3

Staring Back at Me

Reflections as Functions

MATERIALS

Patty paper

Lesson Overview

Students analyze reflections as isometries. They construct a perpendicular bisector of a segment and then conclude that the perpendicular bisector is the line of reflection between the endpoints of the segment. Students investigate reflections as functions using the context from the previous lesson, use function notation to represent geometric reflections, and construct lines of reflection. They combine what they learned in this lesson and the previous lesson to identify sequences of translations and reflections to demonstrate that two figures are congruent.

Geometry

Coordinate and Transformational Geometry

(3) The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

Logical Argument and Constructions

(5) The student uses constructions to validate conjectures about geometric figures. The student is expected to:

(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge.

Proof and Congruence

(6) The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

(A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems.

(C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- The perpendicular bisector of a line segment is a line of reflection between the two endpoints of the segment.
- Reflections are isometries.
- A reflection is a function, R_ℓ, which takes as its input, *P*, the location of a point with respect to some line of reflection, ℓ, and outputs $R_\ell(P)$ or the opposite of the location of *P* with respect to the line of reflection.
- The Perpendicular Bisector Theorem states: "If two points are equidistant from a third point, the third point lies on the perpendicular bisector of the segment connecting the two points."

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Reflecting on Bisecting

Students construct a perpendicular bisector of a line segment and then use patty paper to recognize that the perpendicular bisector is the line of reflection between the endpoints of the line segment. Students are challenged to draw two points in the plane that cannot be reflected one onto the other; this is impossible because any two points in the plane can be mapped onto each other through one reflection.

Develop

Activity 3.1: Reflection Functions

Students consider a different website animation to investigate reflections. They explore the fact that corresponding points of reflections are equidistant from the line of reflection. They also use appropriate notation to complete reflections and construct lines of reflection.

Activity 3.2: Perpendicular Bisector Theorem

Students show that if two points are equidistant from a third point, the third point lies on the perpendicular bisector of the segment connecting the two points.

Day 2

Activity 3.3: Sequences of Isometries Using Translations and Reflections

Students investigate a peer conjecture to conclude that you can always map one congruent line segment onto another using at most two reflections. They also realize the need for translations along with reflections as they describe sequences of isometries to show that two figures are congruent.

Demonstrate

Talk the Talk: But We're Off the Grid

Students perform a sequence of translations and reflections on a coordinate plane and describe the similarities and differences between completing these transformations in the plane and on the coordinate plane. Students also compare geometric reflection functions and algebraic equations which show the reflection of a function.

Getting Started: Reflecting on Bisecting

Facilitation Notes

In this activity, students construct a perpendicular bisector of a line segment and then use patty paper to recognize that the perpendicular bisector is the line of reflection between the endpoints of the line segment. Students are challenged to draw two points in the plane which cannot be reflected one onto the other; this is impossible because any two points in the plane can be mapped onto each other through one reflection.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

The alignment of points *B* and *R* on the patty paper when it is folded on the perpendicular bisector. If they are not in alignment, the crease is not the perpendicular bisector of segment *RB*.

Misconception

Students may not realize that distance is always measured along the perpendicular segment from the point to the perpendicular bisector. If this is not apparent to students, discuss this issue by providing a correct and an incorrect process for determining the distance. Also discuss why the perpendicular segment will always have the shortest length.

Questions to ask

- How did you fold the patty paper to construct the perpendicular bisector of $\overline{RB}$?
- Can any two endpoints of a segment on the patty paper be aligned, then folded to form a perpendicular bisector?
- Is the perpendicular bisector also the line of symmetry for $\overline{RB}$? How do you know?
- Is this relationship true for any two points? Why does this happen?
- Are the angles formed always right angles?
- Is the distance from each point to the perpendicular bisector always the same?

Differentiation strategy

To extend the activity, have students locate a point on the upper portion of the perpendicular bisector of a line segment. Ask the question, "If this point (pre-image) is reflected below the line segment, is the image also on the perpendicular bisector?" Then have them measure the distance the point is to the endpoints of the line segment. Ask other questions, such as, "Are all points on the perpendicular bisector of a line segment equidistant to the endpoints of the segment?" and "Does the reflection preserve this relationship?"

