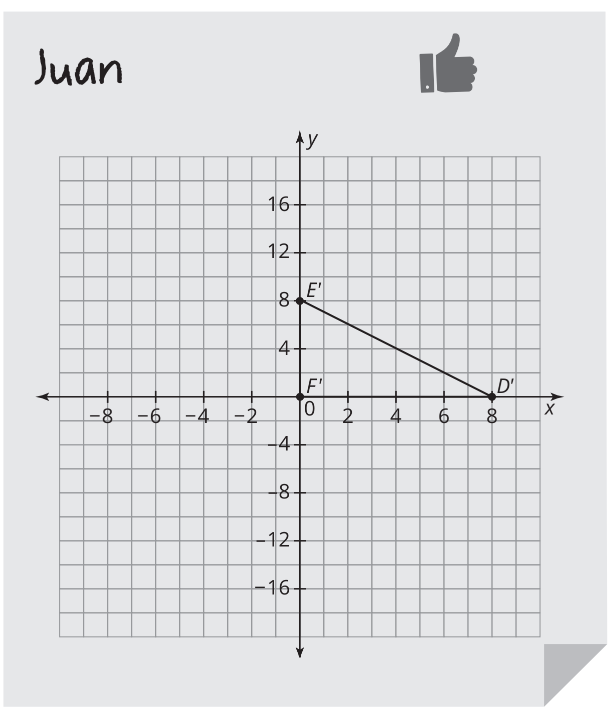
c. Double the length of the base. What are the coordinates of the new triangle? How did this affect the area?

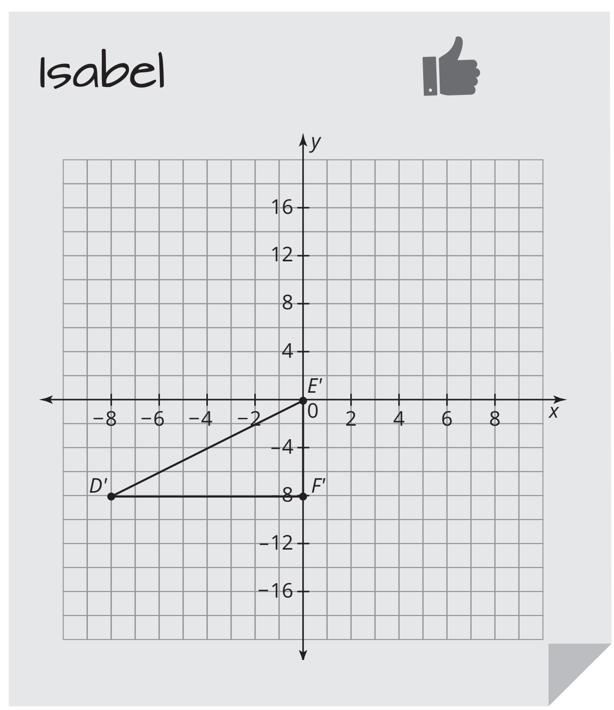
d. Double the length of both the base and the height. How does this affect the area?

11. **Mr. Young gives his class △*DEF* and asks them to determine the area and perimeter. Four of his students decide to first transform the figure and then determine the perimeter and area. Their transformations are shown.**





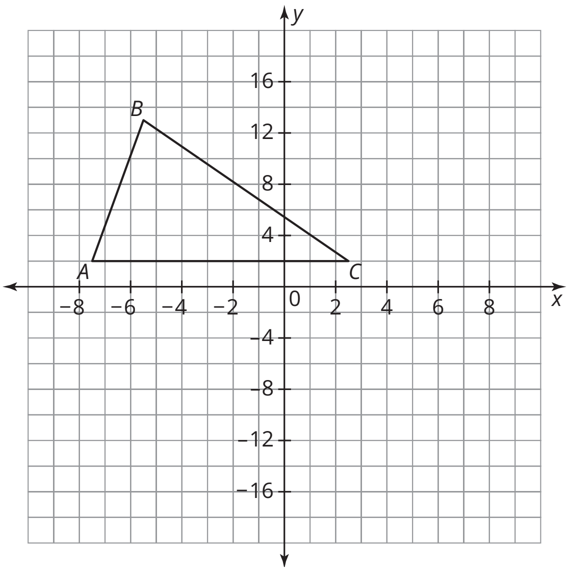




a. Describe the transformation(s) each student made to △*DEF*.

b. Whose method do you think is most efficient? Explain your reasoning.

c. What do you know about the perimeter and area of all the triangles? Explain your reasoning.



Consider $\triangle ABC$ with vertices A (−7.5, 2), B (−5.5, 13), and C (2.5, 2).

