Staring Back at Me

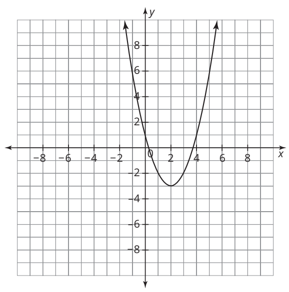
3

Reflections as Functions

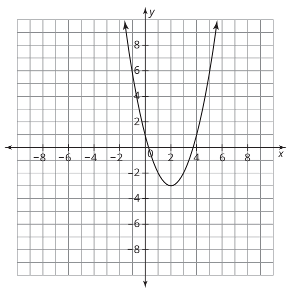
Warm Up

Each coordinate plane shows the graph of $f(x)$. Sketch the graph of $g(x)$.

1. $g(x) = -f(x)$



2. $g(x) = f(-x)$



Learning Goals

- Represent reflections in the plane using patty paper and constructions.
- Describe reflection transformations as functions that take points as inputs and output reflected points.
- Identify reflections as points equidistant to the perpendicular bisector of line segments connecting the pre-image and image points of the reflection.
- Specify a sequence of translations and reflections that will carry a figure onto a congruent figure.

Key Terms

- reflection
- Perpendicular Bisector Theorem
- proof

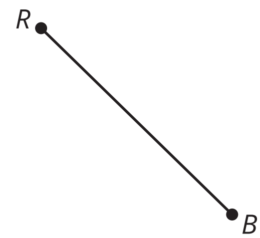
You have learned how to represent the horizontal or vertical reflection of a function. How can you write geometric reflections of figures on the plane as functions?

GETTING STARTED

Reflecting on Bisecting

In previous lessons, you constructed the perpendicular bisector of line segments.

1. Construct a perpendicular bisector of $\overline{RB}$.



2. Use patty paper to trace the diagram you constructed. How can you use patty paper to map point *R* onto point *B*? Explain the transformation you used.

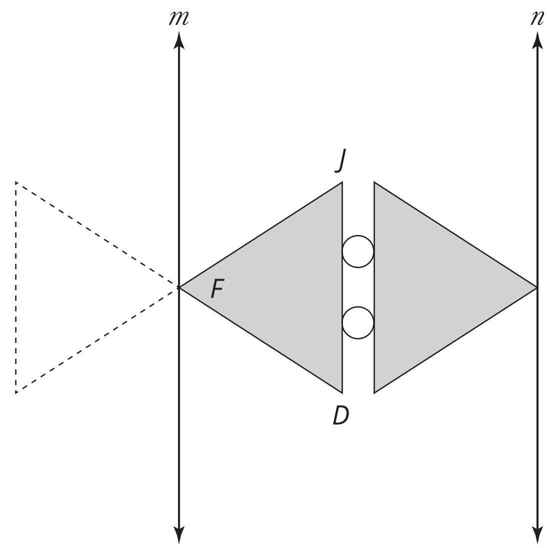
3. Can you draw two points in the plane that cannot be mapped onto each other by the same transformation you used in Question 2? Justify your answer.

ACTIVITY

3.1 Reflection Functions

You saw in the previous lesson that translations can be used in web design and are often written as functions, which take points, distances, and angles as inputs and outputs a new set of points after applying a transformation. The same is true for reflections.

To reveal a message during a game, you have to make two triangles fly open.



1. Consider the reflection of the left triangle, △*FJD*, across line m.

a. Label the points of the image, △*F′J′D′*.

b. What relationship is there among $\overline{JJ'}$ and $\overline{DD'}$? Justify your answer.

c. What relationship is there among $\overline{JJ'}$, $\overline{DD'}$, and line *m*? Justify your answer.

d. Reflect the other triangle across line n. Label the points of the pre-image and image. Compare this reflection with the reflection of the first triangle.

2. **Write equality and congruence statements to compare the corresponding sides of the pre-image and the image.**

A reflection is an isometry.

A **reflection** is a function, R_ℓ, which takes as its input, P, the location of a point with respect to some line of reflection ℓ and outputs $R_\ell(P)$, or the opposite of the location of P with respect to the line of reflection.

