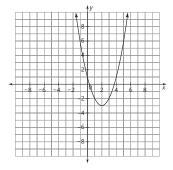
# **Bow Thai**

Translations as Functions

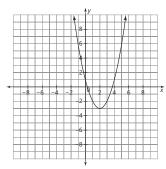
# **Warm Up**

Each coordinate plane shows the graph of f(x). Sketch the graph of g(x).

1. 
$$g(x) = f(x) - 2$$



2. 
$$g(x) = f(x - 3)$$



# **Learning Goals**

- Represent translations on the plane.
- Describe translations as functions that take points on the plane as inputs and produce translated points as outputs.
- · Compare transformations that preserve distance and angles, called isometries, to transformations which are not isometries.

## **Key Terms**

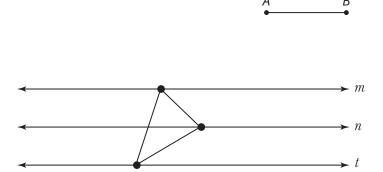
- translation
- isometry

You have learned how to represent the horizontal or vertical translation of a function. How can you write geometric translations of figures on the plane as functions?

#### **Universal Translator**

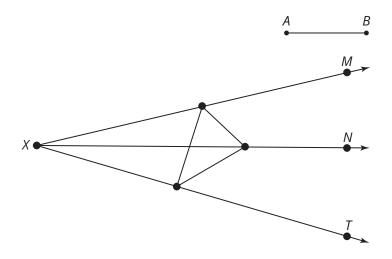
Recall that you used translations in the transformation machine in Lesson 1: *Put Your Input In, Take Your Output Out*. You translated figures along straight lines or line segments.

1. Lines *m*, *n*, and *t* are parallel lines. Draw a translation of each vertex of the triangle along the line the point is located on. Translate each point to the right along the line a distance equal to *AB*. Connect the points to form a triangle.

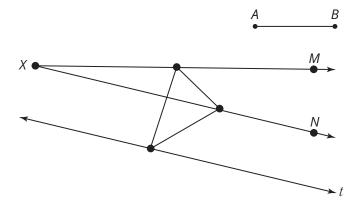


Points *M*, *N*, and *T* are necessary to name each ray.

2. Rays XM, XN, and XT share a vertex point. Draw a translation of each vertex of the triangle along the ray the point is located on. Translate each point to the right along the ray a distance equal to AB. Connect the points to form a triangle.



3. Rays XM and XN share a vertex point, and line t is parallel to  $\overrightarrow{XN}$ . Draw a translation of each vertex of the triangle along the ray or line the point is located on. Translate each point to the right along the ray or line a distance equal to AB. Connect the points to form a triangle.



- 4. Compare the diagrams you created in Questions 1 through 3. Each set of three lines and/or rays makes up a transformation machine.
  - a. Which transformation machine produces a translation of the triangle? Explain your reasoning.
  - b. Which transformation machine produces a dilation of the triangle? Explain your reasoning.



# **Translation Functions**



Transformations are used frequently in web design and game animation and are often written as functions, which take points, distances, and angles as inputs. The functions output a new set of points after applying a transformation. These transformations move objects around on the screen.

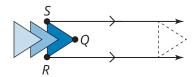


A translation moves a set of points a specified distance in a specified direction along parallel lines.

Suppose you are designing a website banner for a new restaurant. The banner will show three congruent triangles animated from left to right, and then the name will fade in.



1. Consider the translation of the first triangle,  $\triangle SQR$ .



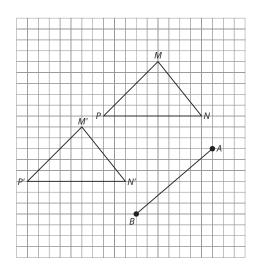
- a. Label the points of the image,  $\triangle S'Q'R'$ .
- b. What relationship is there between  $\overline{SS'}$  and  $\overline{RR'}$ ?
- c. Measure the lengths of the two line segments used in the translation. What do you notice?
- d. What do you know about the distance QQ'? What do you know about the line containing  $\overline{QQ'}$ ?