12. Determine the perimeter of $\triangle ABC$. Round your answer to the nearest hundredth, if necessary.

13. Consider how to determine the area of $\triangle ABC$.

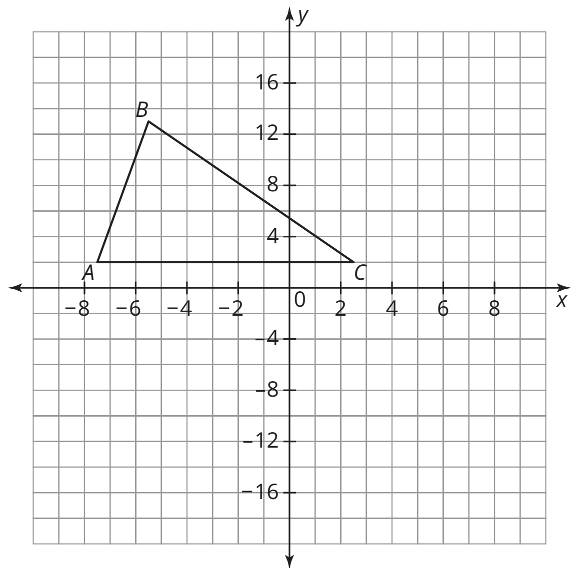
a. What information is needed about $\triangle ABC$ to determine its area?

b. Arlo says that $\overline{AB}$ can be used as the height. Trisha disagrees and says that $\overline{BC}$ can be used as the height. Randy disagrees with both of them and says that none of the line segments currently on the triangle can be used as the height. Who is correct? Explain your reasoning.

c. Draw and label $\overline{BD}$ to represent the height of $\triangle ABC$. Then, determine the height of $\triangle ABC$.

d. Determine the area of $\triangle ABC$.

14. Consider a more efficient way to determine the area and perimeter of $\triangle ABC$.

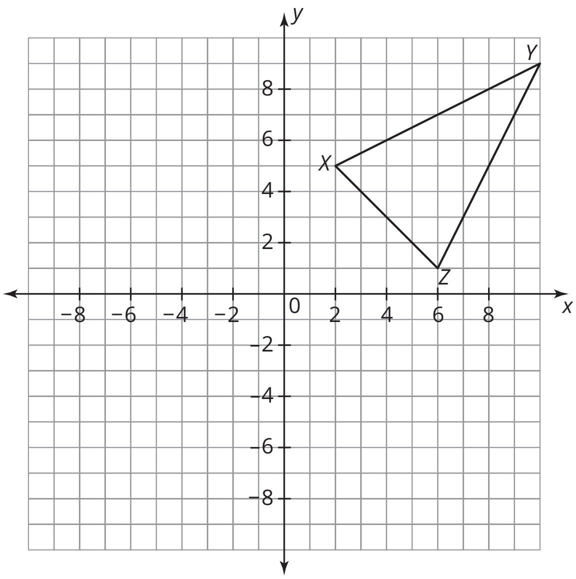


a. Transform $\triangle ABC$ on the coordinate plane. Label the image $A'B'C'$. Describe the transformation(s) completed and explain your reasoning.

b. Determine the perimeter and area of $\triangle A'B'C'$. Round your answer to the nearest hundredth, if necessary.

c. Compare these calculations to your previous calculations. How did the translation change your calculations?

ACTIVITY 5.2 Calculating Heights of Triangles



Consider $\triangle XYZ$ with vertices X (2, 5), Y (10, 9), and Z (6, 1).

1. Determine the perimeter of $\triangle XYZ$. Round your answer to the nearest hundredth, if necessary.

2. To determine the area, you will need to determine the height. How will determining the height of this triangle be different from determining the height of the triangles in previous activities?

3. Jonas wanted to transform the triangle to make the calculations easier. How can you determine that Jonas did not transform the triangle correctly?

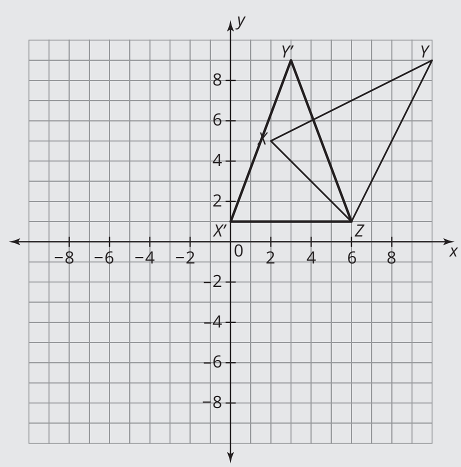
Remember:

The altitude, or height, of a triangle is the perpendicular distance from a vertex to the line containing the opposite side.

