Using a Rectangular Coordinate System Summary

KEY TERMS

- sketch
- draw
- conjecture
- auxiliary line
- construct
- compass
- straightedge
- point
- line
- line segment
- midpoint
- segment bisector
- perpendicular bisector
- diagonal
- transformation
- rigid motion
- translation
- reflection
- rotation
- Distance Formula
- Midpoint Formula
- composite figure
- regular polygon

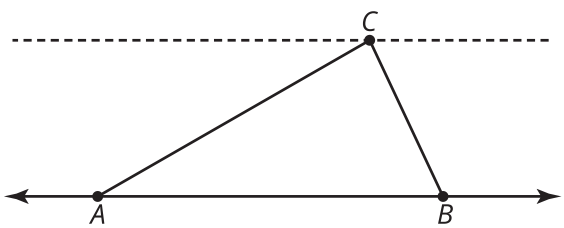
LESSON 1

The Squariest Square

When you **sketch** a geometric figure, you create the figure without tools. Accuracy is not important. When you **draw** geometric figures, you can use tools such as rulers and protractors and the coordinate plane to draw exact lengths and areas.

A **conjecture** is a mathematical statement that appears to be true, but has not been formally proven. You can move from making conjectures and informal arguments to proving that certain mathematical statements must be true. You can use properties and definitions to prove or disprove many conjectures.

An **auxiliary line** is a line or line segment added to a diagram to help in solving or proving a concept. For example, the dashed line drawn parallel to $\overleftrightarrow{AB}$ through point C is an auxiliary line that can be used to reason geometrically about the sum of the measures of the interior angles of a triangle.



LESSON 2

Hip to Be Square

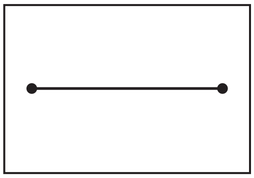
When you **construct** geometric figures, you create exact figures without measurements, using only a **compass** and a **straightedge**. A compass is a tool used to create arcs and circles. A straightedge is a ruler with no numbers.

A **point** is described simply as a location. A point in geometry has no size or shape, but it is often represented using a dot. In a diagram, a point can be labeled using a capital letter. A **line** is described as a straight, continuous arrangement of an infinite number of points. A line has an infinite length, but no width. Arrowheads are used to indicate that a line extends infinitely in opposite directions. In a diagram, a line can be labeled with a lowercase letter positioned next to the arrowhead. A **line segment** is a part of a line between two points on the line, called the endpoints. A distance along a line is the length of a line segment connecting two points on the line. A line segment $\overline{AB}$ has the distance AB.

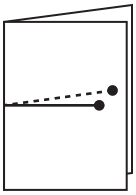
The **midpoint** of a segment is the point that divides the segment into 2 congruent segments. A **segment bisector** is a line, line segment, or ray that divides a line segment into two line segments of equal length. The basic geometric construction used to locate a midpoint of a line segment is called bisecting a line segment.

You can use patty paper to bisect a line segment.

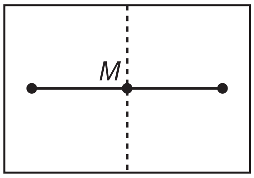
Draw a line on the paper.



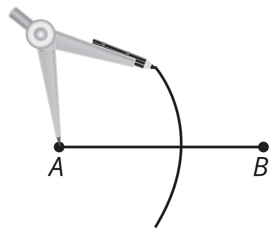
Fold the paper so the endpoints of the line segment lie on top of each other.

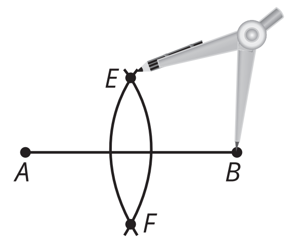


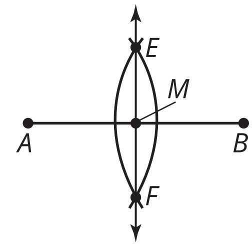
Open the paper. The crease represents the segment bisector, and the midpoint is located where the crease intersects the line segment.



To construct a segment bisector using only a compass and straightedge, you make use of the fact that all the radii of a circle have an equal length.