3. **Consider the reflections from Question 1.**

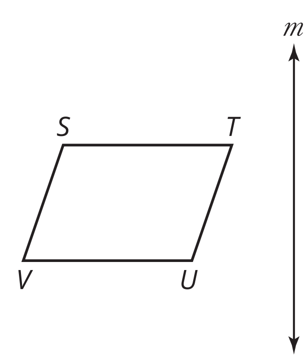
   a. **Write a function of the form $R_\ell(P)$ to describe the reflection of $\triangle FJD$.**

   b. **Explain how your function represents the reflection of every point of $\triangle FJD$.**

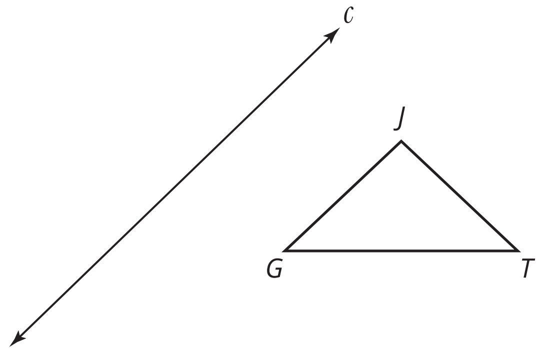
4. **Describe the relationship between corresponding points of a reflection and the line of reflection.**

5. **Complete each reflection using the given function.**

   a. **$R_m(STUV)$**

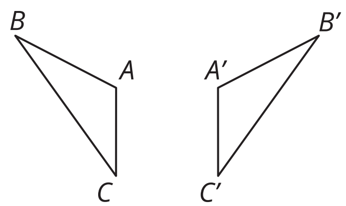


b. $R_c(JTG)$

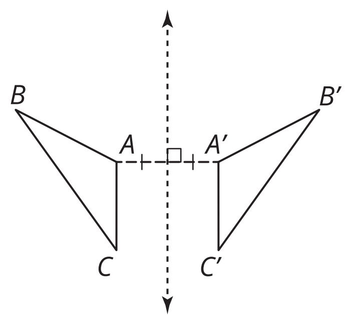


It is possible to construct the line of reflection when given two figures that are reflections of one another.

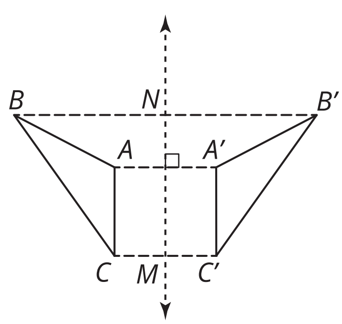
Worked Example



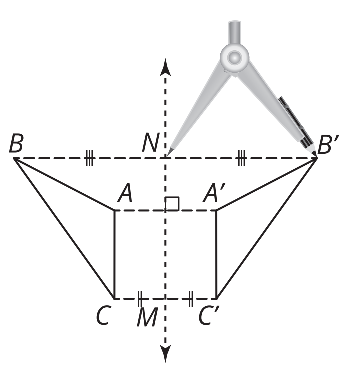
1. Label the vertices.



2. Connect two corresponding points. Construct the perpendicular bisector of the line segment connecting them.



3. Connect remaining corresponding points. Label the intersection of each line segment with the perpendicular bisector.



4. Use a compass to determine whether each intersection point is the midpoint of the line segment connecting corresponding vertices. If that is the case, the perpendicular bisector is the line of reflection. If not, the figures are not reflections of one another.

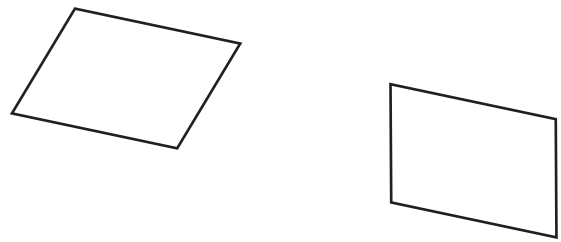
$\overline{CM} \cong \overline{C'M}$

$\overline{BN} \cong \overline{B'N}$

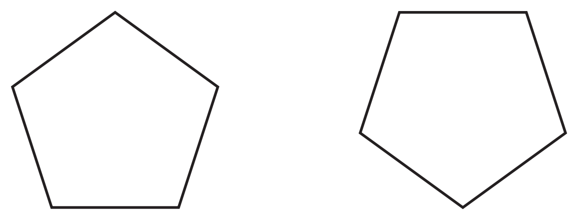
$\overleftrightarrow{MN}$ is the line of reflection.

6. Determine whether the figures are reflections of one another. If so, identify the line of reflection.

a.



b.