Summary

The perpendicular bisector of a line segment is a line of reflection between the two endpoints of the segment.

Activity 3.1
Reflection Functions

DEVELOP

Facilitation Notes

In this activity, students consider a different website animation to investigate reflections. They explore the fact that corresponding points of reflections are equidistant from the line of reflection. They also use appropriate notation to complete reflections and construct lines of reflection.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

Confusion related to the reflection of a triangle when one vertex is located on the line of reflection. When the point reflects onto itself, the location of the pre-image and image are the same, and may lead to issues with labeling. Discuss the fact that a single point can have more than one label.

Questions to ask

- What do you notice about the location of point F and the location of point F'?
- Is point F' the reflection of point F? Can a point be reflected onto itself?
- What situation causes a point to be reflected onto itself?
- How do you know that $\overline{JJ'}$ and $\overline{DD'}$ are parallel?
- How do you know that line m and line n are perpendicular bisectors?
- Are the line segments formed by connecting the pre-image of the other triangle to its image parallel to each other and perpendicular to line n, the line of reflection?
- Does the congruence statement $\triangle FJD \cong \triangle F'J'D'$ describe the relationship between the pre-image and image?

Differentiation strategy

To extend the activity, have students consider the reflection of a figure across a horizontal line or diagonal line of reflection. As in this activity, position a vertex or side of the figure on the line of symmetry. Have students answer questions similar to the questions posed in Questions 1 and 2.

Ask a student to read the definition following Question 2 aloud. Discuss as a class.

Questions to ask

- What does "the opposite of the location of *P* with respect to the line of reflection" mean?
- Under what circumstance will "the opposite of the location of *P* with respect to the line of reflection" mean that the image is located to the left of the pre-image? Above the pre-image?

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

Differentiation strategy

To assist all students, suggest they use patty paper to complete the reflections in Question 5. They can then staple their patty paper next to the question in their book.

Misconceptions

- In Question 5, students may think they are only reflecting vertices; however, by connecting the vertices, they are also reflecting the points that lie between the vertices. To make this clear, you may want to have them label other points on the line segments and demonstrate how they are reflected.
- Students may confuse the equality relationships when constructing a perpendicular bisector to a line segment versus constructing a perpendicular bisector/line of reflection to a figure. In the first situation, all points on the perpendicular bisector are equidistant to the endpoints of the segment. In the second situation, each point (pre-image) and its image are equidistant from the perpendicular bisector/line of reflection.

Questions to ask

- How do you know what to use as the subscript for *R*?
- How do you know what to place in the parentheses? Is there another way to express this?
- How is reflection notation similar to the notation you used to describe translations?
- How is reflection notation different than the notation you used to describe translations?
- Is every point of $\triangle F'J'D'$ a reflection across line *m* of the original corresponding point?
- In a reflection, are the corresponding points of the pre-image and image equidistant from the line of reflection?

- What is the relationship between $\overline{TT'}$, $\overline{SS'}$, $\overline{VV'}$, and $\overline{UU'}$?
- How are $\overline{TT'}$, $\overline{SS'}$, $\overline{VV'}$, and $\overline{UU'}$ related to line *m*?
- What is the relationship between $\overline{TT'}$, $\overline{JJ'}$, and $\overline{GG'}$?
- How are $\overline{TT'}$, $\overline{JJ'}$, and $\overline{GG'}$ related to line *c*?

Differentiation strategy

To extend the activity, have students write a function to describe the reflection of the triangle across line *n*. Have them explain how their function represents the reflection of every point of the pre-image.

Analyze the Worked Example following Question 5 as a class.

Differentiation strategy

To assist all students, suggest they use colored pencils to reenact the steps in the Worked Example.

Questions to ask

- Why do the directions mention to use a compass?
- Why don't you have to create a perpendicular bisector for each pair of corresponding points?

Have students work with a partner or in a group to complete Questions 6 and 7. Share responses as a class.

Differentiation strategy

To assist all students, suggest that they label the vertices in the figures in Question 6.