Construct an Arc

Open the radius of the compass to more than half the length of $\overline{AB}$. Use endpoint *A* as the center and construct an arc.

Construct Another Arc

Keep the compass radius and use point *B* as the center as you construct an arc. Label the points formed by the intersection of the arcs point *E* and point *F*.

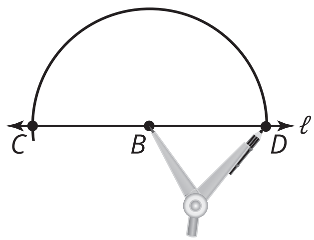
Construct a Line

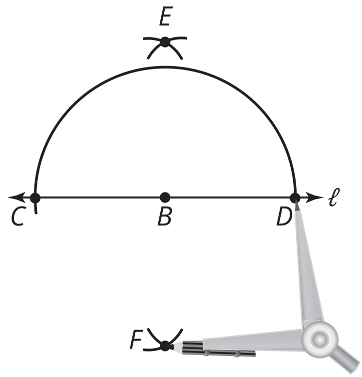
Connect points *E* and *F*. Line segment *EF* is the segment bisector of $\overline{AB}$. The point M represents the midpoint of $\overline{AB}$.

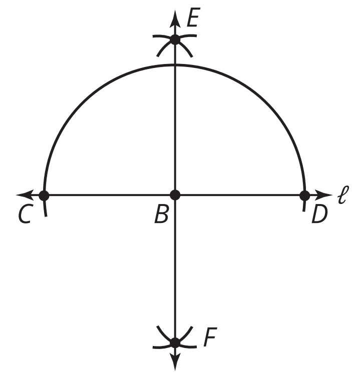
Line *EF* bisects $\overline{AB}$. Point *M* is the midpoint of $\overline{AB}$.

A **perpendicular bisector** is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

You can use a compass and straightedge to create a perpendicular bisector.







Construct an Arc

Use *B* as the center and construct an arc. Label the intersections points *C* and *D*.

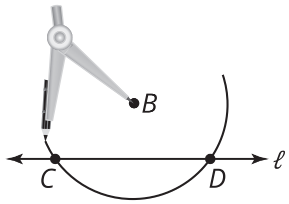
Construct other Arcs

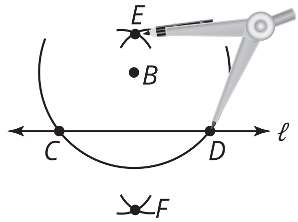
Open the compass larger than the radius. Use *C* and *D* as centers and construct arcs above and below the line. Label the intersection points *E* and *F*.

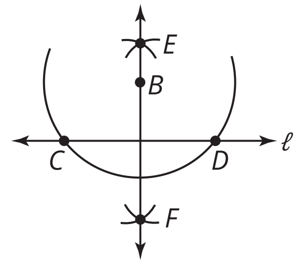
Construct a Line

Use a straightedge to connect points *E* and *F*. Line *EF* is perpendicular to $\overleftrightarrow{CD}$.

You can also construct a perpendicular line through a point not on a line.







Construct an Arc

Use *B* as the center and construct an arc. Label the intersections points *C* and *D*.

→

Construct other Arcs

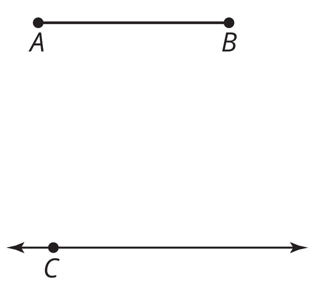
Open the compass larger than the radius. Use *C* and *D* as centers and construct arcs above and below the line. Label the intersection points *E* and *F*.

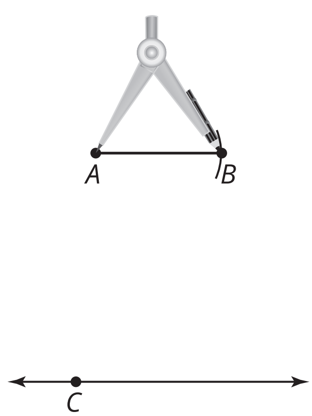
→

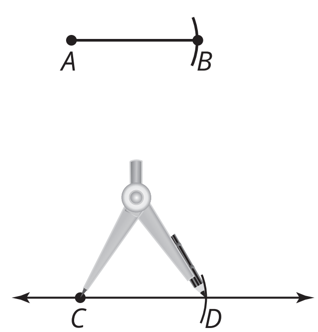
Construct a Line

Use a straightedge to connect points *E* and *F*. Line *EF* is perpendicular to $\overleftrightarrow{CD}$.