Jonas

I can rotate the triangle so the base is on a gridline to determine its height.

The triangle has a height of 8 units.



Let's use $\overline{XY}$ as the base of $\triangle XYZ$. You can draw $\overline{ZW}$ to represent the height. Remember that the height is perpendicular to the base. To determine the length of the height, you need to locate point W, which is located at the intersection of $\overline{XY}$ and $\overline{ZW}$.

Worked Example

Calculate the slope of the base, $\overline{XY}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{10 - 2} = \frac{4}{8} = \frac{1}{2}$$

Determine the slope of the height, $\overline{ZW}$.

$$m = -2$$

You can write equations for $\overleftrightarrow{XY}$ and $\overleftrightarrow{ZW}$ and solve the system to determine where the two lines intersect.

Determine the equations of the lines containing the base and the height.

	Base $\overleftrightarrow{XY}$	Height $\overleftrightarrow{ZW}$
	$X(2, 5)$, $m = \frac{1}{2}$	$Z(6, 1)$, $m = -2$
	$y - y_1 = m(x - x_1)$	$y - y_1 = m(x - x_1)$
	$y - 5 = \frac{1}{2}(x - 2)$	$y - 1 = -2(x - 6)$
	$y = \frac{1}{2}x + 4$	$y = -2x + 13$

Solve the system of equations to determine the coordinates of the point of intersection.

	$\frac{1}{2}x + 4 = -2x + 13$	$y = -2x + 13$
	$\frac{5}{2}x = 9$	$y = -2\left(\frac{18}{5}\right) + 13$
	$x = \frac{18}{5}$	$y = \frac{29}{5}$

Remember:

The slopes of perpendicular lines are opposite reciprocals.

4. Identify the coordinates of the point of intersection. Plot this point on the coordinate plane and label it point W. Draw $\overline{ZW}$ to represent the height.

5. Determine the area of $\triangle XYZ$.

a. Determine the height of the triangle.

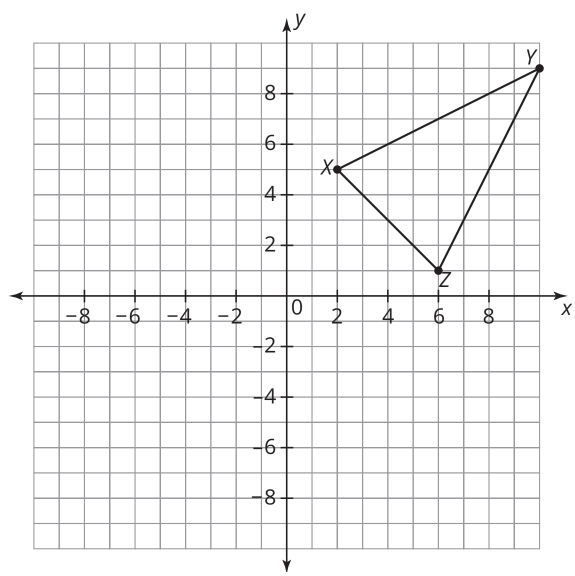
b. Determine the area of the triangle.

You know that any side of a triangle can be thought of as the base of the triangle.

6. Predict whether using a different side as the base will result in a different area of the triangle. Explain your reasoning.

Let's consider your prediction.

7. Triangle *XYZ* is graphed on the coordinate plane. This time consider side $\overline{XZ}$ as the base.

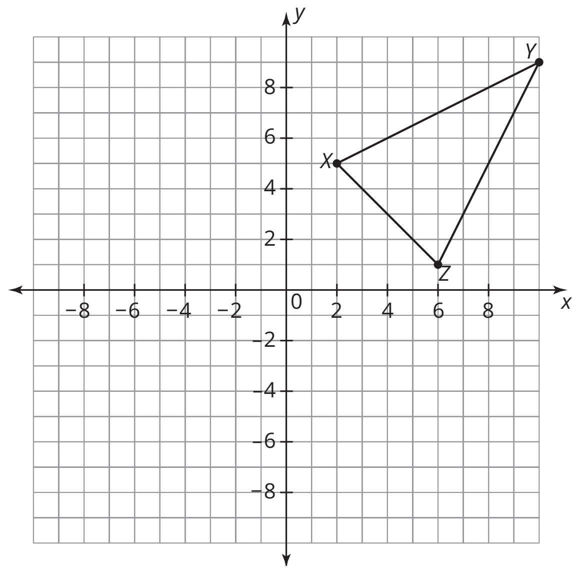


a. Let point *V* represent the intersection point of the height, $\overline{YV}$, and the base. Determine the coordinates of point *V*.

b. Determine the height of $\triangle XYZ$.

c. Determine the area of $\triangle XYZ$.

