5

In and Out and All About

Area and Perimeter on the Coordinate Plane

MATERIALS

Ruler

Lesson Overview

Students calculate the perimeter and area of rectangles and triangles on the coordinate plane. They double dimensions of figures and explain how this affects the area of the figure; they also translate figures on the coordinate plane to more efficiently determine their perimeter and area. Students algebraically determine the non-vertical height of a triangle as they treat each side as the base; they then use the height to calculate the area of the triangle. They conclude that the area of a triangle remains the same regardless of the side considered as the base and the height determined by that base. Next, students divide a composite figure into various known polygons to compute its area. They then consider real-world situations requiring them to calculate the perimeter and area of polygons that lie on a coordinate plane using the Distance Formula and decomposing the polygons into triangles and rectangles. Students determine distances represented as the area under the curve of velocity-time graphs. They investigate how proportional and non-proportional changes in the linear dimensions of a shape affect its perimeter and area. Students develop a strategy for calculating areas of regular polygons.

Geometry
Coordinate and Transformational Geometry

(2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines.

(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point.

(3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

Geometry

Two-dimensional and Three-dimensional Figures

(10) Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:

(B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.

(11) Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:

(A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure.

(B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C,4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- Rigid motion transformations (translations, rotations, and reflections) can be used to change the position of figures on the coordinate plane.
- Performing translations on figures can help to compute perimeter and area more efficiently.
- Non-vertical heights of a figure can be calculated algebraically using formulas, writing equations and solving a system of equations.
- The area of a triangle is the same regardless of what base and height of the triangle are used in the calculation.
- A composite figure is a figure that is formed by combining different shapes.
- Polygons can be divided into a combination of triangles and rectangles to help determine their area.
- The area of a composite figure is determined by dividing the figure into familiar shapes and using the area formulas associated with those shapes.
- The Distance Formula, slope formula, and the Pythagorean Theorem can be used to determine the area of polygons and composite figures on the coordinate plane.
- A velocity-time graph can model acceleration, and distance can be determined by calculating the area under a curve.
- When the dimensions of a plane figure change proportionally by a factor of k, its perimeter changes by a factor of k, and its area changes by a factor of k^2.
- When the dimensions of a plane figure change non-proportionally, its perimeter and area increase or decrease non-proportionally.
- A regular polygon is a polygon with congruent sides and congruent angles.
- A regular n-gon can be decomposed into n congruent triangles.
- The area of a regular n-gon can be calculated by determining the area of one of the n congruent triangles and multiplying by n.

Lesson Structure and Pacing: 4 Days

Day 1

Engage

Getting Started: It's Child's Play

Students analyze a rectangle and a triangle presented on a coordinate grid in the context of a real-world problem. They describe a strategy to determine the area of the rectangle and the triangle.

Develop

Activity 5.1: Perimeter and Area of Figures on the Coordinate Plane

Students calculate the perimeter and area of a rectangle and describe how doubling one dimension affects the area. They then investigate how translating the rectangle can make the calculations of the area and perimeter more efficient. Next, students calculate the perimeter and area of a square and describe how doubling both dimensions affects the area. They calculate the perimeter and area of a right triangle and explain the effect of doubling one or both dimensions. Students analyze different strategies to transform triangles on the coordinate plane to make calculations more efficient. They analyze a different triangle to determine the fact that the height of a triangle must be perpendicular to the base. Students calculate its perimeter and area and then transform the triangle and re-calculate its perimeter and area.

Day 2

Activity 5.2: Calculating Heights of Triangles

Students calculate the area of a triangle with no horizontal or vertical sides. They consider the possibility of rotating the triangle to make the calculations more efficient. A worked example shows the steps necessary to determine the location of the height. They solve for the area of the same triangle three times, each time using a different side as the base. Through practice determining the location of the height of the triangle depending upon its base, students conclude that the area of the triangle remains the same regardless of the base and height used in their calculations.

Day 3

Activity 5.3: Perimeter and Area of a Composite Figure

Students are introduced to the term *composite figure*. They are given a diagram of a composite figure and calculate its perimeter and area in the context of a scenario. Students then compare their methods with their classmates' to determine whether dividing the composite figure into different shapes affects the area and perimeter of the original figure.

Activity 5.4: Solving Problems with Perimeter and Area

Students calculate the perimeter of a pattern modeled by a parallelogram that lies on a coordinate plane. They determine the perimeter of a trapezoid that lies on a coordinate plane and use it to answer a question in the context of a scenario. Students then analyze different ways the trapezoid can be divided into other shapes. They revisit the scenario to calculate the area of the trapezoid to answer another question. Students calculate the perimeter and area of a hexagon to answer questions based on a context. Students analyze scenarios and their velocity-time graphs. They make the connection between the formula *distance* = (*rate*)(*time*) and the product (*rate*)(*time*) modeled by the area under a curve where the base represents time on the x-axis and the height represents the rate (or speed or velocity) on the *y*-axis. Students calculate the distance traveled in each scenario by calculating the area of the polygon constructed with the boundaries on the graph. Students also reverse the process and sketch a velocity-time graph based on a scenario.

Day 4

Activity 5.5: Investigating Changing Dimensions

Students are given 3 different rectangles and asked to complete a table to record the changes in the perimeters and areas of the rectangles after a doubling or tripling of each rectangle's dimensions. They then answer questions and generalize about the effects on perimeter and area of proportional changes to the dimensions of a rectangle. This activity is repeated with non-proportional changes to the dimensions.

Activity 5.6: Area of Regular Polygons

Students develop a strategy for calculating the areas of regular polygons. Students decompose regular polygons and determine that they should calculate the area of one of the n congruent triangles in the n-gon and multiply the area by n to calculate the area of the regular polygon. They practice this strategy by calculating the areas of regular polygons and composite figures with regular polygons.

Demonstrate

Talk the Talk: Vive les Maths!

Students analyze a representation of France mapped onto a coordinate plane and answer questions about population density.

Facilitation Notes

In this activity, students analyze a rectangle and a triangle presented on a coordinate grid in the context of a real-world problem. They describe a strategy to determine the area of the rectangle and the area of the triangle.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

Misconception

Students may assume the base of a triangle must be the bottom of the triangle; for instance, in $\triangle NOP$, they may think the base must be $\overline{PO}$. Any side of a triangle can be considered the base of the triangle.

As students work, look for

An error identifying the height of $\triangle NOP$ when using $\overline{PO}$ as the base. Students may use the length of $\overline{NP}$ as the height of the triangle rather than determining the length of a perpendicular line segment dropped from point N to an extension of $\overline{PO}$.

Questions to ask

- If you were counting blocks, how did you count partial blocks?
- If you used the area formula, how did you determine the base and height for the rectangular playground?
- What is the area formula for a triangle?
- Which side(s) can be considered the base of the triangle?
- How did you calculate the measure of the base? The measure of the height?
- If $\overline{PO}$ is considered the base, how do you determine the height of the triangle?

Differentiation strategies

To extend the activity,

- Have students use their strategies to calculate the area of each play area.
- Have different groups of students calculate the area of the triangle by using a different side of the triangle as the base. Then, have them compare their work and solutions.

Summary

When solving for area of triangles and quadrilaterals, the base and height must be perpendicular segments. It may be necessary to use the distance formula to determine the length of either segment.

DEVELOP

Activity 5.1

Perimeter and Area of Figures on the Coordinate Plane

Facilitation Notes

In this activity, students calculate the perimeter and area of a rectangle and describe how doubling one dimension affects the area. They then investigate how translating the rectangle can possibly make the calculations of the area and perimeter more efficient. Next, students calculate the perimeter and area of a square and describe how doubling both dimensions affects the area. They calculate the perimeter and area of a right triangle and explain the effect of doubling one or both dimensions. Students analyze different strategies to transform triangles on the coordinate plane to make calculations more efficient. They analyze a different triangle to determine the fact that the height of a triangle must be perpendicular to the base. Students calculate its perimeter and area and then transform the triangle and re-calculate its perimeter and area.

Differentiation strategy

To scaffold support, allow students to use a scale of one rather than the different scales labeled on the graphs throughout this activity.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What unit of measure is used to describe the perimeter of rectangle *ABCD*?
- What unit of measure is used to describe the area of rectangle *ABCD*?
- What does the expression *CD*(*CB*) mean?
- How do you know what side of a rectangle to use as its base?
- If Bernice uses $\overline{AB}$ as the base and Horace uses $\overline{CD}$ as the base, will they get the same area?
- What other combinations of sides could be used for the base and height?

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

As students work, look for

A perimeter of 18 units and an area of 18 square units, signifying that the scale of the graph was not taken into account.

Differentiation strategies

To extend the activity,

- Discuss different solution paths to calculate the perimeter and area.

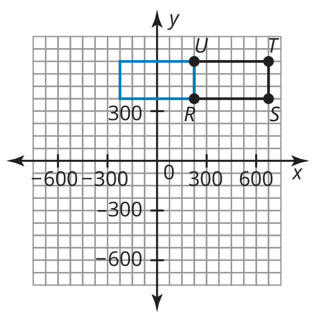
Method 1

$b = 450$ units and
$h = 225$ units
Perimeter $= 2(450) + 2(225)$
$= 1350$ units
Area $= (450)(225)$
$= 101,250$ square units

Method 2

Perimeter $=$ 18 grid spaces
1 grid space $=$ 75 units
Perimeter $= (18)(75) = 1350$ units
Area $=$ 18 grid blocks
1 grid block $= (75)(75)$
$= 5625$ square units
Area $= (18)(5625) = 101,250$ square units

- Have students demonstrate how the area is affected by doubling a dimension using different methods. You may want to revisit these strategies as students complete Questions 9 and 10.

Doubling the Base		
Using Arithmetic	**Using Geometry**	**Using Algebra**
$b = 2(450) = 900$ $h = 225$ Area $= (900)(225)$ $= 202,500$ $\frac{202,500}{101,250} = 2$		Area $=$ base $\times$ height Area $= (2b)(h)$ Area $= 2bh$

Questions to ask

- How is this graph different from the previous graph?
- What is the scale on the *x*-axis and the *y*-axis of this graph?
- How did you determine the scale?
- What were your steps to calculate the perimeter? The area?
- How can you demonstrate that doubling either the height or base of the rectangle also doubles the area?

Have students work with a partner or in a group to complete Questions 6 and 7. Share responses as a class.

Questions to ask

- Do translations preserve size and shape?
- Do all rigid motion transformations preserve both size and shape?
- If translations preserve size, is the length and width of the transformed rectangle equal to the length and width of the original rectangle?

- If the length and width of the rectangle remain the same, does the perimeter and area also remain the same?
- If Shantelle relocates one vertex of rectangle *RSTU* to the origin, how many different ways can Shantelle transform rectangle *RSTU*?

Have students work with a partner or in a group to complete Question 8. Share responses as a class.

Questions to ask

- How would you describe the horizontal and vertical translations of each point when point *R* is relocated at the origin?
- If point *R* is moved to the origin, is it translated up or down?
- If point *R* is moved to the origin, is it translated left or right?
- What is the approximate value of $\sqrt{125}$? $\sqrt{185}$?
- If Shantelle translates point *R* to the origin, what are the coordinates of the other three vertices?