You can duplicate a line segment by constructing an exact copy of the original line segment.







Construct a Starter Line

Use a straightedge to construct a starter line longer than $\overline{AB}$. Label point *C* on the line

→

Measure Length

Set your compass the the length *AB*.

→

Copy Length

Place the compass to *C* max point *D* on the new segment

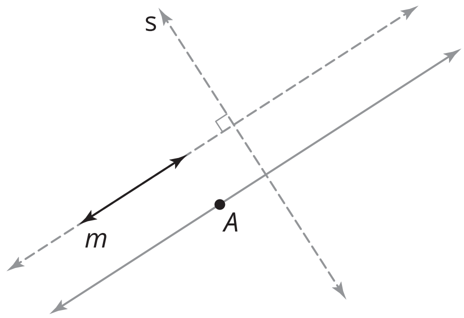
Line segment *CD* is a duplicate of $\overline{AB}$.

A **diagonal** is a line segment joining two vertices of a polygon but is not a side of the polygon.

A **transformation** is the mapping, or movement, of the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of the figure. Three types of rigid motion transformations are translations, reflections, and rotations. A **translation** "slides" a figure up, down, left, or right. A **reflection** "flips" a figure across a line. A **rotation** "spins" a figure about a point.

LESSON 3 Ts and Train Tracks

You can construct parallel lines.

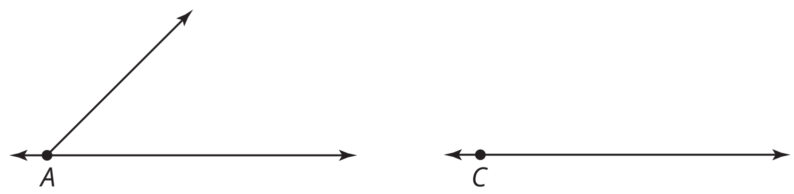


For example, to construct a line parallel to line *m* through point *A*, you can construct a line *s* perpendicular to line *m*. Then, construct a line perpendicular to *m* through point *A*. If two lines are perpendicular to the same line, then they are parallel to each other.

You can also construct parallel lines by duplicating an angle formed by a transversal and a line. You can duplicate an angle using constructions.

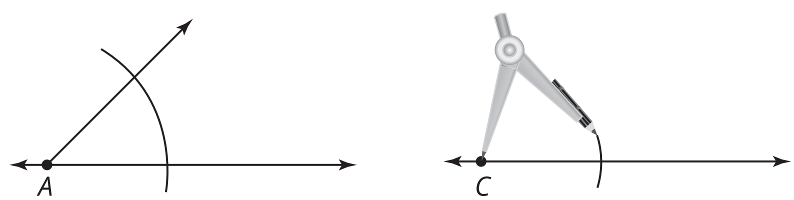
Construct a Starter Line

Use a straightedge to construct a starter line. Label point *C* on the new line.



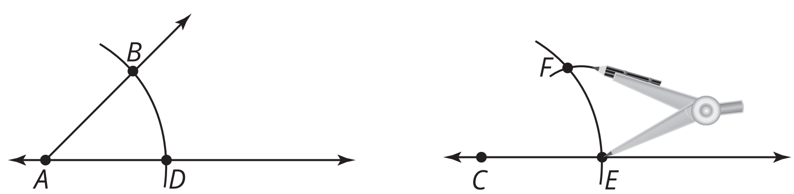
Construct an Arc

Construct an arc with center *A*. Using the same radius, construct an arc with center *C*.



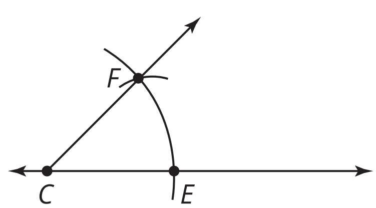
Construct Another Arc

Label points *B*, *D*, and *E*. Construct and arc with radius *BD* and center *E*. Label the intersection *F*.



