Using a Rectangular Coordinate System

Topic 1 Overview

How is *Using a Rectangular Coordinate System* organized?

In this topic, students investigate the properties of squares and use transformations of squares to construct a coordinate plane. They develop strategies for determining the perimeters and areas of rectangles, triangles, parallelograms, and composite plane figures on the coordinate plane. Students also explore the effects of proportional and non-proportional changes to the dimensions of a plane figure on its perimeter and area.

Students begin the course exploring the nature of geometric reasoning. They first analyze three angles formed in a diagram comprising three congruent, adjacent squares. Students use measuring tools to determine the sum of the angle measures and then compare their results with their classmates' results. They learn that investigating with measuring tools is an informal strategy to look at the structure of a figure and develop ideas about its characteristics. Considering a conjecture about the sum of the angles, students then investigate the angle measures with a differently-sized version of the diagram. Students learn that considering more cases is an important step in geometric reasoning. They then use patty paper to explore the diagram without measuring tools, a step towards more formal reasoning. While stopping short of a formal geometric proof, students consider how adding auxiliary lines is helpful in seeing relationships among lines and angles in the diagram. More advanced students should be encouraged to write a formal proof.

Next, students consider the structure of the coordinate plane. Recognizing that it is composed of parallel and perpendicular lines, students construct a coordinate plane using transformations of a square. To do this, students learn basic constructions—constructing perpendicular lines, constructing parallel lines, and duplicating segments. Students use transformations to explore the relationship between perpendicular lines. They use translations to prove that the slopes of parallel lines are equal; they use rotations to prove that if two lines are perpendicular, then the slopes of the lines are negative reciprocals. Students write the equations of lines parallel or perpendicular to given lines at given points.

Students then use a Venn diagram to review properties of triangles and quadrilaterals. To determine the side lengths of figures on the coordinate plane, students derive the Distance Formula from the Pythagorean Theorem. They use the slope criteria for parallel and perpendicular lines to verify parallel sides and perpendicular angles where appropriate. They combine these skills to classify triangles and quadrilaterals on the coordinate plane and to compose a quadrilateral given three of the vertices. The term *midpoint* is defined, and students use the Midpoint Formula to investigate the pattern of shapes that is formed by connecting the midpoints of the sides.

Students continue to practice using the Distance Formula when calculating the area and perimeter of triangles, rectangles and composite figures on the coordinate plane in real-world scenarios.

In solving these problems, students have to synthesize knowledge from multiple domains. They use their knowledge of translations to discover strategies for making these calculations more efficient; they use the slope criteria for perpendicular lines to determine the line upon which the height of a non-right triangle lies; and they solve systems of equations to identify where a perpendicular line intersects the base of the triangle, allowing them to calculate the height of the triangle. Students extend this knowledge to include scenarios comparing speed and time. Given velocity-time graphs, students determine the distance traveled in each scenario by calculating the area of the polygon constructed by the boundaries of the graph.

Students explore proportional and non-proportional changes in dimensions and the effect these changes have on the perimeter and area of a figure. Regular polygons are defined and students find the area of various regular polygons.

What is the entry point for students?

Throughout middle school, students have had extensive experience with linear relationships and know how to calculate the slope of a line. In the previous course, they have translated linear functions.

Students first began classifying quadrilaterals in elementary school. While they have not yet formalized this knowledge through Euclidean proofs, they have a strong sense of how side lengths, angle measures, and side relationships determine the type of quadrilateral. This topic builds on this sense and gives students an opportunity to algebraically determine these relationships and classifications. Beginning in elementary school, students have calculated the area and perimeter of triangles, quadrilaterals, and other figures given side lengths. This topic adds another step to the process by providing only vertices of the figure and requiring students to calculate side lengths before applying area or perimeter formulas. It builds upon students' understanding of the area formulas for squares, rectangles, and triangles; the Pythagorean Theorem; the slope criteria for parallel and perpendicular lines; and methods of solving a system of linear equations. Students synthesize their knowledge, practice the required skills, and apply their understandings to real-world scenarios.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Using a Rectangular Coordinate System* if they can:

- Recognize that slopes of perpendicular lines are negative reciprocals.
- Determine the equation of a line parallel or perpendicular to a given line that passes through a given point.
- Prove the slope criteria for parallel and perpendicular lines and use them to solve problems.
- Use the Distance Formula to determine whether sides of a figure are congruent.
- Use the slope criteria for parallel and perpendicular lines to determine the relationship between sides of a figure.