Differentiation strategy

To scaffold support, provide additional practice by having students translate rectangle *RSTU* three more times, where each translation results in moving a different vertex to the origin.

Have students work with a partner or in a group to complete Questions 9 and 10. Share responses as a class.

Differentiation strategy

To scaffold support, have students use colored pencils to distinguish among the different triangles they create in Question 10.

Questions to ask

- How did you determine the length of a side of square *LMNO*?
- What information is needed to determine the perimeter of the square? The area of the square?
- Does doubling the side length of a square double the area of the square? Why not?
- What could be possible coordinates of the square when both side lengths are doubled?
- Demonstrate using the area formula why the area of the square is quadrupled.
- Did you translate one of the vertices of quadrilateral *LMNO* onto the *x*-axis, the *y*-axis, or the origin? Which vertex?
- What values did you use for the base and height of the triangle?
- How did you determine the measure of the third side of the triangle?
- Demonstrate using the area formula why the area of the triangle is doubled.
- Demonstrate using the area formula why the area of the triangle is quadrupled.

Have students work with a partner or in a group to complete Question 11. Share responses as a class.

Questions to ask

- How is Isabel's transformation different than those of the other students?
- Which student(s) performed only a single vertical translation?
- Which student(s) performed only a single horizontal translation?
- Which student(s) performed both a vertical and horizontal translation?
- How did you determine the distance each triangle was translated?

Have students work with a partner or in a group to complete Questions 12 and 13. Share responses as a class.

Questions to ask

- What is the measure of each side length?
- What relationship must exist between the base and height of a triangle?
- Using $\overline{AC}$ as the base, how is the height of $\triangle ABC$ determined?

Have students work with a partner or in a group to complete Question 14. Share responses as a class.

Differentiation strategy

To scaffold support, provide students patty paper to help them visualize different rigid motion transformations before sketching a transformation on the graph.

As students work, look for

Different translations of $\triangle ABC$. Some students may move a single vertex of the triangle to the origin while others may move vertices to the axes. Share different strategies to accomplish the task.

Questions to ask

- How is the location of your triangle different than the location of the original triangle?
- Why did you decide to move the triangle to that location?
- Did you move any vertex to the origin?
- Did you move any vertex onto the x-axis? Onto the y-axis?
- How could the triangle be translated to easily compute the vertical height?
- If point B was translated to a position on the y-axis, and points A and C translated to positions on the x-axis, could the vertical height of the triangle be easily calculated?
- How can you determine the length of the base and the height of this triangle?

- How would you describe the difference between these calculations and the previous calculations?
- What characteristics makes one calculation method easier than another?

Summary

Doubling the measure of the base or the height of a rectangle, square, or triangle doubles the area of the figure. Doubling the measure of the base and the height of a rectangle, square, or triangle quadruples the area of the figure. Rigid motion transformations can make calculating the base and height of figures on the coordinate plane more efficient.

Activity 5.2
Calculating Heights of Triangles

Facilitation Notes

In this activity, students calculate the area of a triangle with no horizontal or vertical sides. They consider the possibility of rotating the triangle to make the calculations more efficient. A worked example shows the steps necessary to determine the location of the height. They solve for the area of the same triangle three times, each time using a different side as the base. Through practice determining the location of the height of the triangle depending upon its base, students conclude that the area of the triangle remains the same regardless of the base and height used in their calculations.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

The use of patty paper, which may be deceiving. Students should use the Distance Formula to verify that Jonas did not transform the triangle correctly.

Questions to ask

- How is the perimeter of $\triangle XYZ$ determined?
- What formula is used to determine the length of the sides of $\triangle XYZ$?
- What is the length of side $\overline{XY}$? Side $\overline{YZ}$? Side $\overline{XZ}$?
- What type of triangle is $\triangle XYZ$?
- How is the orientation of $\triangle XYZ$ different than the other triangles you have dealt with?
- Can the height of $\triangle XYZ$ be represented using a vertical line segment? Why not?

- What did Jonas do incorrectly?
- How can you use the fact that rigid motions preserve size and shape to show that Jonas did not transform the triangle correctly?
- What are the coordinates of the vertices of $\triangle X'Y'Z$?
- Is $\triangle XYZ \cong \triangle X'Y'Z$?
- How does the length of $\overline{XZ}$ compare to the length of $\overline{X'Z}$?

Ask a student to read the information following Question 3. Analyze the worked example and complete Questions 4 and 5 as a class.

Differentiation strategies

- To scaffold support for all students,
  - Have them number the steps and take notes in the worked example.
  - Discuss how the point-slope formula is derived from the slope formula. Show how the equation for the height can be determined a different way.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$-2 = \frac{y - 1}{x - 6}$$
$$-2(x - 6) = y - 1$$
$$-2x + 12 = y - 1$$
$$-2x + 13 = y$$

- To scaffold support, provide a template with the directions on one side and blanks on the other side so that they can complete the steps with your guidance. Then, have copies of the template available for the remainder of this activity and the Talk the Talk.

Questions to ask

- If $\overline{XY}$ is treated as the base, predict where the height of the triangle should be drawn.
- How do you know where to draw the line segment representing the height of the triangle?
- Why do you need to calculate slope?
- What information do you need to write an equation of a line?
- How is the point-slope formula derived from the slope formula?
- What is the equation for the line that includes the base? The height?
- How do you determine the coordinates of the endpoints of the line segment representing the height of the triangle?
- Why do you have to solve a system of equations?
- How do you determine the height?
- Why is it better to keep your answers in radical form when calculating the area?

Have students work with a partner or in a group to complete Questions 6 through 9. Share responses as a class.

As students work, look for

The realization that because the triangle is isosceles, the bases that have the same measure also have heights with the same measure. Question 5, part (b) and Question 8 both use $A = \frac{1}{2}(\sqrt{80})\left(\sqrt{\frac{720}{25}}\right) = 24$.

Questions to ask

- If $\overline{XZ}$ is treated as the base, where is the height of the triangle located?
- As you change what side of the triangle is considered the base, does the height of the triangle also change? Why or why not?
- How do you know where to draw the line segment representing the height of the triangle?
- What information do you need to write an equation for a line?
- What is the equation for the line that includes the base? The height?
- How do you determine the coordinates of the endpoints of the line segment representing the height of the triangle?
- What is the meaning of the solution to your system of equations?
- Why do you need to know the coordinates for the endpoints of the height?
- Were you surprised by your answer for the area of $\triangle XYZ$? Why or why not?
- If $\overline{YZ}$ is treated as the base, where is the height of the triangle located?
- What calculations did you use to determine the area?
- Why do your calculations for the area in Question 8 appear similar to those in Question 5, part (b)?
- How does knowing the height when $\overline{XY}$ is the base help you know the height when $\overline{YZ}$ is the base?
- Why should the two heights be the same length?
- Why are all the areas equal to 24 square units?
- Thinking about the area formula, if the three bases of a triangle are different lengths and the areas are the same value, what does this imply about the heights?

Differentiation strategy

To extend the activity, perform a similar activity with a rectangle drawn on a coordinate plane where none of its sides are vertical or horizontal segments.

Summary

The area of a triangle remains the same regardless of the side considered the base. Any side of a triangle can be considered the base, and the triangle's height is identified by the location of that base.

Activity 5.3
Perimeter and Area of a Composite Figure

Facilitation Notes

In this activity, students are introduced to the term *composite figure*. They are given a diagram of a composite figure and calculate its perimeter and area in the context of a scenario. Students then compare their methods with their classmates' to determine whether dividing the composite figure into different shapes affects the area and perimeter of the original figure.

Ask a student to read the introduction and definition aloud. Discuss as a class.

Differentiation strategy

To scaffold support, discuss how the mathematical term composite is related to other terms to which they may be familiar, such as an English, art, or music composition.

Questions to ask

- What is a composite figure?
- What shapes can you visualize in this composite figure?
- How would you describe the orientation of the sides of the composite figure on the coordinate plane? Which segments have a vertical orientation? Which segments have a horizontal orientation?
- Which segments are neither vertically or horizontally orientated? What is the most efficient way to determine their length?
- How many sides does this composite figure have?

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

Failure to consider the scale.

Questions to ask

- Do you need to use the Distance Formula to calculate the length of each of the nine sides of the composite figure? Why or why not?
- Which sides require use of the Distance Formula?
- What unit of measure is used to describe the perimeter?
- What is sod?
- What shapes did you use to divide the Carter's irregular backyard?
- Is there more than one way to divide this composite figure into familiar shapes? How?
- Did you use a trapezoid? Or did you use rectangles and triangles only? Why did you make that decision?
- Would transforming the composite figure be helpful? Why or why not?

- Which solution method was most efficient? Why?
- Is there a solution method you prefer more than your own? If so, why?

Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

Differentiation strategies

- To scaffold support for all students, draw a square yard on the board in actual size. Divide the square yard into nine square feet to demonstrate the conversion factor and help students visualize the size.
- To scaffold support with Question 4,
  - Provide guidance for the unit conversion or provide costs in terms of dollars per yard and dollars per square yard.
  - Remove the 10% from the problem situation. Calculate the cost of ordering only the materials needed to complete the job.

As students work, look for

- Correct unit conversions for square yards to square feet.
- Inclusion of the cost for the extra 10% of materials.

Questions to ask

- How many extra feet of fencing does Carter want to order?
- How many extra square feet of sod does Carter want to order?
- How did you calculate the total cost?
- When did you deal with the 10% in your calculations?
- Is there a more efficient way to calculate the total cost? If so, how?

Summary

The area of a composite figure can be determined by calculating the areas of the shapes that combine to create it and adding those areas together.

Activity 5.4
Solving Problems with Perimeter and Area

Facilitation Notes

In this activity, students calculate the perimeter of a pattern modeled by a parallelogram that lies on a coordinate plane. They determine the perimeter of a trapezoid that lies on a coordinate plane and use it to answer a question in the context of a scenario. Students then analyze different ways the trapezoid can be divided into other shapes. They revisit the scenario to calculate the area of the trapezoid to answer another question. Students calculate the perimeter and area of a hexagon in order to answer questions based on a context. Students analyze scenarios and their velocity-time graphs. They make the connection between the formula *distance* = (*rate*)(*time*) and the product (*rate*)(*time*) modeled by the area under a curve where the base represents time on the x-axis and the height represents the rate (or speed or velocity) on the y-axis. Students calculate the distance traveled in each scenario by calculating the area of the polygon constructed with the boundaries on the graph. Students also reverse the process and sketch a velocity-time graph based on a scenario.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Differentiation strategies

- To scaffold support, help them strategize by suggesting they identify the coordinates of each of the four vertices, then use the vertices to sketch right triangles using each side of the parallelogram as a hypotenuse.
- To extend the activity, ask students to calculate the area of the parallelogram.

Questions to ask

- Did you need to determine the perimeter of parallelogram *ABCD* or the area of parallelogram *ABCD*?
- Did you use the Distance Formula to determine the length of sides of parallelogram *ABCD*?
- What coordinates did you use in the Distance Formula?
- Did you use a right triangle to estimate the length of any side of parallelogram *ABCD*?
- Can the perimeter of parallelogram *ABCD* be determined by solving for the length of two consecutive sides? How?