#### Worked Example

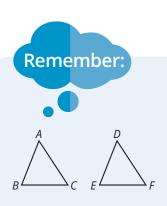
A translation can be measured as a directed line segment.

 $\triangle MNP$  was translated to produce  $\triangle M'N'P'$ . The triangle was translated a distance equal to the distance between points A and B. It was translated in the direction from point A to point B.

So,  $\overline{AB}$  is the directed line segment used to measure this translation.



- 2. Suppose each grid square is 1 unit  $\times$  1 unit.
  - a. What is the distance from point A to point B?
  - b. Compare the distance AB with the distances MM', NN', and PP'. What do you notice?
  - c. Can you draw another directed line segment on the grid which defines the translation of  $\triangle MNP$  to  $\triangle M'N'P'$ ? If so, draw the segment on the grid. Explain your thinking.
- 3. Write equality and congruence statements to compare the corresponding sides and angles of the pre-image  $\triangle MNP$  and the image  $\triangle M'N'P'$ .



If  $\triangle ABC \cong \triangle DEF$ , then:

 $\overline{AB} \cong \overline{DE}$ AB = DE $\angle ABC \cong \angle DEF$  $m \angle ABC = m \angle DEF$ 



A function is a rule that assigns exactly one output to each input.

Inputs to functions do not have to be numbers. They can be points, too.

A **translation** is a function, *T*, which takes as its input a set of pre-image points and outputs a set of image points. The pre-image points are translated a distance of AB in the direction AB. For example, a translation of point P could be expressed as  $T_{AB}(P)$ , or P'. A translation is an example of an *isometry*. An **isometry** is a rigid motion transformation that preserves size and shape.

### Worked Example

A translation function can represent the distance and direction of the translation using a line or line segment, or a parallel line or line segment.

$$T_{AB}(P) = P'$$
 $P''$ 
 $P = P'$ 
 $T_{AC}(P) = P''$ 
 $P = P'$ 

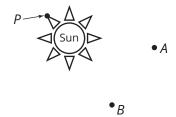
4. Identify how the distance and direction of the translation are specified in each of the functions.

- 5. Consider the translation of the website banner from Question 1.
  - a. Use the notation from the Worked Example to describe the translation of  $\triangle SQR$ .
  - b. Explain how your function represents the translation of every point of  $\triangle SQR$ .

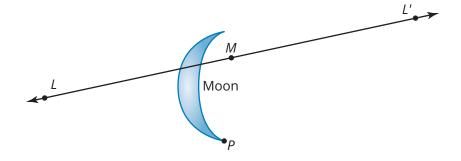
6. Greta says that the exact same function can be used for every triangle in the animated web banner. Is Greta correct? Explain your thinking, and then draw the translations to justify your answer.



- 7. Complete each translation using the given function.
  - a.  $T_{AB}(Sun)$



b.  $T_{LM}$  (Moon)



8. Explain why you can use parallel lines when describing translations.

# Determining Congruence Using Translations



When users click on the menu of Bow Thai, copies of 3 triangles will move from the corners and top of the web page to the center to form the background behind the word "Menu" as shown.



- 1. Write and draw translation functions to show how each triangle will move on the page.
- 2. Are the triangles all congruent? Explain why or why not.

The owner of Bow Thai is thinking about using smaller triangles on the sides of the menu web page. She still wants the triangles to move and merge to form the triangle background behind the word "Menu."

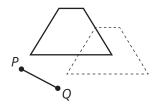


3. Would these transformations be isometries? Demonstrate why or why not and explain your process.

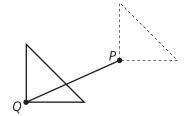
# TALK the TALK

### Isometries on the Menu

- 1. Describe how to distinguish between an isometry and a transformation that is not an isometry.
- 2. Write a function to describe each translation. Each solid figure is a pre-image, and each dashed figure is an image.
  - a.



b.



3. What similarities and differences are there between a geometric translation function and an algebraic equation which shows the translation of a function?