8. Triangle *XYZ* is graphed on the coordinate plane. Determine the area of $\triangle XYZ$ using side $\overline{YZ}$ as the base.



9. Compare the three areas you determined for $\triangle XYZ$. Was your prediction in Question 6 correct?

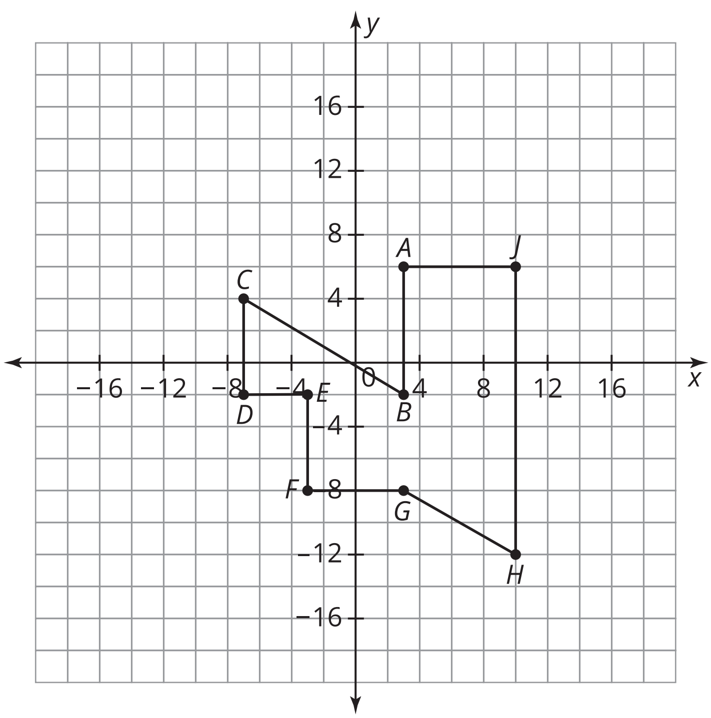
ACTIVITY
5.3

Perimeter and Area of a Composite Figure

The method you used to determine the perimeter of a rectangle or triangle can be used with any polygon. You can use the Distance Formula to calculate the distance between any set of vertices and then add the lengths of all the sides.

You can determine the area of a *composite figure* by dividing the figure into a combination of rectangles and triangles. A **composite figure** is a figure that is formed by combining different shapes.

Carter has an irregular backyard because it backs onto the foothill of a mountain and is very rocky. The composite figure graphed on the coordinate plane represents the flat area of Carter's backyard. Each interval of the coordinate plane represents two yards.



1. **Carter will install fencing all around the flat area of his backyard. Determine the amount of fencing he needs to the nearest whole yard.**

2. **Carter wants to lay grass sod in the flat area of his backyard. Determine the total area of sod he needs.**

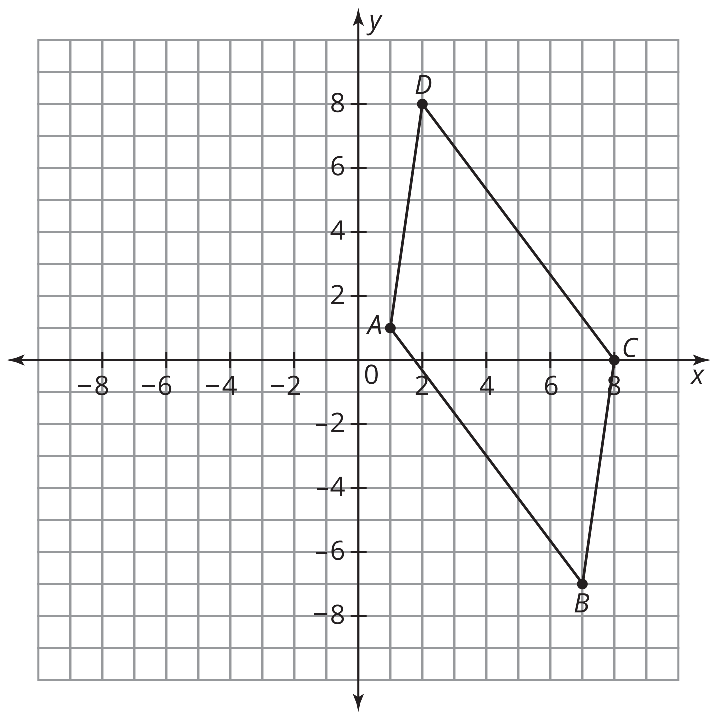
3. **Compare the method you used to determine the area of sod Carter needs to your classmates' methods. If you had a different way of dividing up the composite figure, did your answers differ? Explain why or why not.**