- Use the Midpoint Formula to bisect the side of a figure.
- Write the equation for a line that is perpendicular to a given side of a figure and passes through the opposite vertex.
- Calculate the perimeter of a figure on the coordinate plane using the Distance Formula.
- Compute the area of a figure on the coordinate plane using the coordinates of the figure, the Distance Formula, and the appropriate area formula.
- Decompose given composite figures into a set of non-overlapping triangles and rectangles.

Why is *Using a Rectangular Coordinate System* important?

The coordinate plane provides structure that allows students to analyze and classify the properties of figures algebraically. Provided with only vertices for the given shapes, students must take the initial step of calculating side lengths using the Distance Formula. Proving relationships and performing geometric measurements algebraically integrates geometry and algebra. As with much of high school geometry, students formalize concepts that they investigated informally in elementary and middle school.

How do the activities in *Using a Rectangular Coordinate System* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

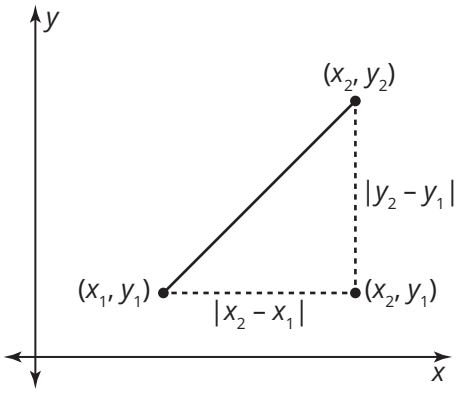
In this topic, students use tools appropriately to accurately construct basic geometric shapes. They attend to precision as they use clear reasoning to classify shapes and to accurately calculate lengths and relationships of sides and perimeters and areas of figures. Students use reasoning as they seek efficient strategies for making these calculations. They make use of structure as they recognize the significance of perpendicular lines through a given vertex when calculating the area of a triangle given any side as the base. They use the structure of geometric shapes to decompose composite figures into sets of non-overlapping triangles and rectangles.

Materials Needed

Compasses
Patty paper
Protractors
Rulers
Straightedges

New Tools and Notation

The Distance Formula states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the distance d between (x_1, y_1) and (x_2, y_2) is calculated using the formula given.



$$\text{Distance Formula}$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Midpoint Formula states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Learning Together

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	The Squariest Square: From Informal to Formal Geometric Thinking	G.4A G.5A	2	Through a series of activities, students consider the range of geometric reasoning from informal to formal. To start, students attempt to sketch a "perfect" square and discuss the properties of a square. They analyze a diagram with three squares, create specific angles within the squares, use a protractor to determine their measures, and compare the sum of the measures with their classmates' results. They consider a conjecture about the sum of the measures. To determine whether this conjecture holds true in a second case, they measure the angles in a larger version of the diagram. To move towards generalization, students use patty paper to further analyze the conjecture that the angle measures sum to 90°. The diagram is then expanded through rigid motions to create other geometric properties that students can consider to formally verify the proof, although this final step is not required. They conclude that informal reasoning involves measurements, while formal reasoning involves properties.
2	Hip to Be Square: Constructing a Coordinate Plane	G.3C G.5B G.5C	2	Students consider how a coordinate plane can be constructed using squares. They start by completing geometric constructions using patty paper or a compass and a straightedge. They analyze worked examples to construct perpendicular lines, perpendicular bisectors, and duplicated line segments. Students construct a square and then describe how rigid motions can be applied to create a coordinate plane. They then describe rigid motions that can be used to create two-dimensional shapes on a coordinate plane. Students also relate a sequence of translations to the slope of a line.