Construct a Ray

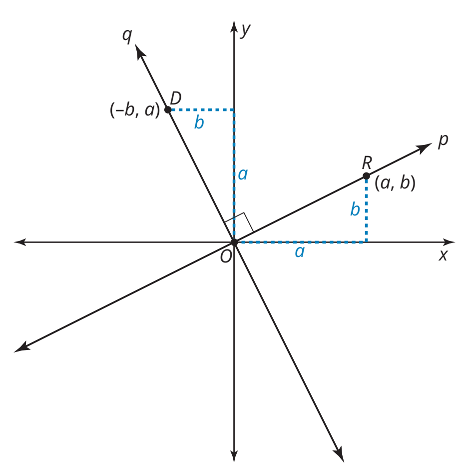
Construct ray *CF*. $\angle BAD \cong \angle FCE$.



Perpendicular lines or line segments form a right angle at the point of intersection. You can think of perpendicular lines as a line and its rotation 90° about a point, which is also the point of intersection.

The product of the slopes of perpendicular lines is -1.

For example, consider the graph.



The slope of line *q* is the negative reciprocal of the slope of line *p* because $\frac{b}{a} \cdot \frac{a}{-b} = -1$.

The slope of any horizontal line is 0 since no matter the change for x, there is 0 change for y. The slope of any vertical line is undefined since no matter the change for y, there is 0 change for x.

All horizontal lines are parallel to each other since their slopes are equal and all vertical lines are parallel since their slopes are equal. A horizontal and a vertical line are always perpendicular to each other.

For example, to write an equation for a line that passes through the point (−4, −2) and is perpendicular to the line $y = 3$, first determine that the line given by $y = 3$ is a horizontal line. Therefore, a line that is perpendicular to $y = 3$ is a vertical line. A vertical line that passes through the point (−4, −2) has the equation $x = -4$.

You can use what you know about the slopes of perpendicular lines and slope-intercept form to write the equation of a perpendicular line.

For example, consider the line $y = 4x - 1$. Write the equation of the line that passes through the point (−4, 2) and is perpendicular to $y = 4x - 1$.

The slope of $y = 4x - 1$ is 4. The slope of the line perpendicular to $y = 4x - 1$ must have a slope of $-\frac{1}{4}$, since $4 \cdot -\frac{1}{4} = -1$.

Using the given point and slope-intercept form, you can set up an equation to solve for b, the y-intercept.

$$2 = -\frac{1}{4}(-4) + b$$
$$2 = 1 + b$$
$$b = 1$$

Therefore, the equation of the line that passes through the point (−4, 2) and is perpendicular to $y = 4x - 1$ is $y = -\frac{1}{4}x + 1$.

Since a line and its translation are parallel to each other, the slopes of parallel lines are equal.

You can use what you know about the slopes of parallel lines and slope-intercept form to write the equation of a parallel line.

For example, consider the line $y = 4x - 1$. Write the equation of the line that passes through the point (−4, 2) and is parallel to $y = 4x - 1$.

The slope of $y = 4x - 1$ is 4. The slope of the line parallel to $y = 4x - 1$ must also have a slope of 4.

Using the given point and slope-intercept form, you can set up an equation to solve for b, the y-intercept.

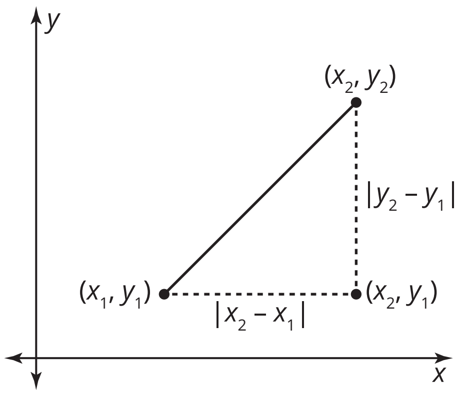
$$2 = 4(-4) + b$$
$$2 = -16 + b$$
$$18 = b$$

Therefore, the equation of the line that passes through the point (−4, 2) and is parallel to $y = 4x - 1$ is $y = 4x + 18$.