4. **Fencing costs $5.45 per foot and sod costs $0.62 per square foot. To allow for measurement error, Carter plans to buy an extra 10% of both materials. How much will it cost Carter to purchase these materials?**

ACTIVITY **5.4**

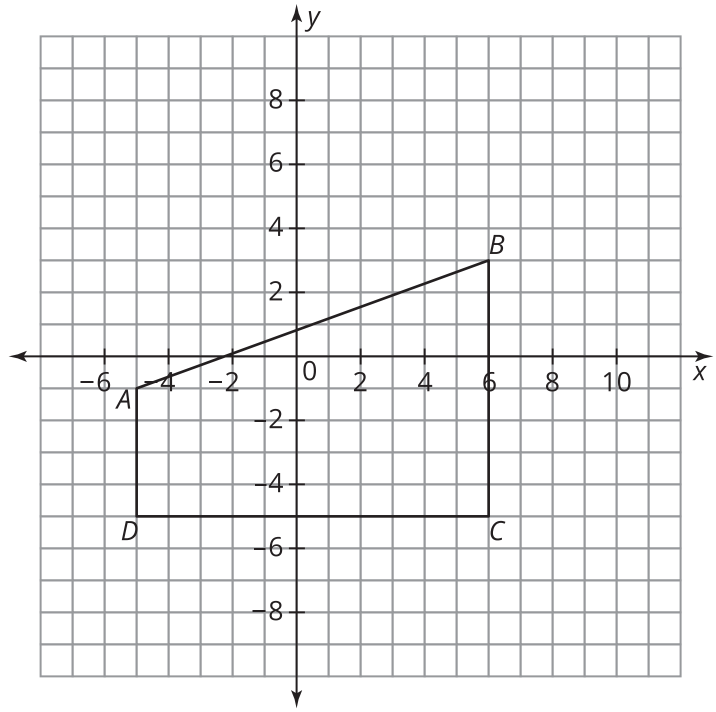
Solving Problems with Perimeter and Area

A pattern for a quilt patch is drawn on a coordinate plane, where each interval represents one inch. Parallelogram *ABCD* represents the patch.



1. Bryce is in charge of buying the ribbon that will be sewn around the outside of each patch. How many inches of ribbon are needed for each patch?

Aida's bedroom is on the top floor of her house. In her room, the roof slants downward, creating two congruent trapezoid-shaped walls. One of the walls in her room is represented on the coordinate plane by quadrilateral *ABCD*. Each interval on the coordinate plane represents one foot.



Aida and Marco are going to paint the two walls and want to place a strip of painter's tape along each edge of the walls so the paint does not touch any other wall, the ceiling, or the floor.

2. What is the length of painter's tape (to the nearest whole foot) that Aida and Marco need to cover the edges of both walls?

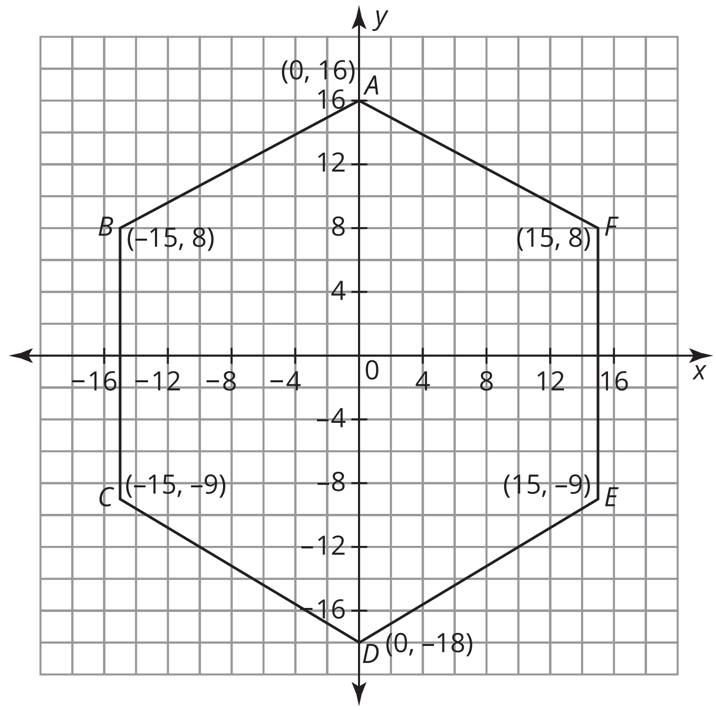
Ask yourself:

How can you use a transformation of trapezoid *ABCD* on the coordinate plane as part of your strategy?