Ask a student to read the scenario after Question 1 aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 2 through 4. Share responses as a class.

As students work, look for

Consideration of one wall, rather than two walls.

Questions to ask

- Is this question asking you to determine the perimeter or the area of the trapezoid?
- Did you use a transformation as a strategy to solve this problem? If so, where did you place the trapezoid?
- Which two sides of the trapezoid are parallel?
- Do you need to use the Distance Formula to calculate the length of each of the four sides of the trapezoid? Why or why not?

- What other methods can be used to determine the length of each of the four sides of the trapezoid?
- What is the area of the right triangle formed by adding the segment parallel to $\overline{DC}$ through point *A*? What is the area of the rectangle?
- What is the area of the right triangle formed by connecting points *A* and *C*? What is the area of the isosceles triangle?
- How many square feet does one fourth of a gallon of paint cover?
- What is the area of the two walls that Aida needs to paint?

Differentiation strategies

To extend the activity,

- Ask students to determine an alternate strategy to calculate the area of the trapezoid.
  - Add area to the trapezoid *ABCD* to create rectangle *BCDE*; then, subtract the area of the newly created $\triangle ABE$ to determine the area of the trapezoid.
  - Use the formula for the area of a trapezoid, $A = \frac{1}{2}h(b_1 + b_2)$.
- Have students analyze the area of a trapezoid formula to determine how it was developed.

Ask a student to read the scenario after Question 4 aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 5 and 6. Share responses as a class.

As students work, look for

- Errors associated with the signs used inside the radicals when substituting the values of the coordinates of the vertices of the hexagon into the Distance Formula.
- Different strategies, such as using triangles and rectangles or six congruent triangles.

Questions to ask

- Is this question asking you to determine the perimeter or the area of the hexagon?
- Do you need to use the Distance Formula to calculate the length of each of the six sides of the hexagon? Why or why not?
- Is the hexagon equilateral? How can you tell?
- How did you divide this composite figure?
- What is another way to divide this composite figure into familiar shapes?
- Is one strategy to calculate the area more efficient than the other methods? If so, why?

Differentiation strategy

To extend the activity, have students divide the hexagon into six congruent triangles. Have them define variables and write a general formula for the area of the hexagon. Then, ask students to rewrite the formula using p, the perimeter of the hexagon, as one of their variables.

Have students work with a partner or in a group to complete Questions 7 and 8. Share responses as a class.

Questions to ask

- Use the graph to interpret what is going on in the scenario.
- What unit of measurement is used to describe the time?
- What unit of measurement is used to describe the speed?
- What is the relationship between distance, rate, and time?
- Using the graph, is the same unit used to describe the time and speed?
- What are the dimensions of the rectangle shown on the graph?
- What is the area of the rectangle drawn on the graph?

Have students work with a partner or in a group to complete Questions 9 and 10. Share responses as a class.

As students work, look for

Errors associated with use of minutes and hours on the graph of the scenario. If students do not realize they are using two different measures of time on the same graph, their answers will be incorrect. Look for students to change minutes into hours on the x-axis or miles per hour into miles per minute on the y-axis.

Questions to ask

- What unit of measurement is used to describe the time?
- What unit of measurement is used to describe the speed?
- Using the graph, is the same unit used to describe the time and speed?
- How did you deal with the different units of time?
- Where can you draw a line segment that will divide the trapezoid into a triangle and a rectangle?
- What are the dimensions of the rectangle?
- What is the area of the rectangle?
- What is the length of the base and the height of the triangle?
- What is the area of the triangle?

Have students work with a partner or in a group to complete Question 11. Share responses as a class.

As students work, look for

Appropriate use of units.

Questions to ask

- What unit of measurement is used to describe the time?
- What unit of measurement is used to describe the speed?
- Using the graph, is the same unit used to describe the time and speed?
- How did you deal with the different units of time?
- Where can you draw a line segment that will divide the trapezoid into a triangle and a rectangle?
- What are the dimensions of the rectangle?
- What is the area of the rectangle?
- What is the length of the base and the height of the triangle?
- What is the area of the triangle?

Summary

The perimeter of any polygon can be determined using the Distance Formula to calculate the length of its sides. A composite figure can be divided into a combination of familiar shapes to calculate its area. The graphic representation of distance on a velocity-time graph is the area under a curve.

Activity 5.5

Proportional and Non-Proportional Changes in Dimensions

Facilitation Notes

In this activity, students are given 3 different rectangles and asked to complete a table to record the changes in the perimeters and areas of the rectangles after a doubling or tripling of each rectangle's dimensions. They then answer questions and generalize about the effects on perimeter and area of proportional changes to the dimensions of a rectangle. This activity is repeated with non-proportional changes to the dimensions.

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Questions to ask

- What happens to the perimeter when the dimensions are doubled? tripled?
- Do you think the pattern of changes to the perimeter will apply if the dimensions are increased by a factor of 4? Explain.
- What is the perimeter of a 4×10 rectangle?

- What is the perimeter of a 2 × 5 rectangle?
- How do the dimensions and perimeter of a 2 × 5 rectangle compare to the dimensions and perimeter of a 4 × 10 rectangle?
- How do these changes relate to the doubling factor (2) and tripling factor (3)?
- What general statement can you make about the new perimeter of a rectangle whose original dimensions have been multiplied by a factor *k*?
- What happens to the area when the dimensions are doubled? tripled?
- Do you think the pattern of changes to the area will apply if the dimensions are increased by a factor of 4? Explain.
- What is the area of a 4 × 10 rectangle?
- What is the area of a 2 × 5 rectangle?
- How do the dimensions and area of a 2 × 5 rectangle compare to the dimensions and area of a 4 × 10 rectangle?
- What general statement can you make about the new area of a rectangle whose original dimensions have been multiplied by a factor *k*?

Have students complete Questions 4 through 7 with a partner. Then have students share their responses as a class.

Questions to ask

- What happens to the perimeter when 2 inches are added to the dimensions? 3 inches are added to the dimensions?
- Is this change proportional or non-proportional? Explain.
- If the same number of *x* units are added to each of the four sides of the rectangle, what is the total change to the perimeter in terms of *x*?
- If the same number of *x* units are subtracted from each of the four sides of the rectangle, what is the total change to the perimeter in terms of *x*?
- Does a non-proportional change to the dimensions of a rectangle have the same affect on the perimeter of the resulting rectangle as a proportional change to the dimensions of a rectangle?
- What happens to the area when 2 inches are added to the dimensions? 3 inches are added to the dimensions?
- Do you see a pattern in the areas of the resulting rectangles after a non-proportional change to the dimensions?
- What operation(s) represent(s) a proportional change? a non-proportional change?

Summary

Proportional and non-proportional changes in a figure's dimensions affect its perimeter and area. For proportional changes, a rectangle with base b and height h will change its perimeter by a factor of k and its area by a factor of k^2 given that its original dimensions are multiplied by a factor of k. When the dimensions of a rectangle change non-proportionally, the resulting area will change, but there is not a clear cut pattern of increase or decrease. Given that a rectangle's original dimensions change by x, its perimeter will change by a factor of $4x$.

Activity 5.6
Areas of Regular Polygons

Facilitation Notes

In this activity, students develop a strategy for calculating the areas of regular polygons. They decompose regular polygons and determine that they should calculate the area of one of the n congruent triangles in the n-gon and multiply the area by n to calculate the area of the regular polygon. They practice this strategy by calculating the areas of regular polygons and composite figures with regular polygons.

Students need a ruler for this activity. Have a class discussion about how to locate the center of the pentagon. Emphasize that the distance from each vertex to the center should be the same.

Ask students to work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Do the five triangles formed in the regular pentagon have the same area? How do you know?
- What is the base length of each triangle in relation to the regular pentagon?
- What is the formula for the area of a triangle?
- Into how many congruent triangles can you divide a hexagon? an octogon? an n-gon?
- If you only know the length of the sides of a regular polygon, can you determine its area using this strategy? Explain.

Ask students to work with a partner or in a group to complete Questions 3 through 9. Share responses as a class.

Questions to ask

- What shape is this polygon?
- How many sides does this polygon have?
- Are all of the sides of this polygon congruent? How do you know?
- This regular polygon is decomposed into how many congruent triangles?
- What information is needed to determine the area of each triangle? Is this information provided?
- Given the area of this hexagon, what strategy can be used to solve for the height of one of the congruent triangles?
- What strategy can be used to calculate the area of the shaded region in this octagon?
- Is this shaded region considered to be an annulus? Why or why not?
- What is the formula for determining the area of a trapezoid? Do you have the necessary information to compute the area?

Differentiation strategy

To extend the activity, ask students where the perimeter is evident in the area formula.

Summary

A regular polygon has all sides congruent and all angles congruent. You can decompose a regular polygon into a number of congruent triangles equal to the number of sides of the polygon.

Talk the Talk: Vive les Maths!

DEMONSTRATE

Facilitation Notes

In this activity, students analyze a representation of France mapped onto a coordinate plane and answer questions associated with population density.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To scaffold support, help students strategize by suggesting they identify the coordinates of each of the six vertices before calculating the lengths. Then use the vertices to sketch right triangles using each segment of the border as a hypotenuse.

As students work, look for

The use of different strategies to estimate the area of the region.

Questions to ask

- What is the shape of the region?
- What method did you use to compute the approximate length of the coastline?

- What method did you use to compute the approximate area?
- How was the population of France determined? Did you use a conversion? How?
- How many segments define the region?
- Did you use the Distance Formula to determine the length of any borders? Which borders?
- What coordinates did you use in the Distance Formula?
- What shapes were used to determine the area of the region?

Summary

A composite figure can be divided into a combination of familiar shapes to solve real-world problems.

Warm Up Answers

1. 7.2
2. 16.1
3. 7.9

5

In and Out and All About

Area and Perimeter on the Coordinate Plane

Warm Up

Determine the distance between each set of points. Round your answer to the nearest tenth, if necessary.

1. (2, −3) and (−4, 1)

2. (−4.75, −8.5) and (3.25, 5.5)

3. $\left(\frac{5}{4}, \frac{9}{4}\right)$ and (0, 10)

Learning Goals

- Determine the perimeter and area of rectangles and triangles on the coordinate plane.
- Use transformations to discover efficient strategies to determine the perimeter and area of rectangles and triangles.
- Determine the perimeter and the area of composite figures on a coordinate plane.
- Use the Distance Formula to solve real-world problems involving perimeters of parallelograms, trapezoids, and hexagons.
- Decompose polygons—including trapezoids and hexagons—to solve real-world problems involving area.
- Calculate area under a curve to determine distance in an acceleration model.

Key Term

- composite figure

You have used the Distance Formula and the slope formula to classify geometric figures on the coordinate plane. How can you use these same formulas to determine the perimeter and area of polygons on the coordinate plane?