Questions to ask

- How did you know what vertices were corresponding vertices?
- Do the segments formed by connecting the corresponding vertices appear to be parallel?
- Is the perpendicular bisector the same for all of the segments formed by connecting the corresponding vertices? How do you know?
- Are the all of the midpoints of the segments formed by connecting the corresponding vertices also on the perpendicular bisector? Why does this happen?
- How do you know the pentagon image did not result from a reflection of the pre-image?
- What rigid motion would map the pentagon onto itself?
- How do you know the triangle image did not result from a reflection of the pre-image?
- What rigid motion would map the triangle onto itself?

Differentiation strategy

To extend the activity, have students write the translation function that would map the triangle onto itself for Question 6, part (c).

Summary

A reflection is a function, R_ℓ, which takes as its input, P, the location of a point with respect to some line of reflection, ℓ, and outputs $R_\ell(P)$, the opposite of the location of P with respect to the line of reflection.

Activity 3.2

Perpendicular Bisector Theorem

Facilitation Notes

In this activity, students show that if two points are equidistant from a third point, the third point lies on the perpendicular bisector of the segment connecting the two points.

Ask a student to read the introduction aloud. Have students work with a partner or in a group to complete Question 1.

Questions to ask

- Did you locate point P above or below $\overline{RB}$?
- Does the location of point P affect the relationship between point P and points R and B?

Read the definition following Question 1 and analyze the worked example as a class.

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

Questions to ask

- Is a reflection a rigid motion transformation?
- What do rigid motion transformations preserve?

Ask a student to read the definition aloud following Question 2. Discuss as a class.

Questions to ask

- State the perpendicular bisector theorem in your own words.
- How can you use the concepts of midpoints to make sense of this theorem?
- How is this theorem related to the construction of a perpendicular bisector?

Have students work with a partner or in a group to complete Question 3. Share responses as a class.

As students work, look for

The use of patty paper or a compass and straightedge to bisect the angle.

Questions to ask

- What information in the first statement in Question 3 lets you know that point *Q* lies on the perpendicular bisector?
- How did you bisect the angle?
- Where is the location of point *B*′?

Summary

The Perpendicular Bisector Theorem states: "If two points are equidistant from a third point, the third point lies on the perpendicular bisector of the segment connecting the two points."

Activity 3.3

Sequences of Isometries Using Translations and Reflections

Facilitation Notes

In this activity, students investigate a peer conjecture to conclude that you can always map one congruent line segment onto another using at most two reflections. They also realize the need for translations along with reflections as they describe sequences of isometries to show that two figures are congruent.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategies

To scaffold support,

- Discuss how the phrase *preserves distances and angle measures* in the introduction means the same as *preserving size and shape* in the definition of an isometry presented in the previous lesson.
- Allow students to use patty paper rather than draw diagrams to justify their reasoning for Question 1.

As students work, look for

Different strategies to respond to Question 2. Some students may identify all the midpoints first, then realize they are collinear and lie on the perpendicular bisector. Other students may draw a perpendicular bisector first, and then identify each midpoint as the intersection of the line segment connecting two corresponding points and the perpendicular bisector.

Questions to ask

- What is a way to generalize your steps to show Sunita is correct?
- Using patty paper containing two congruent line segments, how could you map one endpoint of the first line segment onto one endpoint of the second line segment?
- Once the first two endpoints are mapped onto each other, how could you use a second reflection to map the remaining two endpoints on top of each other?
- Explain how the segments might appear if only one reflection is required?
- How did you determine the location of the midpoints? Is there another way to do this?
- Why are the midpoints of $\overline{GM}$, $\overline{HN}$ and $\overline{JP}$ collinear?
- How do you know the midpoints lie on the perpendicular bisector?

Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

Differentiation strategies

- To scaffold support with Question 3, part (b), tell students that two reflections and one translation are required. Then, let them determine how to complete those transformations to map one triangle onto the other.
- To extend the activity, have students use patty paper to demonstrate their sequences of translations and reflections.

Questions to ask

- What two transformations could you use to demonstrate the two figures in Question 3, part (a), are congruent?
- What three transformations could you use to demonstrate the two figures in Question 3, part (b), are congruent?
- What direction does your translation need to occur?
- Where do you need to place the line of reflection?
- Does the order of the transformations make a difference? Why or why not?
- If Miguel translated $\triangle ABC$ first, would the midpoints have aligned on the perpendicular bisector?
- How would you describe the translation Lina is suggesting?
- If the translation is performed after the reflection, the midpoints of the segments align, but they do not align on the same perpendicular bisector. Why?
- If the translation is performed before the reflection, the midpoints of the segments align on the same perpendicular bisector. Why?