LESSON 4

Where Has Polly Gone?

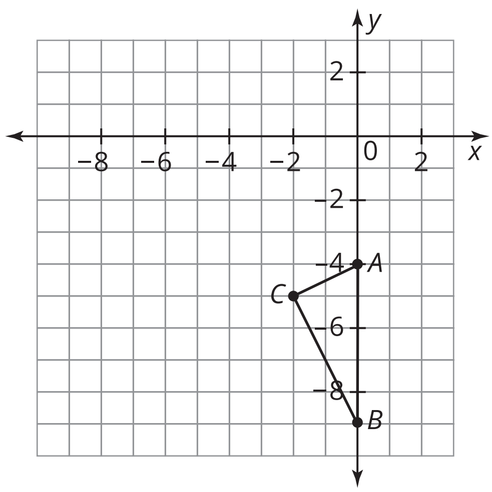
You can use the Pythagorean Theorem to calculate the distance between two points on the coordinate plane. This method can be written as the Distance Formula. The **Distance Formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the distance d between (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Note that absolute value symbols are used because length is always positive.

You can apply the Distance Formula to determine the lengths of the sides of polygons on the coordinate plane.

For example, classify $\triangle ABC$ as scalene, isosceles, or equilateral by determining AC, CB, and AB.
Since points A and B have the same x-value, determine the length of AB by determining the absolute value of the difference in the y-values.



$$AB = |-4 - (-9)|$$
$$AB = 5$$

Then, use the Distance Formula to determine AC and CB.

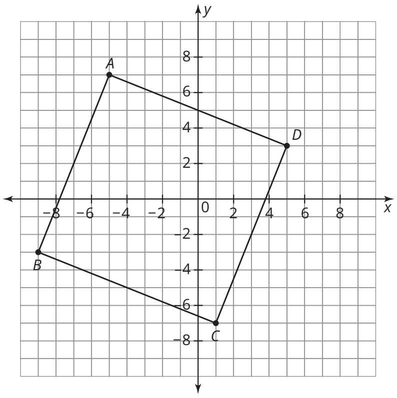
$$d = \sqrt{(2)^2 + (1)^2} \qquad d = \sqrt{(4)^2 + (2)^2}$$
$$d = \sqrt{5} \qquad d = \sqrt{20}$$
$$AC = \sqrt{5} \qquad CB = \sqrt{20}$$

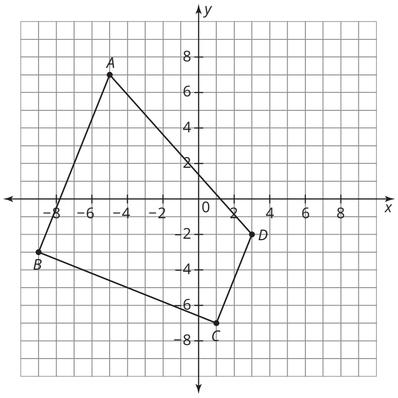
Since all sides of the triangle are different lengths, the triangle is scalene.

The slopes of adjacent sides of a polygon can also be used to determine if the sides form a right angle since the slopes of perpendicular lines are negative reciprocals of each other. For example, in $\triangle ABC$, $\overline{AC}$ has a slope of $\frac{1}{2}$ and $\overline{BC}$ has a slope of -2. Since $\frac{1}{2}$ and -2 are negative reciprocals, the line segments form a right angle and $\triangle ABC$ can be classified as a right scalene triangle.

Given three points of a quadrilateral, the fourth point can be determined using the Distance Formula and the characteristics of the specific quadrilateral.

For example, given points *A*, *B*, and *C*, point *D* can be placed at (5, 3) to create a square or at (3, −2) to create a trapezoid.





To determine the coordinates of the midpoint of a segment on the coordinate plane, you can use the Midpoint Formula. The **Midpoint Formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

LESSON 5 In and Out and All About

The Distance Formula can be used to determine the perimeter and area of rectangles and triangles on the coordinate plane. The perimeter of a geometric figure is calculated by adding the side lengths. The formula for the area of a rectangle is $A = bh$, and the formula for the area of triangle is $A = \frac{1}{2}bh$, where *A* represents area, *b* represents the base, and *h* represents height.