Answer

1. Sample answers.

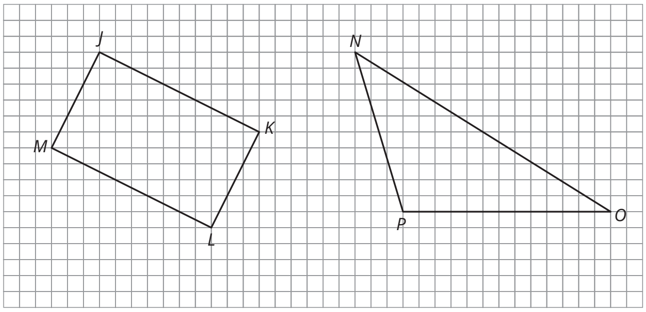
   I can count the number of blocks inside of each shape, then convert square units to square feet.

   I can use the formulas for the area of a rectangle, $A = bh$, and the area of a triangle, $A = \frac{1}{2}bh$. I can get the measurements for b and h by using the Distance Formula, where each unit represents one square foot.

GETTING STARTED

It's Child's Play

A city uses a coordinate grid to map out the locations of two play areas at the park that need to be covered with a rubber surface to prevent injuries. Rectangle *JKLM* represents an area under a swing set and $\triangle NOP$ represents an area under a play structure. Each square on the coordinate grid represents one square foot.



1. Describe a way you can use the grid to determine the area of rectangle *JKLM* and $\triangle NOP$.

ELL Tip

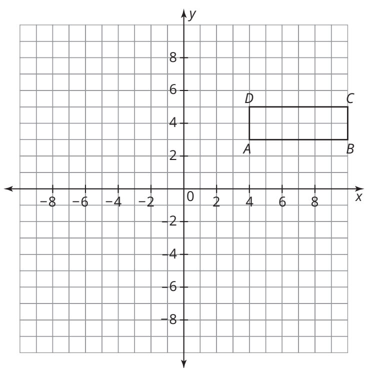
A non-mathematical phrase that is used in this section is *prevent injuries*. Explain each term separately by first defining each term. Define *prevent* as *to stop from doing something*. Define *injury* as *hurt or damage*. Read aloud the first sentence in the section, "A city uses a coordinate grid to map out the locations of two play areas at the park that need to be covered with a rubber surface to *prevent injuries*." Explain that in the context of the sentence, the term *prevent injuries* means that *the rubber surface is meant to make sure that children do not get hurt while playing*.

ACTIVITY
5.1

Perimeter and Area of Figures on the Coordinate Plane

Previously, you classified geometric figures on the coordinate plane by examining the lengths and relationships of their sides. Now, you will determine the perimeter and the area of geometric figures.

1. Consider rectangle *ABCD*.



a. Determine the perimeter of rectangle *ABCD*.

b. Determine the area of rectangle *ABCD*.

Remember:

The perimeter of a geometric figure is calculated by adding the side lengths.

Remember:

The formula for area of a rectangle is $A = bh$, where A represents the area, b represents the base, and h represents the height.

Answers

1a. The perimeter of rectangle *ABCD* is 16 units.

1b. The area of rectangle *ABCD* is 12 square units.

Answers

2. Horace is correct, $\overline{CD}$ can be one of the bases. In fact, any of the sides of the rectangle can be considered a base.
3. The perimeter of rectangle *RSTU* is 1,350 units.

   The area of rectangle *RSTU* is 101,250 square units.
4. Doubling the height of the rectangle doubles the area.
5. Doubling the base of the rectangle doubles the area.

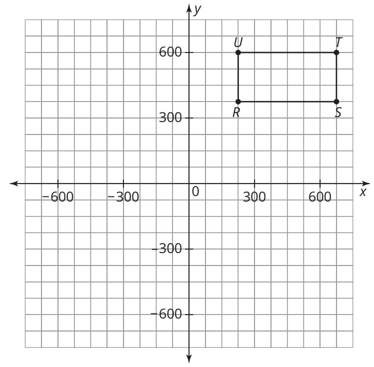
2. Horace says that he determined the area of rectangle *ABCD* by determining the product *CD*(*CB*). Bernice says that Horace is incorrect because he needs to use the base of the rectangle and that the base is $\overline{AB}$, not $\overline{CD}$. Horace responded by saying that $\overline{CD}$ is one of the bases. Who's correct? Explain your reasoning.

When a rectangle is graphed along gridlines, you can determine the perimeter and area by simply counting units or square units on the coordinate plane. This is true if all coordinates are integers. If they are fractions or decimals, it presents a challenge.

Analyze rectangle *RSTU* on the coordinate plane shown.

Think about:

Notice the intervals along the axes.



3. Calculate the perimeter and area of rectangle *RSTU*.

4. How would doubling the height of the rectangle affect the area?

5. How would doubling the length of the base of the rectangle affect the area?

Shantelle used another strategy to determine the perimeter and area of rectangle *RSTU* from the previous activity.

Shantelle

If I translate rectangle *RSTU* to have at least one point of image *R′S′T′U′* on the origin, it is easier to calculate the perimeter and area of rectangle *RSTU* because one of the points will have coordinates (0, 0).

6. How do you know a translation of rectangle *RSTU* will have the same area and perimeter as the pre-image *RSTU*? Explain your reasoning.

7. Explain why Shantelle's rationale is correct.

8. Translate rectangle *RSTU* so that point *R* is located at the origin.

a. List the coordinates of rectangle *R′S′T′U′*.

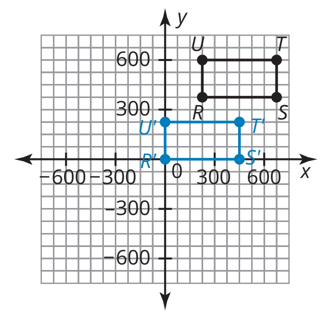
b. Determine the perimeter and area of *R′S′T′U′*. What do you notice?

Answers

6. Transformations preserve size and shape, so the two figures are congruent.

7. Shantelle's rationale is correct because by translating rectangle *RSTU* to the origin, it will be more efficient to calculate the side lengths because one point is on the origin.

8a.



R′ (0, 0), *S′* (450, 0), *T′* (450, 225), *U′* (0, 225)

8b. The perimeter of rectangle *R′S′T′U′* is 1350 units.

The area of *R′S′T′U′* is 101,250 square units.

The perimeter and area of *R′S′T′U′* are equal to the perimeter and area of *RSTU*.

ELL Tip

Assess students' prior knowledge of the term *rationale*. If they are unfamiliar with the term, define it as *a set of reasons or logic used to explain an action or belief*. Provide examples of synonyms for *rationale*, which include *reasons, explanations, arguments,* and *justifications*. Read the sentence in Question 7 aloud, "Explain why Shantelle's *rationale* is correct." Clarify any remaining misunderstandings about the term *rationale* in the context of the sentence.

Answers

9a. The perimeter of square *LMNO* is approximately 53.67 units.

The area of square *LMNO* is 180 square units.

9b. Sample answer.

The area of the square with doubled side lengths is four times the area of the original square.

The new coordinates of the square are (−8, −16), (−32, −4), (−8, 8) and (16, −4).

9c. Sample answer.

I could translate point *L* 8 units to the right and 4 units up to the origin to simplify the perimeter and area calculations.

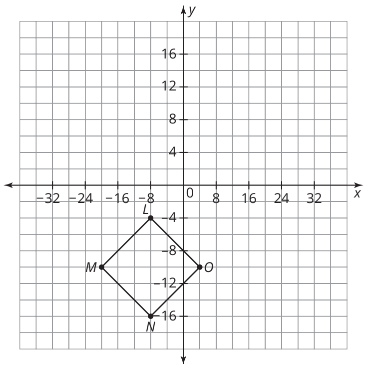
Think about:

Is the quadrilateral a square, a rectangle, or a rhombus?

When the sides of a rectangle do not lie on the gridlines of the coordinate plane, you can use the Distance Formula to determine the lengths of the sides.

9. Consider quadrilateral *LMNO*.

a. Determine the perimeter and area of quadrilateral *LMNO*. Round your answer to the nearest hundredth, if necessary.



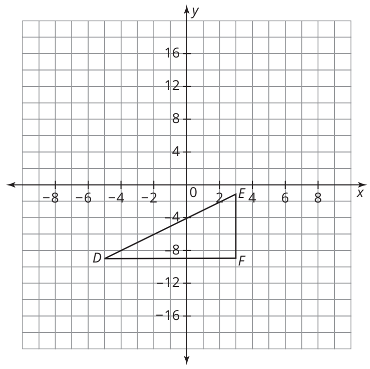
b. Double the side lengths of quadrilateral *LMNO*. How does this affect the area? What are the new coordinates?

c. Describe how you could translate quadrilateral *LMNO* to make the perimeter and area calculations more efficient.

6 • TOPIC 1: Using a Rectangular Coordinate System

10. **Consider $\triangle DEF$ with vertices $D\,(-5, -9)$, $E\,(3, -1)$, and $F\,(3, -9)$.**

a. Determine the perimeter and area of $\triangle DEF$. Round your answer to the nearest hundredth, if necessary.



b. Double the height. What are the coordinates of the new triangle? How did this affect the area?

c. Double the length of the base. What are the coordinates of the new triangle? How did this affect the area?

d. Double the length of both the base and the height. How does this affect the area?

Answers

10a. Perimeter of $\triangle DEF \approx 27.31$ units

Area of $\triangle DEF =$ 32 square units

10b. Sample answer.

The coordinates of the new triangle with doubled height are $D\,(-5, -9)$, $E'\,(3, 7)$, and $F\,(3, -9)$.

The area of the triangle is doubled to 64 square units.

10c. Sample answer.

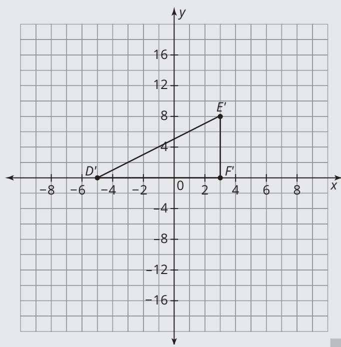
The coordinates of the new triangle with the doubled base are $D'\,(-8, -9)$, $E'\,(8, -1)$, and $F'\,(8, -9)$.

The area of the triangle is doubled to 64 square units.

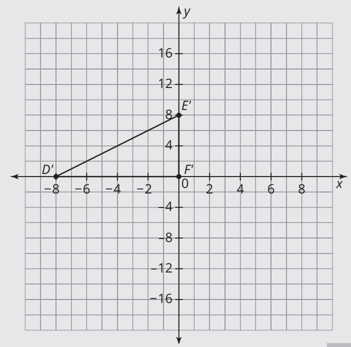
10d. The area of the triangle is quadrupled to 128 square units.

11. Mr. Young gives his class △*DEF* and asks them to determine the area and perimeter. Four of his students decide to first transform the figure and then determine the perimeter and area. Their transformations are shown.