Summary

A sequence of translations and reflections can be used to show that two plane figures are congruent.

Talk the Talk: But We're Off the Grid

DEMONSTRATE

Facilitation Notes

In this activity, students perform a sequence of translations and reflections on a coordinate plane and describe the similarities and differences between completing these transformations in the plane and on the coordinate plane. Students also compare geometric reflection functions and algebraic equations which show the reflection of a function.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Is $\triangle PQR$ the image or the pre-image?
- Is $\triangle PQR$ reflected across the x-axis or the y-axis?
- Is $\triangle PQR$ translated up 4 units or down 4 units?
- Can $\triangle PQR$ be translated down 4 units and then reflected across the y-axis?
- Can $\triangle PQR$ be reflected across the y-axis and then translated down 4 units?
- Is it easier to transform shapes on or off the coordinate plane? Why?
- Do both reflections of functions and reflection of figures move a set of points across a line?
- When reflecting functions, what does $y = -f(x)$ mean?
- When reflecting functions, what does $y = f(-x)$ mean?
- Do geometric reflection functions move the set of points across horizontal lines? Vertical lines? Diagonal lines? Any lines?

Summary

When you translate a geometric figure on a coordinate plane, the movements can be quantified by units on the grid. When you translate a geometric figure in the plane, the movements are identified by directed line segments. When you reflect a geometric figure on a coordinate plane, the line of reflection can be the equation of any line on the coordinate plane. When you reflect a geometric figure in a plane, the line of reflection can be any line in the plane.

NOTES

3

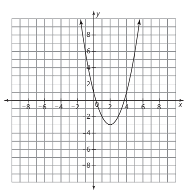
Staring Back at Me

Reflections as Functions

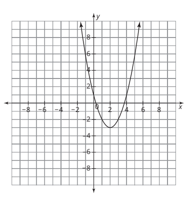
Warm Up

Each coordinate plane shows the graph of $f(x)$. Sketch the graph of $g(x)$.

1. $g(x) = -f(x)$



2. $g(x) = f(-x)$



Learning Goals

- Represent reflections in the plane using patty paper and constructions.
- Describe reflection transformations as functions that take points as inputs and output reflected points.
- Identify reflections as points equidistant to the perpendicular bisector of line segments connecting the pre-image and image points of the reflection.
- Specify a sequence of translations and reflections that will carry a figure onto a congruent figure.

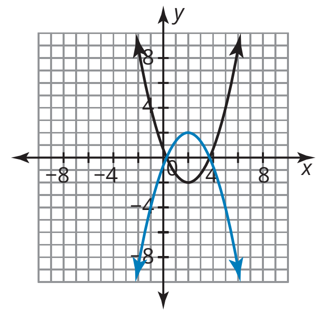
Key Terms

- reflection
- Perpendicular Bisector Theorem
- proof

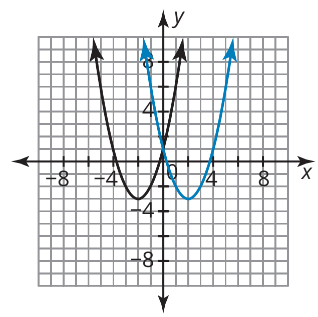
You have learned how to represent the horizontal or vertical reflection of a function. How can you write geometric reflections of figures on the plane as functions?

Warm Up Answers

1.

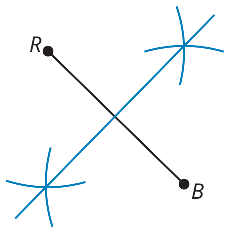


2.



Answers

1.



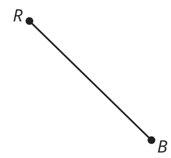
2. I can fold the patty paper along the perpendicular bisector to map point *B* onto point *R*. Points *B* and *R* are reflections of each other across the line containing the perpendicular bisector.
3. This is impossible. Any two points in the plane are reflections of each other across some line.

GETTING STARTED

Reflecting on Bisecting

In previous lessons, you constructed the perpendicular bisector of line segments.