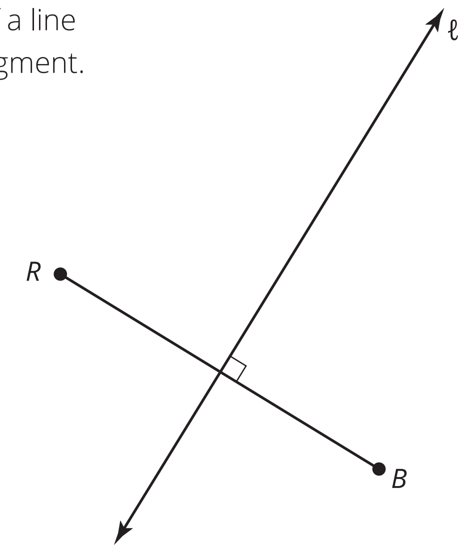


c.

7. How do you know whether two figures are not reflections of one another?

ACTIVITY 3.2 Perpendicular Bisector Theorem

In the Getting Started, you saw that the perpendicular bisector of a line segment is the line of reflection between the endpoints of the segment. Consider $\overline{RB}$ with perpendicular bisector ℓ.



1. Label point *P* anywhere on ℓ. What do you notice about the relationship between point *P* and points *R* and *B*?

A **proof** is a series of statements and corresponding reasons forming a valid argument that starts with a hypothesis and arrives at a conclusion.

You can prove that the endpoints of a line segment are equidistant from any point on the perpendicular bisector.

Worked Example

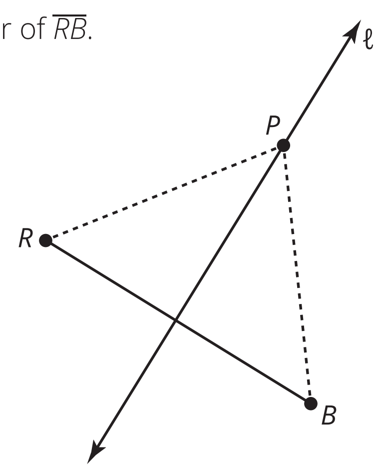
Given: Line ℓ is the perpendicular bisector of $\overline{RB}$.
Point P is on ℓ.

Prove: $PR = PB$

The reflection of point P across line ℓ is point P by the definition of reflection.

Because line ℓ is the perpendicular bisector of $\overline{RB}$, the reflection of point R across line ℓ is point B.

Therefore, $PR = PB$.



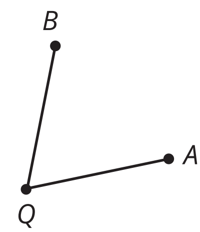
2. Provide a reason why $PR = PB$.

The **Perpendicular Bisector Theorem** states: "Any point on the perpendicular bisector of a segment is equidistant from the endpoints of that segment." Remember that a theorem is a statement that can be demonstrated to be true by accepted mathematical operations and arguments.

Let's consider the relationship between endpoints of a line segment and any point on the perpendicular bisector from another perspective.

Suppose you have two points that are equidistant from a third point. You can show that the third point lies on the perpendicular bisector of the segment connecting the points.

3. Consider point *Q*, which is equidistant from points *A* and *B*.



a. Draw $\overrightarrow{QP}$ so that it bisects $\angle BQA$. This makes $\angle BQP$ and $\angle PQA$ congruent angles. Label the intersection of $\overrightarrow{QP}$ and $\overline{AB}$ as point *C*.

b. Describe the location of $\overline{QB'}$ if $\overline{QB}$ is reflected across $\overrightarrow{QP}$. What does this tell you about the distances *BC* and *CA*? Explain your thinking.

c. Explain how you know that $\overrightarrow{QP}$ is a perpendicular bisector of $\overline{AB}$.

ACTIVITY
3.3

Sequences of Isometries Using Translations and Reflections

You have learned that reflections and translations are isometries, which means that they preserve distances and angle measures.

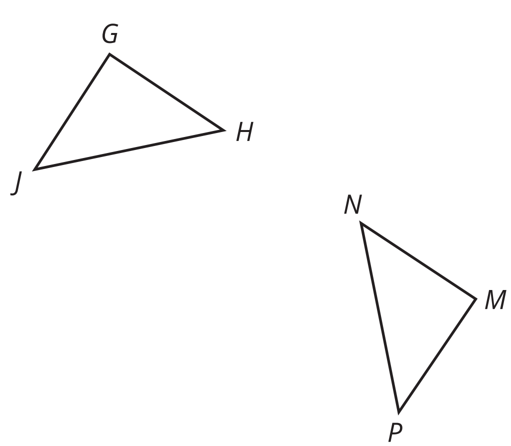
1. Sunita made a conjecture about reflections. She said that you can always map one congruent line segment onto the other using at most two reflections.