3. Marco says he can draw a diagonal to divide trapezoid *ABCD* into a right and an isosceles triangle to determine the area of the trapezoid. Aida says she can draw a horizontal line segment to divide trapezoid *ABCD* into a rectangle and a right triangle. Who's correct? Explain your reasoning.

4. One gallon of paint covers approximately 400 square feet. Aida estimates she has about one fourth of a gallon of paint remaining of the color she wants to use. Does she have enough paint for both walls? Explain your reasoning.

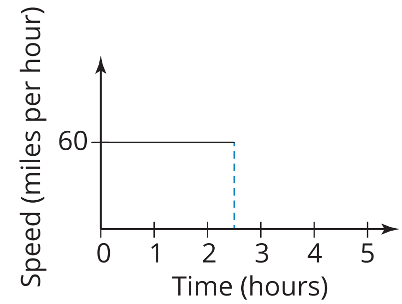
Emma and Kevin are designing a gazebo for the local park. The polygon shown on the coordinate plane represents the base of the gazebo. Each interval on the coordinate plane represents two feet.



5. The base of the gazebo needs to be built with lengths of lumber around the outside to support the floorboards. What is the length of lumber needed for the outside of the base?

6. How many square feet of floorboards are needed for the base of the gazebo? Describe how you determined your answer and show your work.

The graph shows the constant speed of a car on the highway over the course of 2.5 hours.

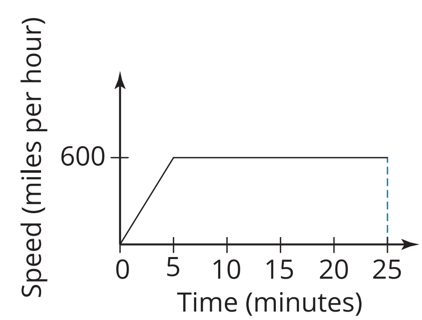


7. Describe how you could calculate the distance the car traveled in 2.5 hours using what you know about area.

8. How far did the car travel in 2.5 hours?

The graph you used is called a velocity-time graph. In a velocity-time graph, the area under the line or curve gives the distance.

The graph shown describes the speed and the time of a passenger jet's ascent.



9. How can you use the graph to determine the distance the jet has traveled in 25 minutes?

10. Determine the distance the jet has traveled:

a. in the first 5 minutes.

b. in 25 minutes.

11. Consider the ascent of a passenger jet.

a. Draw a velocity-time graph to model the ascent of a passenger jet using the information given.

- **The jet took 7 minutes to reach a top speed of 600 miles per hour.**
- **The jet continued to travel at a constant speed of 600 miles per hour.**
- **The jet left the airport 4 hours ago.**

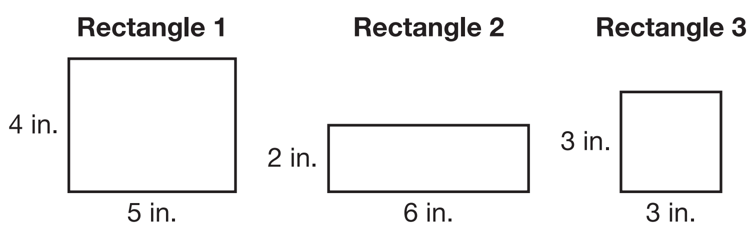
b. How many miles has the jet traveled?

ACTIVITY **5.5**

Proportional and Non-Proportional Changes in Dimensions

In the activity *Perimeter and Area of Figures on the Coordinate Plane,* you investigated how doubling one or both of the dimensions of a figure affected its area. Now let's investigate how both proportional and non-proportional changes in a figure's dimensions affect its perimeter and area.

1. Consider the following rectangles with the dimensions shown.



Complete the table to determine how doubling or tripling each rectangle's base and height affects its perimeter and area. The information for Rectangle 1 has been done for you.

		Original Rectangle	Rectangle Formed by Doubling Dimensions	Rectangle Formed by Tripling Dimensions
Rectangle 1	Linear Dimensions	b = 5 in. h = 4 in.	b = 10 in. h = 8 in.	b = 15 in. h = 12 in.
	Perimeter (in.)	2(5 + 4) = 18	2(10 + 8) = 36	2(15 + 12) = 54
	Area (in.^{2})	5(4) = 20	10(8) = 80	15(12) = 180
Rectangle 2	Linear Dimensions			
	Perimeter (in.)			
	Area (in.^{2})			
Rectangle 3	Linear Dimensions			
	Perimeter (in.)			
	Area (in.^{2})			

Remember:

Multiplying side lengths by a number is a proportional change.