1. Construct a perpendicular bisector of $\overline{RB}$.



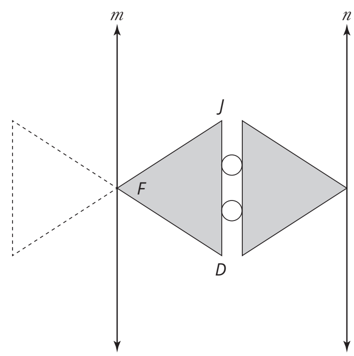
2. Use patty paper to trace the diagram you constructed. How can you use patty paper to map point *R* onto point *B*? Explain the transformation you used.

3. Can you draw two points in the plane that cannot be mapped onto each other by the same transformation you used in Question 2? Justify your answer.

ACTIVITY
3.1 Reflection Functions

You saw in the previous lesson that translations can be used in web design and are often written as functions, which take points, distances, and angles as inputs and outputs a new set of points after applying a transformation. The same is true for reflections.

To reveal a message during a game, you have to make two triangles fly open.



1. Consider the reflection of the left triangle, △*FJD*, across line *m*.

a. Label the points of the image, △*F′J′D′*.

b. What relationship is there among $\overline{JJ'}$ and $\overline{DD'}$? Justify your answer.

c. What relationship is there among $\overline{JJ'}$, $\overline{DD'}$, and line *m*? Justify your answer.

d. Reflect the other triangle across line *n*. Label the points of the pre-image and image. Compare this reflection with the reflection of the first triangle.

Answers

1a. Check students' drawings.

1b. The line segments are parallel to each other.

1c. The points on line *m* are midpoints of the line segments. Line *m* is perpendicular to the line segments. So, line *m* is the perpendicular bisector of the segments connecting the pre-image and image points of the reflection.

1d. Check students' drawings. The line segments connecting this pre-image and image are also parallel to one another and are perpendicular to the line of reflection.

ELL Tip

Assess students' prior knowledge of the term *reveal*. Define *reveal* as *to share previously unknown information, or secrets, with others*. Synonyms for *reveal* include *give out, release, tell, divulge,* and *disclose*. Read aloud the third sentence in the activity, "To *reveal* a message during a game, you have to make two triangles fly open." Ensure students' understanding of both applications of *reveal* in the sentence.

Answers

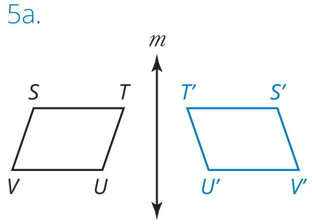
2. $FJ = F'J'$
   $JD = J'D'$
   $DF = D'F'$
   $\overline{FJ} \cong \overline{F'J'}$
   $\overline{JD} \cong \overline{J'D'}$
   $\overline{DF} \cong \overline{D'F'}$

3a. $R_m(\triangle FJD) = \triangle F'J'D'$

3b. Every point of $\triangle F'J'D'$ is a reflection across line m of the original corresponding point.

4. The corresponding points of the pre-image and image in a reflection are equidistant from the line of reflection.

5a.



2. Write equality and congruence statements to compare the corresponding sides of the pre-image and the image.

> A reflection is an isometry.

A **reflection** is a function, R_ℓ, which takes as its input, P, the location of a point with respect to some line of reflection ℓ and outputs $R_\ell(P)$, or the opposite of the location of P with respect to the line of reflection.

3. Consider the reflections from Question 1.

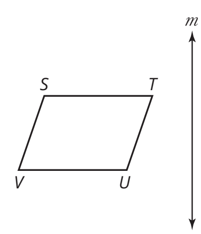
a. Write a function of the form $R_\ell(P)$ to describe the reflection of $\triangle FJD$.

b. Explain how your function represents the reflection of every point of $\triangle FJD$.