Lesson	Lesson Name	TEKS	Days	Highlights
3	Ts and Train Tracks: Parallel and Perpendicular Lines	G.2C G.5A G.5B G.5C	3	Students investigate segments on a coordinate grid and use patty paper to create parallel and perpendicular segments. They then construct parallel lines off the coordinate plane and graph parallel lines on the coordinate plane. Students identify perpendicular lines on the coordinate plane, use a rigid motion transformation to demonstrate that their slopes are negative reciprocals, and extend their understanding of perpendicular lines to include horizontal and vertical lines. They provide an explanation to demonstrate that if two lines are parallel, then their slopes are equal.
4	Where Has Polly Gone?: Classifying Shapes on the Coordinate Plane	G.2B G.9B	3	Students use a Venn diagram to sort quadrilaterals and triangles based on shared properties. They are introduced to the Distance Formula and use it to calculate the lengths of sides of triangles and quadrilaterals on the coordinate plane. Students also use the slope formula to determine whether opposite sides of a quadrilateral are parallel and whether consecutive sides of a quadrilateral are perpendicular. They use these skills to classify triangles and quadrilaterals that lie on a coordinate plane, or determine the fourth point of a quadrilateral when given three points. Students are then introduced to the Midpoint Formula and use it to classify secondary figures formed when connecting the midpoints of consecutive sides of quadrilaterals. Finally, students consider translations as a strategy to identify the coordinates that create quadrilaterals with parallel sides.

Lesson	Lesson Name	TEKS	Days	Highlights
5	In and Out and All About: Area and Perimeter on the Coordinate Plane	G.2B G.2C G.3C G.10B G.11B G.11A	4	Students calculate the perimeter and area of rectangles and triangles on the coordinate plane. They double dimensions of figures and explain how this affects the area of the figure; they also translate figures on the coordinate plane to more efficiently determine their perimeter and area. Students algebraically determine the non-vertical height of a triangle as they treat each side as the base; they then use the height to calculate the area of the triangle. They conclude that the area of a triangle remains the same regardless of the side considered as the base and the height determined by that base. Next, students divide a composite figure into various known polygons to compute its area. They then consider real-world situations requiring them to calculate the perimeter and area of polygons that lie on a coordinate plane using the Distance Formula and decomposing the polygons into triangles and rectangles. Students determine distances represented as the area under the curve of velocity-time graphs. They investigate how proportional and non-proportional changes in the linear dimensions of a shape affect its perimeter and area. Students develop a strategy for calculating areas of regular polygons.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: G.4A, G.5A **LESSON 1** **The Squariest Square** GETTING STARTED ACTIVITY 1	**LESSON 1** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	TEKS: G.3C, G.5B, G.5C **LESSON 2** **Hip to Be Square** GETTING STARTED ACTIVITY1	**LESSON 2** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
TEKS: G.2C, G.5A, G.5B, G.5C **LESSON 3** **Ts and Train Tracks** GETTING STARTED ACTIVITY 1	**LESSON 3** continued ACTIVITY 2 ACTIVITY 3	**LESSON 3** continued ACTIVITY 4 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: G.2B, G.9B **LESSON 4** **Where Has Polly Gone?** GETTING STARTED ACTIVITY 1
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
LESSON 4 continued ACTIVITY 2 ACTIVITY 3 ACTIVITY 4	**LESSON 4** continued ACTIVITY 5 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: G.2B, G.2C, G.3C, G.10B, G.11A, G.11B **LESSON 5** **In and Out and All About** GETTING STARTED ACTIVITY 1	**LESSON 5** continued ACTIVITY 2
Day 16	**Day 17**	**Day 18**	**Day 19**	
LESSON 5 continued ACTIVITY 3 ACTIVITY 4	**LESSON 5** continued ACTIVITY 5 ACTIVITY 6 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**	

Assessments

There is one assessment aligned to this topic: End of Topic assessment.