To determine the area of a triangle, first determine the height of the triangle. The altitude, or height, of a triangle is the perpendicular distance from a vertex to the line containing the opposite side, represented by a line segment.

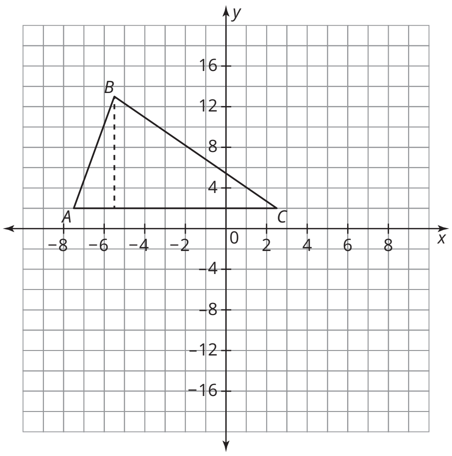
For example, again consider $\triangle ABC$ with vertices A (−7.5, 2), B (−5.5, 13), and C (2.5, 2). The height of the triangle can be created using a line segment from vertex B to base $\overline{AC}$.

The height is $13 - 2 = 11$ units.

The length of the base, $\overline{AC}$, is $|-7.5 - 2.5| = 10$ units.

$A = \frac{1}{2}(11)(10) = 55$

The area of the triangle is 55 square units.

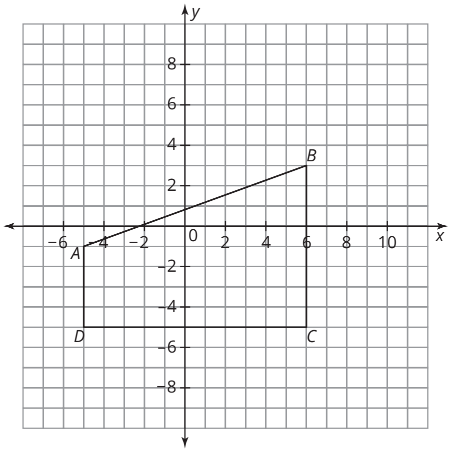


When the base of a triangle is not horizontal, the points that can be used to create a height perpendicular to the base must be identified algebraically by determining the equation of the line containing the base and the equation of the line perpendicular to the base that passes through the opposite vertex, and then solving the system of equations.

You can translate shapes on the coordinate plane to make the process of determining the perimeter and area more efficient. Translations are rigid motions that preserve the size and shape of a figure. The pre-image and the image are congruent because in a translation, all vertices must be rigidly moved from one location to another location.

You can determine the area of a **composite figure** by dividing the figure into a combination of rectangles and triangles. A composite figure is a figure that is formed by combining different shapes.

For example, suppose Aida has a bedroom that is on the top floor of her house. The roof slants downward, creating two congruent trapezoid shaped walls. One of the walls in her room is represented on the coordinate plane by quadrilateral $ABCD$. Each interval on the coordinate plane represents one foot.



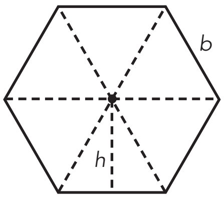
You can determine the area of quadrilateral $ABCD$ by breaking it into a triangle and a rectangle and determining the area of each.

The area of the triangle with vertices at (−5, −1), (6, 3), and (6, −1), is 22 square feet. The area of the rectangle with vertices at (−5, −1), (6, −1), (6, −5), and (−5, −5) is 44 square feet.

The area of the wall is 22 square feet + 44 square feet = 66 square feet.

A **regular polygon** is a polygon with all sides congruent and all angles congruent. The area of a regular polygon can be thought of as the area of a composite shape. To calculate the area of a regular polygon, decompose the polygon into congruent triangles, calculate the area of one triangle, and multiply the area of the triangle by the number of congruent triangles that comprise the polygon.

The area of a regular hexagon can be determined if you know the side length of the hexagon and the height of one of the six congruent triangles you decompose the shape into.



$$A = \frac{1}{2}bh(6) = 3bh$$