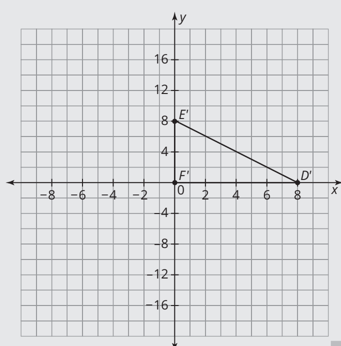
Michael



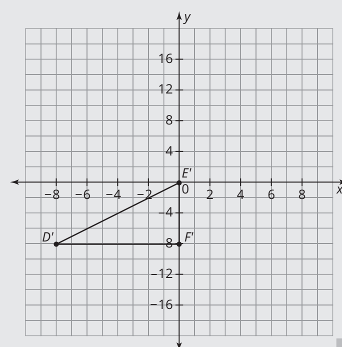
Angelica



Juan



Isabel



a. Describe the transformation(s) each student made to △*DEF*.

b. Whose method do you think is most efficient? Explain your reasoning.

c. What do you know about the perimeter and area of all the triangles? Explain your reasoning.

Answers

11a. Michael vertically translated △*DEF* up 9 units.

Angelica vertically translated △*DEF* up 9 units, then horizontally translated it to the left 3 units.

Juan vertically translated △*DEF* up 9 units, horizontally translated it to the left 3 units, and reflected it over the *y*-axis.

Isabel horizontally translated △*DEF* to the left 3 units then vertically translated, 1 unit.

11b. Sample answer.

Juan's method is most efficient because it transforms the triangle into Quadrant I, so all the coordinates are positive and two line segments of the triangle are on the axes.

11c. The perimeters of all the triangles will be equal, and the areas of all the triangles will be equal. Rigid motion transformations preserve size and shape, so the figures are congruent.

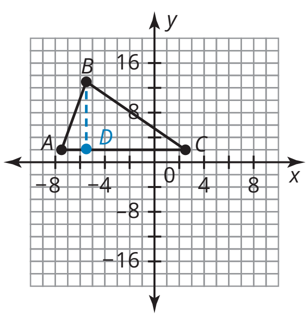
Answers

12. The perimeter of $\triangle ABC$ is approximately 34.78 units.

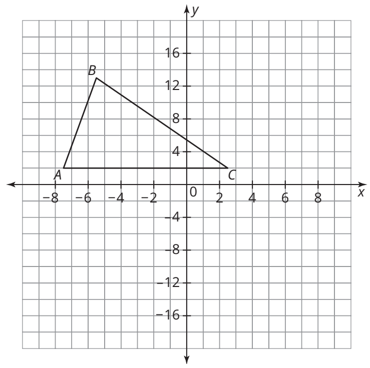
13a. To determine the area of $\triangle ABC$, I need to know the measures of the base and the height.

13b. Randy is correct. The height of a triangle must be a perpendicular line from a vertex to a line containing the base of the triangle. Neither $\overline{AB}$ nor $\overline{BC}$ can be used as the height because neither of them are perpendicular to any line containing a base.

13c. The height of $\triangle ABC$ is 11 units.



13d. The area of $\triangle ABC$ is 55 square units.



Consider $\triangle ABC$ with vertices A (−7.5, 2), B (−5.5, 13), and C (2.5, 2).

12. Determine the perimeter of $\triangle ABC$. Round your answer to the nearest hundredth, if necessary.

13. Consider how to determine the area of $\triangle ABC$.

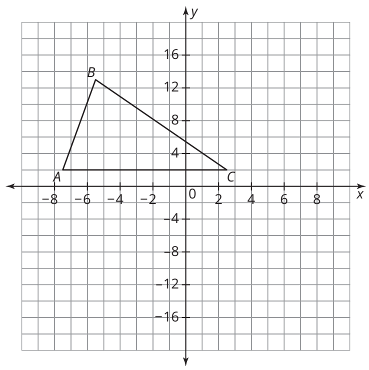
a. What information is needed about $\triangle ABC$ to determine its area?

b. Arlo says that $\overline{AB}$ can be used as the height. Trisha disagrees and says that $\overline{BC}$ can be used as the height. Randy disagrees with both of them and says that none of the line segments currently on the triangle can be used as the height. Who is correct? Explain your reasoning.

c. Draw and label $\overline{BD}$ to represent the height of $\triangle ABC$. Then, determine the height of $\triangle ABC$.

d. Determine the area of $\triangle ABC$.

14. Consider a more efficient way to determine the area and perimeter of △*ABC*.



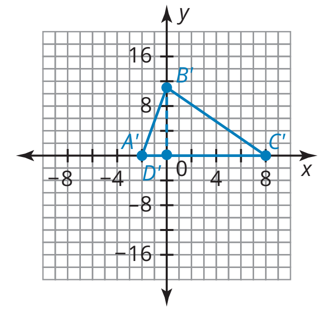
a. Transform △*ABC* on the coordinate plane. Label the image *A′B′C′*. Describe the transformation(s) completed and explain your reasoning.

b. Determine the perimeter and area of △*A′B′C′*. Round your answer to the nearest hundredth, if necessary.

c. Compare these calculations to your previous calculations. How did the translation change your calculations?

Answers

14a. Sample answer.



I vertically translated the triangle down two units so the base $\overline{AC}$ is on the *x*-axis. I then horizontally translated the triangle to the right 5.5 units so point *B* is on the *y*-axis.

14b. $P = \sqrt{125} + \sqrt{185} + 10 \approx 34.78$

The perimeter of triangle *A′B′C′* is approximately 34.78 units.

$A = \frac{1}{2}(10)(11) = 55$

The area of triangle *A′B′C′* is 55 square units.

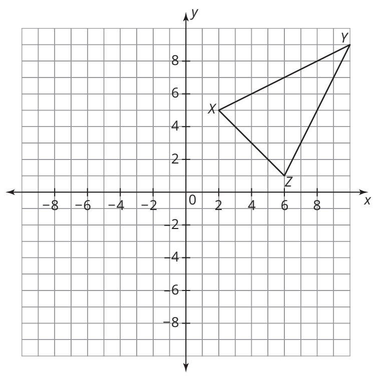
14c. Sample answer. The calculations were somewhat simpler, but rigid motion transformations are more helpful with rectangles than triangles.

Answers

1. $P = \sqrt{80} + \sqrt{80} + \sqrt{32} \approx 23.55$
   The perimeter of $\triangle XYZ$ is approximately 23.55 units.
2. This triangle does not have a base that is a horizontal line segment. I will have to use the slope formula to determine the height for a given base.
3. Jonas did not rotate the vertices of the triangle correctly. If Jonas had performed the rotation correctly, then the side lengths of the image and pre-image would be congruent, since rigid motion transformations preserve size and shape. The length of side $\overline{X'Z}$ is 6 units and the length of $\overline{XZ}$ is $\sqrt{32}$ units, so the sides are not congruent. The lengths of sides $\overline{X'Y'}$ and $\overline{Y'Z}$ are both $\sqrt{73}$ units, but the lengths of sides $\overline{XY}$ and $\overline{YZ}$ are $\sqrt{80}$ units, so these sides are also not congruent.

ACTIVITY **5.2**

Calculating Heights of Triangles



Consider $\triangle XYZ$ with vertices X (2, 5), Y (10, 9), and Z (6, 1).

1. Determine the perimeter of $\triangle XYZ$. Round your answer to the nearest hundredth, if necessary.

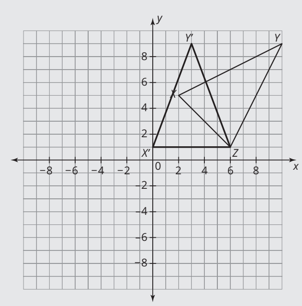
2. To determine the area, you will need to determine the height. How will determining the height of this triangle be different from determining the height of the triangles in previous activities?

3. Jonas wanted to transform the triangle to make the calculations easier. How can you determine that Jonas did not transform the triangle correctly?

Remember:

The altitude, or height, of a triangle is the perpendicular distance from a vertex to the line containing the opposite side.

Jonas
I can rotate the triangle so the base is on a gridline to determine its height.
The triangle has a height of 8 units.



Let's use $\overline{XY}$ as the base of $\triangle XYZ$. You can draw $\overline{ZW}$ to represent the height. Remember that the height is perpendicular to the base. To determine the length of the height, you need to locate point W, which is located at the intersection of $\overline{XY}$ and $\overline{ZW}$.

Worked Example

Calculate the slope of the base, $\overline{XY}$.	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{10-2} = \frac{4}{8} = \frac{1}{2}$	
Determine the slope of the height, $\overline{ZW}$.	$m = -2$	

You can write equations for $\overleftrightarrow{XY}$ and $\overleftrightarrow{ZW}$ and solve the system to determine where the two lines intersect.

Determine the equations of the lines containing the base and the height.	Base $\overleftrightarrow{XY}$ $X(2, 5)$, $m = \frac{1}{2}$ $y - y_1 = m(x - x_1)$ $y - 5 = \frac{1}{2}(x - 2)$ $y = \frac{1}{2}x + 4$	Height $\overleftrightarrow{ZW}$ $Z(6, 1)$, $m = -2$ $y - y_1 = m(x - x_1)$ $y - 1 = -2(x - 6)$ $y = -2x + 13$
Solve the system of equations to determine the coordinates of the point of intersection.	$\frac{1}{2}x + 4 = -2x + 13$ $\frac{5}{2}x = 9$ $x = \frac{18}{5}$	$y = -2x + 13$ $y = -2\left(\frac{18}{5}\right) + 13$ $y = \frac{29}{5}$

Remember:

The slopes of perpendicular lines are opposite reciprocals.

4. Identify the coordinates of the point of intersection. Plot this point on the coordinate plane and label it point *W*. Draw $\overline{ZW}$ to represent the height.

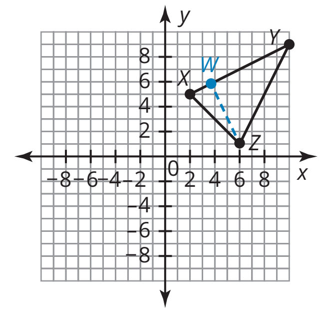
5. Determine the area of $\triangle XYZ$.

a. Determine the height of the triangle.

b. Determine the area of the triangle.

Answers

4. The coordinates are $\left(\frac{18}{5}, \frac{29}{5}\right)$.



5a. $ZW = \sqrt{\frac{144}{25} + \frac{576}{25}} = \sqrt{\frac{720}{25}}$

5b. $A = \frac{1}{2}(\sqrt{80})\left(\sqrt{\frac{720}{25}}\right) = 24$

The area of $\triangle XYZ$ is 24 square units.

Answers

6. Sample answer.
No. I do not think using a different side as the base will matter because the size and shape of the triangle do not change.

7a. Slope of $\overline{XZ} = -1$
Slope of height $\overline{YV} = 1$
Equation of base $\overline{XZ}$:
$(y - 5) = -1(x - 2)$
$y = -x + 7$

Equation of height $\overline{YV}$:
$(y - 9) = 1(x - 10)$
$y = x - 1$

Solution of the system of equations:

$x - 1 = -x + 7$ $\quad y = x - 1$
$2x = 8$ $\quad y = 4 - 1$
$x = 4$ $\quad y = 3$

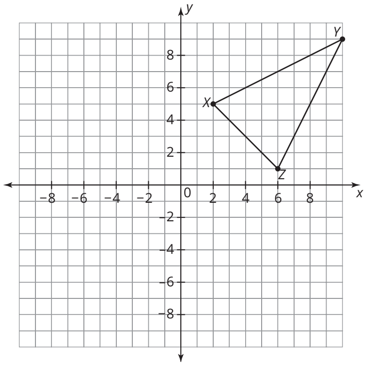
The coordinates of point V are (4, 3).