2. Describe how a proportional change in the linear dimensions of a rectangle affects its perimeter.

a. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 2?

b. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 3?

c. What would happen to the perimeter of a rectangle when its dimensions increase by a factor of 4?

d. Describe how you think the perimeter of the resulting rectangle would compare to the perimeter of a 4×10 rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the perimeter of the resulting rectangle.

e. In terms of k, can you generalize change in the perimeter of a rectangle with base b and height h, given that its original dimensions are multiplied by a factor k?

3. **Describe how a proportional change in the linear dimensions of a rectangle affects its area.**

   a. **What happens to the area of a rectangle when its dimensions increase by a factor of 2?**

   b. **What happens to the area of a rectangle when its dimensions increase by a factor of 3?**

   c. **What would happen to the area of a rectangle when its dimensions increase by a factor of 4?**

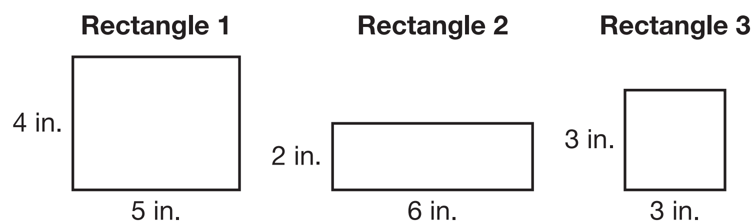
   d. **Describe how you think the area of the resulting rectangle would compare to the area of a 4×10 rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the area of the resulting rectangle.**

   e. **In terms of *k*, can you generalize change in the area of a rectangle with base *b* and height *h*, given that its original dimensions are multiplied by a factor *k*?**

Non-proportional change to linear dimensions of a two-dimensional figure involves adding or subtracting from the side lengths.

4. Do you think a non-proportional change in the linear dimensions of a two-dimensional figure will have the same effect on perimeter and area as proportional change? Explain your reasoning.

5. Consider the following rectangles with the dimensions shown.



Complete the table to determine how adding two or three inches to each rectangle's base and height affects its perimeter and area The information for Rectangle 1 has been done for you.

		Original Rectangle	Rectangle Formed by Adding 2 Inches to Dimensions	Rectangle Formed by Adding 3 Inches to Dimensions
Rectangle 1	Linear Dimensions	$b = 5$ in. $h = 4$ in.	$b = 7$ in. $h = 6$ in.	$b = 8$ in. $h = 7$ in.
	Perimeter (in.)	$2(5 + 4) = 18$	$2(7 + 6) = 26$	$2(8 + 7) = 30$
	Area (in.^{2})	$5(4) = 20$	$7(6) = 42$	$8(7) = 56$
Rectangle 2	Linear Dimensions			
	Perimeter (in.)			
	Area (in.^{2})			
Rectangle 3	Linear Dimensions			
	Perimeter (in.)			
	Area (in.^{2})			

6. Describe how a non-proportional change in the linear dimensions of a rectangle affects its perimeter.

a. What happens to the perimeter of a rectangle when 2 inches are added to its dimensions?

b. What happens to the perimeter of a rectangle when 3 inches are added to its dimensions?

c. What would happen to the perimeter of a rectangle if 4 inches are added to its dimensions?

d. Describe how you think the perimeter of the resulting rectangle would compare to the perimeter of a 4×10 rectangle if the dimensions of the original rectangle were reduced by 2 inches. Then, determine the perimeter of the resulting rectangle.

e. Given that a rectangle's original dimensions change by x units, generalize the change in the perimeter in terms of x.

7. **Describe how a non-proportional change in the linear dimensions of a rectangle affects its area.**

   a. **What happens to the area of a rectangle when its dimensions increase by 2 inches?**

   b. **What happens to the area of a rectangle when its dimensions increase by a factor of 3?**

   c. **What would happen to the area of a rectangle when its dimensions increase by a factor of 4?**

d. Describe how you think the area of the resulting rectangle would compare to the area of a 4 × 10 rectangle if the dimensions of the original rectangle were reduced by 2 inches. Then, determine the area of the resulting rectangle.

e. What happens to the area when the dimensions of a rectangle change non-proportionally?

f. Summarize the change in a rectangle's area caused by a non-proportional change in its linear dimensions and the change in a rectangle's area caused by a proportional change in its linear dimensions.

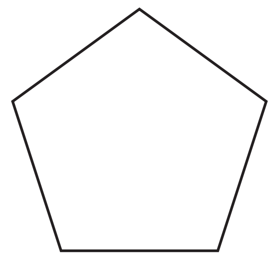
ACTIVITY

5.6

Areas of Regular Polygons

A **regular polygon** is a polygon with all sides congruent and all angles congruent. The area of a regular polygon can be thought of as the area of a composite shape.

1. A regular pentagon is shown.



a. Locate and place a point at the center of the regular pentagon. From the center point, draw line segments to connect the point with each vertex of the pentagon.

b. Describe the new polygons formed by adding these line segments.

c. What information do you need to calculate the area of each new polygon?