Do you think Sunita is correct? Draw examples or counterexamples to justify your reasoning.

Remember:

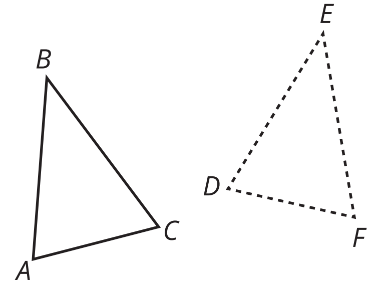
Any point that is equidistant from two other points lies on the perpendicular bisector between the two points.

2. Triangle *GHJ* is a reflection of $\triangle MNP$. Describe the locations of the midpoints of $\overline{GM}$, $\overline{HN}$, and $\overline{JP}$.



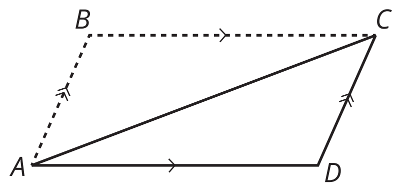
3. Describe and sketch the sequence of translations and reflections that shows that the two figures in each pair are congruent. Images are shown with dashed lines.

a.

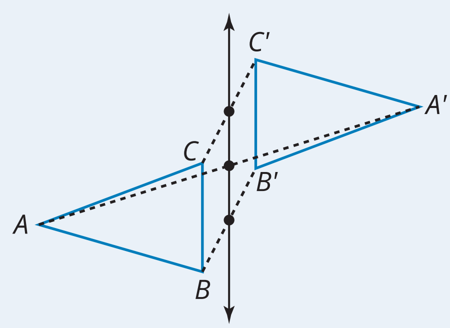


Sides that have the same number of arrowhead markings are parallel to one another.

b.



4. Miguel was investigating a transformation of △*ABC* to △*A′B′C′* and discovered that the midpoints of the segments connecting corresponding points were collinear, but the line was not a perpendicular bisector of each segment. He thought that this must not be a reflection.



Lina disagreed. She said that you can translate first to show that the midpoints lie on a perpendicular bisector.
Who is correct? Explain your reasoning.

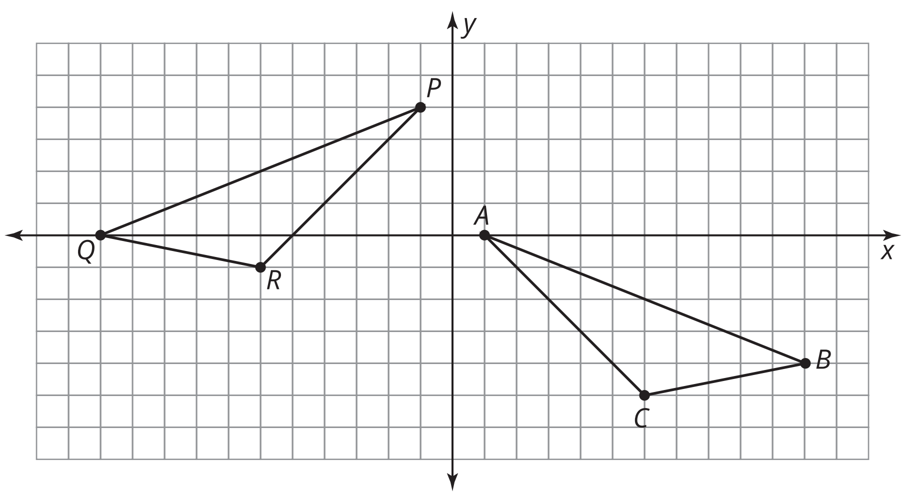
NOTES

TALK the TALK

But We're Off the Grid

In this activity, you translated and reflected figures in the plane. Let's compare those transformations with the same ones performed on a coordinate plane.

1. **Describe the sequence of translations and reflections which maps the pre-image triangle, $\triangle PQR$, to the image triangle, $\triangle ABC$ on the coordinate plane.**



2. **How are translating and reflecting geometric figures in the plane different from performing these transformations on the coordinate plane? How are they the same?**

3. **What similarities and differences are there between a geometric reflection function and an algebraic equation that shows the reflection of a function?**