You know that any side of a triangle can be thought of as the base of the triangle.

6. Predict whether using a different side as the base will result in a different area of the triangle. Explain your reasoning.

Let's consider your prediction.

7. Triangle *XYZ* is graphed on the coordinate plane. This time consider side $\overline{XZ}$ as the base.



a. Let point *V* represent the intersection point of the height, $\overline{YV}$, and the base. Determine the coordinates of point *V*.

b. **Determine the height of $\triangle XYZ$.**

c. **Determine the area of $\triangle XYZ$.**

Answers

7b. $YV = \sqrt{6^2 + 6^2} = \sqrt{72}$

7c. $A = \frac{1}{2}(\sqrt{32})(\sqrt{72}) = 24$

The area of $\triangle XYZ$ is 24 square units.

Answers

8. Slope of base $\overline{YZ} = 2$

   Slope of height $\overline{XT} = -\frac{1}{2}$

   Equation of base $\overline{YZ}$:

   $(y - 1) = 2(x - 6)$

   $y = 2x - 11$

   Equation of height $\overline{XT}$:

   $(y - 5) = -\frac{1}{2}(x - 2)$

   $y = -\frac{1}{2}x + 6$

   Solution of the system of equations:

   $2x - 11 = -\frac{1}{2}x + 6$

   $\frac{5}{2}x = 17$

   $x = \frac{34}{5}$

   $y = 2x - 11$

   $y = 2\left(\frac{34}{5}\right) - 11$

   $y = \frac{13}{5}$

   The coordinates of point T are $\left(\frac{34}{5}, \frac{13}{5}\right)$.

   $XT = \sqrt{\frac{576}{25} + \frac{144}{25}} = \sqrt{\frac{720}{25}}$

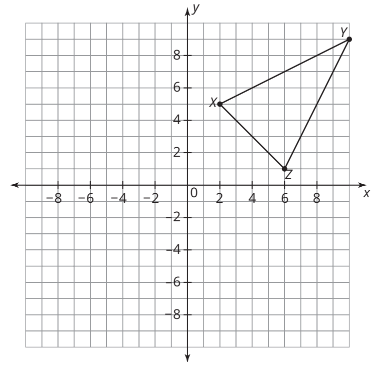
   $A = \frac{1}{2}(\sqrt{80})\left(\sqrt{\frac{720}{25}}\right) = 24$

   The area of triangle XYZ is 24 square units.

9. Sample answer.

   Yes. My prediction was correct. The areas are the same no matter which side of the triangle I used as a base.

8. Triangle *XYZ* is graphed on the coordinate plane. Determine the area of $\triangle XYZ$ using side $\overline{YZ}$ as the base.



9. Compare the three areas you determined for $\triangle XYZ$. Was your prediction in Question 6 correct?

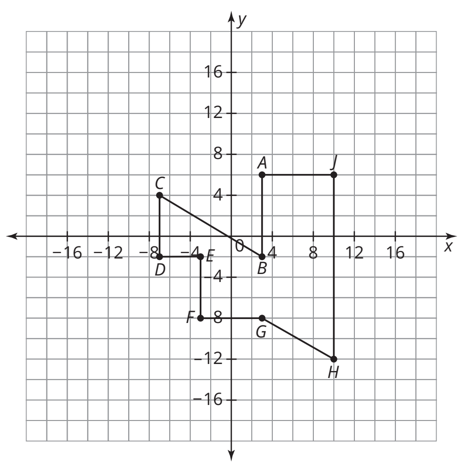
ACTIVITY
5.3

Perimeter and Area of a Composite Figure

The method you used to determine the perimeter of a rectangle or triangle can be used with any polygon. You can use the Distance Formula to calculate the distance between any set of vertices and then add the lengths of all the sides.

You can determine the area of a *composite figure* by dividing the figure into a combination of rectangles and triangles. A **composite figure** is a figure that is formed by combining different shapes.

Carter has an irregular backyard because it backs onto the foothill of a mountain and is very rocky. The composite figure graphed on the coordinate plane represents the flat area of Carter's backyard. Each interval of the coordinate plane represents two yards.



Answers

1. $P = 8 + \sqrt{136} + 6 + 4 + 6 + 6 + \sqrt{65} + 18 + 7 \approx 74.7$

   Carter needs 75 yards of fencing.
2. Carter needs 178 square yards of sod.
3. Sample answer.

   No matter how the composite figure was divided, the total area did not change. This is because the shape of the composite figure does not change regardless of how it is divided.
4. It will cost Carter \$2441.44 to purchase the materials.

   Sample answer.

   Fencing: $75(1.10) = 82.5$ yards
   $82.5(3) = 247.5$ feet
   $(247.5)(\$5.45) = \$1348.88$

   Sod: $178(1.10) = 195.80$ square yards
   $(195.80)(9) = 1762.2$ feet.
   $(1762.2)(\$0.62) = \$1092.56$

1. Carter will install fencing all around the flat area of his backyard. Determine the amount of fencing he needs to the nearest whole yard.

2. Carter wants to lay grass sod in the flat area of his backyard. Determine the total area of sod he needs.

3. Compare the method you used to determine the area of sod Carter needs to your classmates' methods. If you had a different way of dividing up the composite figure, did your answers differ? Explain why or why not.

4. Fencing costs \$5.45 per foot and sod costs \$0.62 per square foot. To allow for measurement error, Carter plans to buy an extra 10% of both materials. How much will it cost Carter to purchase these materials?

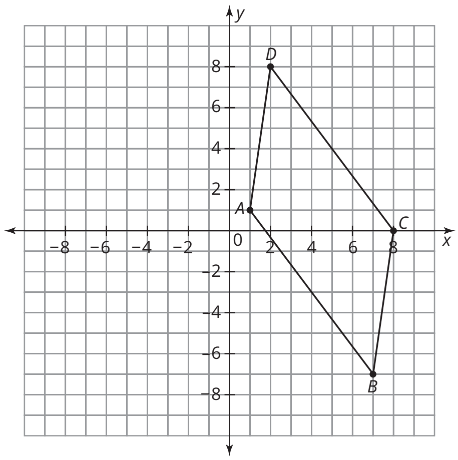
ELL Tip

Determine whether students are familiar with the term *sod*. If not, define *sod* as *the surface of the ground with grass growing on it*. Read aloud the sentence in Question 2, "Carter wants to lay grass *sod* in the flat area of his backyard." Discuss the context of *sod* as it is used in this sentence. Explain that *sod* is the layer of grass that Carter wants to plant. Clarify any remaining misunderstandings about the term *sod* in the context of the scenario used in the activity.

ACTIVITY **5.4**

Solving Problems with Perimeter and Area

A pattern for a quilt patch is drawn on a coordinate plane, where each interval represents one inch. Parallelogram *ABCD* represents the patch.



1. Bryce is in charge of buying the ribbon that will be sewn around the outside of each patch. How many inches of ribbon are needed for each patch?

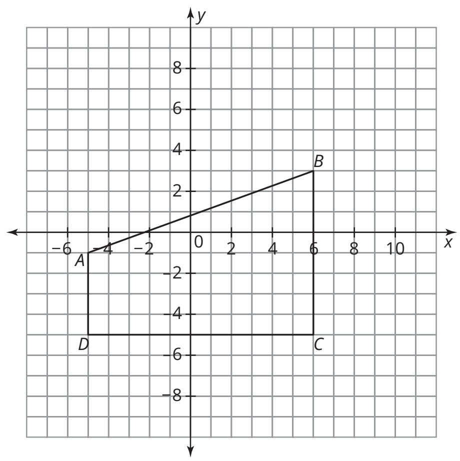
Answer

1. The perimeter of the parallelogram is approximately 34.14 inches, so Bryce needs at least 35 inches of ribbon for each patch.

Answer

2. 70 feet

Aida's bedroom is on the top floor of her house. In her room, the roof slants downward, creating two congruent trapezoid-shaped walls. One of the walls in her room is represented on the coordinate plane by quadrilateral *ABCD*. Each interval on the coordinate plane represents one foot.



Aida and Marco are going to paint the two walls and want to place a strip of painter's tape along each edge of the walls so the paint does not touch any other wall, the ceiling, or the floor.

2. What is the length of painter's tape (to the nearest whole foot) that Aida and Marco need to cover the edges of both walls?

Ask yourself:

How can you use a transformation of trapezoid *ABCD* on the coordinate plane as part of your strategy?

3. Marco says he can draw a diagonal to divide trapezoid *ABCD* into a right and an isosceles triangle to determine the area of the trapezoid. Aida says she can draw a horizontal line segment to divide trapezoid *ABCD* into a rectangle and a right triangle. Who's correct? Explain your reasoning.

4. One gallon of paint covers approximately 400 square feet. Aida estimates she has about one fourth of a gallon of paint remaining of the color she wants to use. Does she have enough paint for both walls? Explain your reasoning.

Answers

3. Both are correct.

   Marco can draw diagonal *AC* and divide the trapezoid into an isosceles triangle with a base of 8 feet and a height of 11 feet and a right triangle with a height of 4 feet and a base of 11 feet.

   Aida can also draw a horizontal line segment from vertex *A* to create a right triangle with a base of 11 feet and a height of 4 feet and a rectangle with a base of 11 feet and a height of 4 feet.

   According to both calculations, the area of the trapezoid is 66 square feet.

4. No; Aida does not have enough paint. Each wall has an area of 66 square feet, so she needs enough paint to cover 132 square feet.

   One fourth of a gallon of paint covers only 100 square feet.

Answers

5. $P \approx 6(17.5) \approx 105$

   A total of approximately 105 feet of lumber is needed to build the outside of the base of the gazebo.

6. Explanations may vary.

   I divided the hexagon into 4 triangles and a rectangle. I determined the area of each piece then added them together.

   Area of each triangle:

   $A = \frac{1}{2}bh$

   $A = \frac{1}{2}(15)(8)$

   $A = 60$

   $60 \cdot 4 = 240$

   Area of rectangle:

   $A = bh$

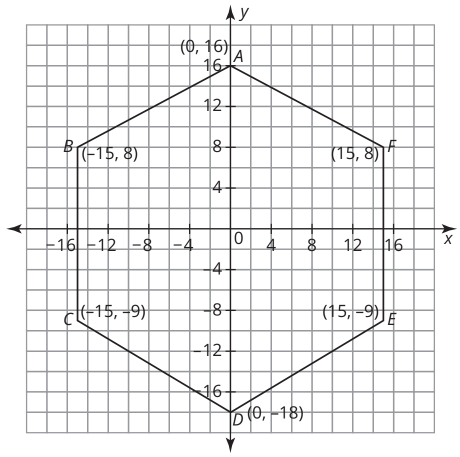
   $A = (30)(17)$

   $A = 510$

   $240 + 510 = 750$

   A total of 750 square feet of floorboards is needed for the base of the gazebo.

Emma and Kevin are designing a gazebo for the local park. The polygon shown on the coordinate plane represents the base of the gazebo. Each interval on the coordinate plane represents two feet.