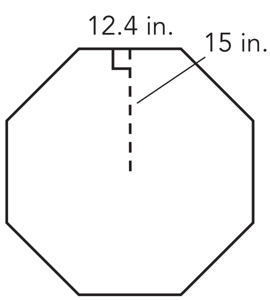
d. What formula is used to calculate the area of each new polygon?

e. Describe a strategy to determine the area of the entire regular pentagon.

2. How can you use this strategy to determine the area of other regular polygons?

For each regular polygon, draw in the congruent triangles and then calculate the area.

3.



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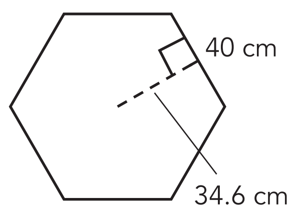
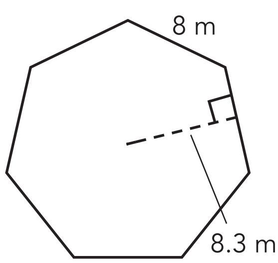
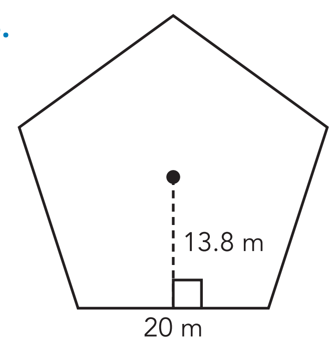


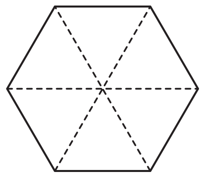
5.



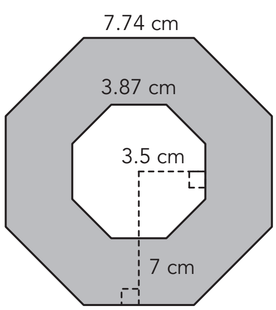
6.



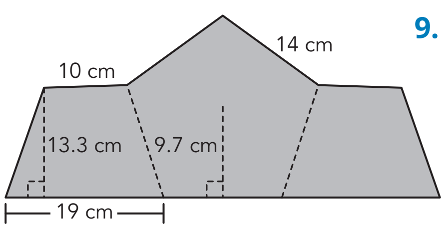
Solve each problem involving regular polygons.



7. A regular hexagon has an area of 540 cm^2, and its side lengths are each 15 cm. Calculate the height of the congruent triangles in the hexagon.



8. A regular octagon has an octagonal hole as shown. Calculate the area of the shaded region.

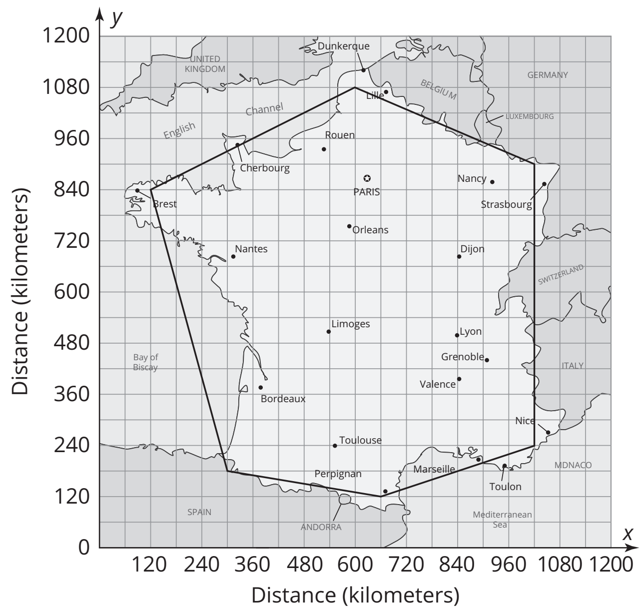


9. Calculate the area of the figure composed of a regular pentagon and 2 congruent trapezoids.

TALK the TALK

Vive les Maths!

Eva is using a map to estimate the area of France. She thinks the country looks like a hexagon and draws the polygon shown to approximate its shape.



1. **Determine which statements are true. Justify your answers.**
   - **The coastline of France is greater than 5000 km.**

   - **The coastline of France is less than 5000 km.**

   - **The coastline of France is approximately 5000 km.**

   - **The area of France is greater than 1,000,000 sq km.**

   - **The area of France is less than 1,000,000 sq km.**

   - **The area of France is approximately 1,000,000 sq km.**

NOTES

NOTES

2. If the population of France is approximately 104 people per square kilometer, how many people live in France?