5. The base of the gazebo needs to be built with lengths of lumber around the outside to support the floorboards. What is the length of lumber needed for the outside of the base?

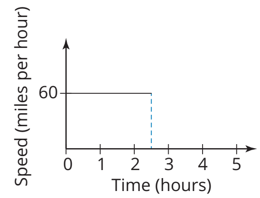
6. How many square feet of floorboards are needed for the base of the gazebo? Describe how you determined your answer and show your work.

ELL Tip

Two non-mathematical terms that appear in this activity are *gazebo* and *lumber*. Define a *gazebo* as *a small, open structure that has a roof*. Define *lumber* as *wood that is commonly used in construction*. Provide examples of synonyms for *gazebo*, such as *pavilion, summerhouse,* and *platform*. Synonyms for *lumber* include *wood, timber,* and *boards*. Discuss the connection between *gazebo* and *lumber* as they are used in the activity. Provide pictures of a *gazebo* and *lumber* if possible.

The graph shows the constant speed of a car on the highway over the course of 2.5 hours.

7. Describe how you could calculate the distance the car traveled in 2.5 hours using what you know about area.

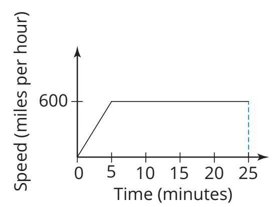


8. How far did the car travel in 2.5 hours?

The graph you used is called a velocity-time graph. In a velocity-time graph, the area under the line or curve gives the distance.

The graph shown describes the speed and the time of a passenger jet's ascent.

9. How can you use the graph to determine the distance the jet has traveled in 25 minutes?



Answers

7. Sample answer.

   Distance equals rate multiplied by time.

   Because the *y*-axis represents the rate and the *x*-axis represents the time, the area of the rectangle represents the distance traveled.

8. The car traveled 150 miles.

   $\frac{60 \text{ miles}}{1 \text{ hour}} \times \frac{2.5 \text{ hours}}{1} =$ 150 miles

9. I can calculate the area of the region enclosed by the graph from 0 to 25 minutes to determine the distance the jet traveled.

ELL Tip

Assess students' prior knowledge of the term *ascent*. Define *ascent* as *a climb or increase in something*. Discuss other forms and usage of *ascent* with which students may be familiar, such as *ascending numbers*. Remind students that a commonly used antonym for *ascent* is *descent*, which means *a decrease in something*. Read aloud the sentence above Question 9 in the activity, "The graph shown describes the speed and the time of a passenger jet's *ascent*." Discuss the context of *ascent* in this sentence and clarify any remaining misunderstandings of the term.

Answers

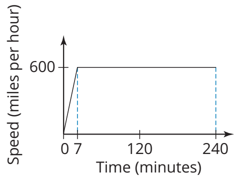
10a. The jet has traveled 25 miles in 5 minutes.

$\frac{1}{2}(5)(10) = 25$

10b. The jet has traveled 225 miles in 25 minutes.

$\frac{1}{2}(5)(10) + (20)(10) = 225$

11a.



11b. The jet traveled 2365 miles in 4 hours.

$\frac{1}{2}(7)(10) + (233)(10) = 2365$

10. Determine the distance the jet has traveled:

a. in the first 5 minutes.

b. in 25 minutes.

11. Consider the ascent of a passenger jet.

a. Draw a velocity-time graph to model the ascent of a passenger jet using the information given.
- **The jet took 7 minutes to reach a top speed of 600 miles per hour.**
- **The jet continued to travel at a constant speed of 600 miles per hour.**
- **The jet left the airport 4 hours ago.**

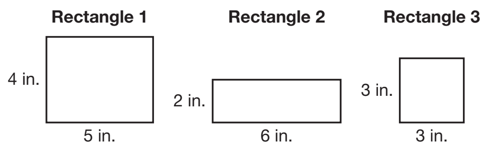
b. How many miles has the jet traveled?

ACTIVITY **5.5**

Proportional and Non-Proportional Changes in Dimensions

In the activity *Perimeter and Area of Figures on the Coordinate Plane,* you investigated how doubling one or both of the dimensions of a figure affected its area. Now let's investigate how both proportional and non-proportional changes in a figure's dimensions affect its perimeter and area.

1. Consider the following rectangles with the dimensions shown.



Complete the table to determine how doubling or tripling each rectangle's base and height affects its perimeter and area. The information for Rectangle 1 has been done for you.

		Original Rectangle	Rectangle Formed by Doubling Dimensions	Rectangle Formed by Tripling Dimensions
Rectangle 1	Linear Dimensions	$b = 5$ in. $h = 4$ in.	$b = 10$ in. $h = 8$ in.	$b = 15$ in. $h = 12$ in.
	Perimeter (in.)	$2(5 + 4) = 18$	$2(10 + 8) = 36$	$2(15 + 12) = 54$
	Area (in.^{2})	$5(4) = 20$	$10(8) = 80$	$15(12) = 180$
Rectangle 2	Linear Dimensions			
	Perimeter (in.)			
	Area (in.^{2})			
Rectangle 3	Linear Dimensions			
	Perimeter (in.)			
	Area (in.^{2})			

ELL Tip

Ask students what they think is meant by a *proportional change*. Some students may use the term *scale factor*. Encourage students to ask other students when unfamiliar terms are used. Guide students to conclude that a *proportional change* is a result of a multiplication. Then ask if this multiplication can only apply to one dimension or if it should apply to two dimensions.

Answers

1. Rectangle 2 (Original Rectangle)
   Linear Dimensions: $b = 6$ in.; $h = 2$ in.
   Perimeter (in.): $2(6 + 2) = 16$
   Area (in.^{2}): $6(2) = 12$

   Rectangle 2 (Rectangle Formed by Doubling Dimensions)
   Linear Dimensions: $b = 12$ in.; $h = 4$ in.
   Perimeter (in.): $2(12 + 4) = 32$
   Area (in.^{2}): $12(4) = 48$

   Rectangle 2 (Rectangle Formed by Tripling Dimensions)
   Linear Dimensions: $b = 18$ in.; $h = 6$ in.
   Perimeter (in.): $2(18 + 6) = 48$
   Area (in.^{2}): $18(6) = 108$

   Rectangle 3 (Original Rectangle)
   Linear Dimensions: $b = 3$ in.; $h = 3$ in.
   Perimeter (in.): $2(3 + 3) = 12$
   Area (in.^{2}): $3(3) = 9$

   Rectangle 3 (Rectangle Formed by Doubling Dimensions)
   Linear Dimensions: $b = 6$ in.; $h = 6$ in.
   Perimeter (in.): $2(6 + 6) = 24$
   Area (in.^{2}): $6(6) = 36$

   Rectangle 3 (Rectangle Formed by Tripling Dimensions)
   Linear Dimensions: $b = 9$ in.; $h = 9$ in.
   Perimeter (in.): $2(9 + 9) = 36$
   Area (in.^{2}): $9(9) = 81$

Answers

2a. When the dimensions of a rectangle increase by a factor of 2, the resulting perimeter is 2 times greater than the original perimeter.

2b. When the dimensions of a rectangle increase by a factor of 3, the resulting perimeter is 3 times greater than the original perimeter.

2c. When the dimensions of a rectangle increase by a factor of 4, the resulting perimeter will be 4 times greater than the original perimeter.

2d. The perimeter of the resulting rectangle, 14 inches, would be half of the perimeter of the original rectangle, 28 inches.

Dimensions of Resulting Rectangle (inches):

Base $= 4 \cdot \frac{1}{2} = 2$

Height $= 10 \cdot \frac{1}{2} = 5$

Perimeter of Resulting Rectangle (inches): $2(2 + 5) = 14$

2e. The perimeter of a rectangle with base *b* and height *h* will change by a factor of *k*, given that its original dimensions are multiplied by a factor *k*.

Remember:

Multiplying side lengths by a number is a proportional change.

2. Describe how a proportional change in the linear dimensions of a rectangle affects its perimeter.

a. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 2?

b. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 3?

c. What would happen to the perimeter of a rectangle when its dimensions increase by a factor of 4?

d. Describe how you think the perimeter of the resulting rectangle would compare to the perimeter of a 4 × 10 rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the perimeter of the resulting rectangle.

e. In terms of *k*, can you generalize change in the perimeter of a rectangle with base *b* and height *h*, given that its original dimensions are multiplied by a factor *k*?

3. **Describe how a proportional change in the linear dimensions of a rectangle affects its area.**

   a. **What happens to the area of a rectangle when its dimensions increase by a factor of 2?**

   b. **What happens to the area of a rectangle when its dimensions increase by a factor of 3?**

   c. **What would happen to the area of a rectangle when its dimensions increase by a factor of 4?**

   d. **Describe how you think the area of the resulting rectangle would compare to the area of a 4 × 10 rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the area of the resulting rectangle.**

   e. **In terms of *k*, can you generalize change in the area of a rectangle with base *b* and height *h*, given that its original dimensions are multiplied by a factor *k*?**

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Answers

3a. When the dimensions of a rectangle increase by a factor of 2, the resulting area is 4 times greater than the original area.

3b. When the dimensions of a rectangle increase by a factor of 3, the resulting area is 9 times greater than the original area.

3c. When the dimensions of a rectangle increase by a factor of 4, the resulting area will be 16 times greater than the original area.

3d. The area of the resulting rectangle, 10 square inches, would be one-fourth of the area of the original rectangle, 40 square inches.

Dimensions of Resulting Rectangle square (inches):

Base $= 4 \cdot \frac{1}{2} = 2$

Height $= 10 \cdot \frac{1}{2} = 5$

Area of Resulting Rectangle square (inches square):
$2(5) = 10$

3e. The area of a rectangle with base b and height h will change by a factor of k^2, given that its original dimensions are multiplied by a factor k.

Answers

4. Answers will vary.
5. Rectangle 2 (Original Rectangle)
   Linear Dimensions:
   $b = 6$ in.; $h = 2$ in.
   Perimeter (in.):
   $2(6 + 2) = 16$
   Area (in.^{2}): $6(2) = 12$

   Rectangle 2 (Rectangle Formed by Adding 2 Inches to Dimensions)
   Linear Dimensions:
   $b = 8$ in.; $h = 4$ in.
   Perimeter (in.):
   $2(8 + 4) = 24$
   Area (in.^{2}): $8(4) = 32$

   Rectangle 2 (Rectangle Formed by Adding 3 inches to Dimensions)
   Linear Dimensions:
   $b = 9$ in.; $h = 5$ in.
   Perimeter (in.):
   $2(9 + 5) = 28$
   Area (in.^{2}): $9(5) = 45$

   Rectangle 3 (Original Rectangle) Linear Dimensions:
   $b = 3$ in.; $h = 3$ in.
   Perimeter (in.):
   $2(3 + 3) = 12$
   Area (in.^{2}): $3(3) = 9$

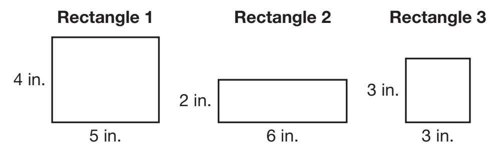
   Rectangle 3 (Rectangle Formed by Adding 2 Inches to Dimensions)
   Linear Dimensions:
   $b = 5$ in.; $h = 5$ in.
   Perimeter (in.):
   $2(5 + 5) = 20$
   Area (in.^{2}): $5(5) = 25$

   Rectangle 3 (Rectangle Formed by Adding 3 Inches to Dimensions) Linear Dimensions:
   $b = 6$ in.; $h = 6$ in.
   Perimeter (in.):
   $2(6 + 6) = 24$
   Area (in.^{2}): $6(6) = 36$

Non-proportional change to linear dimensions of a two-dimensional figure involves adding or subtracting from the side lengths.

4. Do you think a non-proportional change in the linear dimensions of a two-dimensional figure will have the same effect on perimeter and area as proportional change? Explain your reasoning.

5. Consider the following rectangles with the dimensions shown.



Complete the table to determine how adding two or three inches to each rectangle's base and height affects its perimeter and area The information for Rectangle 1 has been done for you.

		Original Rectangle	Rectangle Formed by Adding 2 Inches to Dimensions	Rectangle Formed by Adding 3 Inches to Dimensions
Rectangle 1	Linear Dimensions	$b = 5$ in. $h = 4$ in.	$b = 7$ in. $h = 6$ in.	$b = 8$ in. $h = 7$ in.
	Perimeter (in.)	$2(5 + 4) = 18$	$2(7 + 6) = 26$	$2(8 + 7) = 30$
	Area (in.^{2})	$5(4) = 20$	$7(6) = 42$	$8(7) = 56$
Rectangle 2	Linear Dimensions			
	Perimeter (in.)			
	Area (in.^{2})			
Rectangle 3	Linear Dimensions			
	Perimeter (in.)			
	Area (in.^{2})			

6. **Describe how a non-proportional change in the linear dimensions of a rectangle affects its perimeter.**

   a. **What happens to the perimeter of a rectangle when 2 inches are added to its dimensions?**

   b. **What happens to the perimeter of a rectangle when 3 inches are added to its dimensions?**

   c. **What would happen to the perimeter of a rectangle if 4 inches are added to its dimensions?**

   d. **Describe how you think the perimeter of the resulting rectangle would compare to the perimeter of a 4×10 rectangle if the dimensions of the original rectangle were reduced by 2 inches. Then, determine the perimeter of the resulting rectangle.**

   e. **Given that a rectangle's original dimensions change by *x* units, generalize the change in the perimeter in terms of *x*.**

Answers

6a. When 2 inches are added to the dimensions of a rectangle, the resulting perimeter is 8 inches greater than the original perimeter.

6b. When 3 inches are added to the dimensions of a rectangle, the resulting perimeter is 12 inches greater than the original perimeter.

6c. If 4 inches were added to the dimensions of a rectangle, then its perimeter would increase by 16 inches.

6d. The perimeter of the resulting rectangle, 20 inches, would be 8 inches less than the perimeter of the original rectangle, 28 inches.

Dimensions of Resulting Rectangle (inches):

Base $= 4 - 2 = 2$

Height $= 10 - 2 = 8$

Perimeter of Resulting Rectangle (inches): $2(2 + 8) = 20$

6e. Given that a rectangle's original dimensions change by *x*, the perimeter of the resulting rectangle will be $4x$ more or less than the perimeter of the original rectangle, depending on whether *x* was added or subtracted from the original rectangle.

Answers

7a. Answers will vary.

When the dimensions of a rectangle increase by 2 inches, the resulting area increases, but I do not see a clear cut pattern of increase as was the case with proportional change in linear dimensions.

7b. Answers will vary.

When the dimensions of a rectangle increase by 3 inches, the resulting area increases, but I do not see a clear cut pattern of increase as was the case with proportional change in linear dimensions.

7c. Answers will vary.

When the dimensions of a rectangle increase by 4 inches, the resulting area will increase, but there will probably not be a clear cut pattern of increase as was the case with proportional change in linear dimensions.

7. Describe how a non-proportional change in the linear dimensions of a rectangle affects its area.

a. What happens to the area of a rectangle when its dimensions increase by 2 inches?

b. What happens to the area of a rectangle when its dimensions increase by a factor of 3?

c. What would happen to the area of a rectangle when its dimensions increase by a factor of 4?

d. **Describe how you think the area of the resulting rectangle would compare to the area of a 4 × 10 rectangle if the dimensions of the original rectangle were reduced by 2 inches. Then, determine the area of the resulting rectangle.**

e. **What happens to the area when the dimensions of a rectangle change non-proportionally?**

f. **Summarize the change in a rectangle's area caused by a non-proportional change in its linear dimensions and the change in a rectangle's area caused by a proportional change in its linear dimensions.**

Answers

7d. Answers will vary.

The area of the resulting rectangle, 16 square inches, is less than the area of the original rectangle, 40 square inches. I do not see a clear cut pattern of decrease as was the case with proportional change in linear dimensions.

Dimensions of Resulting Rectangle (inches):
Base $= 4 - 2 = 2$
Height $= 10 - 2 = 8$

Area of Resulting Rectangle (square inches):
$2(8) = 16$

7e. Answers will vary.

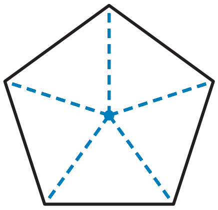
When the dimensions of a rectangle change non-proportionally, the resulting area will change, but there is not a clear cut pattern of increase or decrease as was the case with proportional change in linear dimensions.

7f. When the dimensions of a rectangle change non-proportionally, the resulting area will change, but there is not a clear cut pattern of increase or decrease as was the case with proportional change in linear dimensions.

When the dimensions of a rectangle change proportionally by a factor of k, its area changes by a factor of k^2.

Answers

1a.



1b. When the 5 line segments are added, the pentagon is divided into 5 congruent triangles.

1c. The information I need to calculate the area of each triangle is the length of the base of the triangle and the height of the triangle.

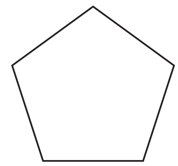
1d. The formula used to calculate the area of a triangle is $A = \frac{1}{2}(b)(h)$.

ACTIVITY 5.6

Areas of Regular Polygons

A **regular polygon** is a polygon with all sides congruent and all angles congruent. The area of a regular polygon can be thought of as the area of a composite shape.

1. A regular pentagon is shown.



a. Locate and place a point at the center of the regular pentagon. From the center point, draw line segments to connect the point with each vertex of the pentagon.

b. Describe the new polygons formed by adding these line segments.

c. What information do you need to calculate the area of each new polygon?

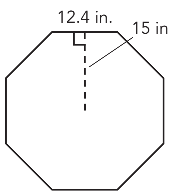
d. What formula is used to calculate the area of each new polygon?

e. Describe a strategy to determine the area of the entire regular pentagon.

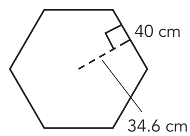
2. How can you use this strategy to determine the area of other regular polygons?

For each regular polygon, draw in the congruent triangles and then calculate the area.

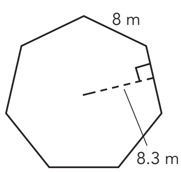
3.



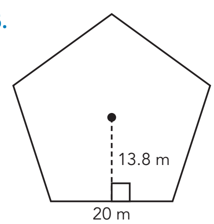
4.



5.



6.

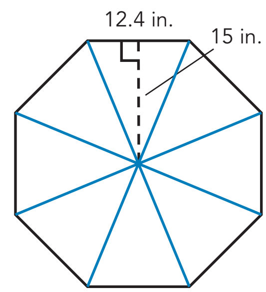


Answers

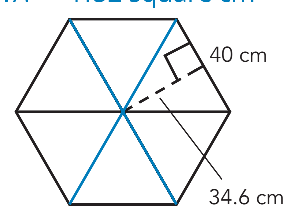
1e. To determine the area of the regular pentagon, I can multiply the area of one of the triangles by 5.

2. I can divide any regular polygon into a number of congruent triangles equal to the number of sides of the polygon. I can determine the area of one of the triangles and multiply that area by the number of triangles I divided the polygon into.

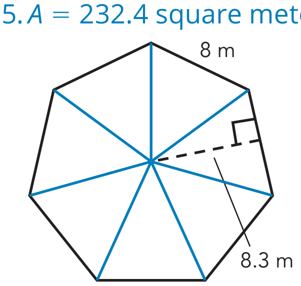
3. A = 744 square inches



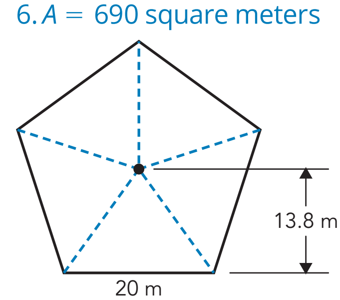
4. A = 4152 square cm



5. A = 232.4 square meters



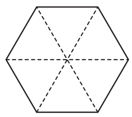
6. A = 690 square meters



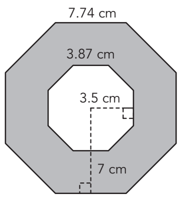
Answers

7. The height of the congruent triangles in the hexagon is 12 cm.
8. The area of the shaded region is 162.54 square centimeters.
9. The area of the figure is 725.2 square centimeters.

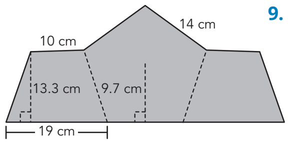
Solve each problem involving regular polygons.



7. A regular hexagon has an area of 540 cm², and its side lengths are each 15 cm. Calculate the height of the congruent triangles in the hexagon.



8. A regular octagon has an octagonal hole as shown. Calculate the area of the shaded region.



9. Calculate the area of the figure composed of a regular pentagon and 2 congruent trapezoids.

Answer

1. The total coastlines and borders of France is less than 5000 km. The area of France is less than 1,000,000 square km.

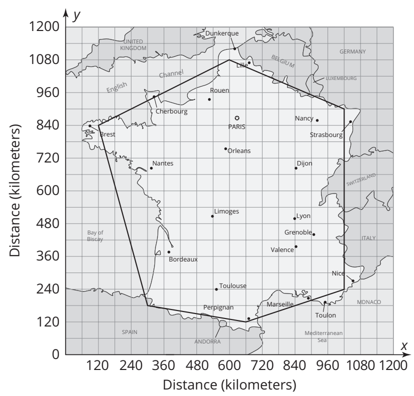
   Students' explanations will vary.

NOTES

TALK the TALK

Vive les Maths!

Eva is using a map to estimate the area of France. She thinks the country looks like a hexagon and draws the polygon shown to approximate its shape.



1. **Determine which statements are true. Justify your answers.**
   - **The coastline of France is greater than 5000 km.**
   - **The coastline of France is less than 5000 km.**
   - **The coastline of France is approximately 5000 km.**
   - **The area of France is greater than 1,000,000 sq km.**
   - **The area of France is less than 1,000,000 sq km.**
   - **The area of France is approximately 1,000,000 sq km.**

Answer

2. Approximately $666{,}000 \times 104$, or 69,264,000 people live in the country of France.

NOTES

2. If the population of France is approximately 104 people per square kilometer, how many people live in France?

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