4. Describe the relationship between corresponding points of a reflection and the line of reflection.

5. Complete each reflection using the given function.

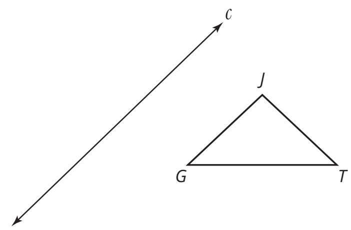
a. $R_m(STUV)$



ELL Tip

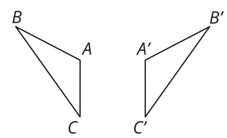
Discuss the phrase *with respect to*. Remind students that although *respect* is a multiple-meaning word that is commonly used to mean *a great admiration of someone or something*, in the context of the phrase, *respect* means *a particular point or detail*. In the paragraph above Question 3 in the activity, read aloud the part of the sentence, "...the location of a point *with respect to* some line of reflection...". Explain the context of the phrase and clarify any misunderstandings students may have about its meaning.

b. $R_c(JTG)$

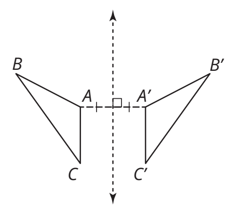


It is possible to construct the line of reflection when given two figures that are reflections of one another.

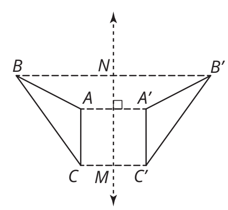
Worked Example



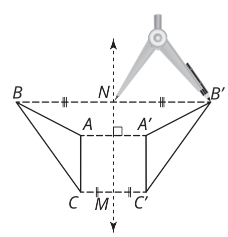
1. Label the vertices.



2. Connect two corresponding points. Construct the perpendicular bisector of the line segment connecting them.



3. Connect remaining corresponding points. Label the intersection of each line segment with the perpendicular bisector.



4. Use a compass to determine whether each intersection point is the midpoint of the line segment connecting corresponding vertices. If that is the case, the perpendicular bisector is the line of reflection. If not, the figures are not reflections of one another.

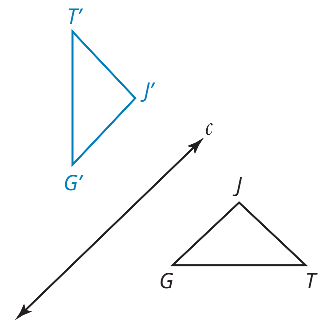
$\overline{CM} \cong \overline{C'M}$

$\overline{BN} \cong \overline{B'N}$

$\overleftrightarrow{MN}$ is the line of reflection.

Answer

5b.



Answers

6a. Check students' lines of reflection. The figures are reflections of one another.

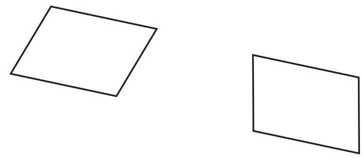
6b. The pentagons are not reflections of one another.

6c. The triangles are not reflections of one another.

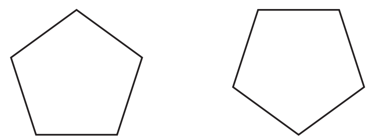
7. Two figures are not reflections of one another if the perpendicular bisector of one pair of corresponding vertices is not also the perpendicular bisector of each pair of corresponding vertices.

6. Determine whether the figures are reflections of one another. If so, identify the line of reflection.

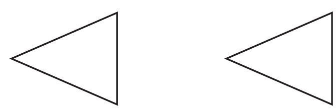
a.



b.



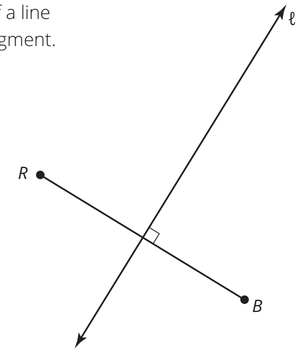
c.



7. How do you know whether two figures are not reflections of one another?

ACTIVITY
3.2 Perpendicular Bisector Theorem

In the Getting Started, you saw that the perpendicular bisector of a line segment is the line of reflection between the endpoints of the segment. Consider $\overline{RB}$ with perpendicular bisector ℓ.



1. Label point *P* anywhere on ℓ. What do you notice about the relationship between point *P* and points *R* and *B*?

A **proof** is a series of statements and corresponding reasons forming a valid argument that starts with a hypothesis and arrives at a conclusion.

You can prove that the endpoints of a line segment are equidistant from any point on the perpendicular bisector.

Worked Example

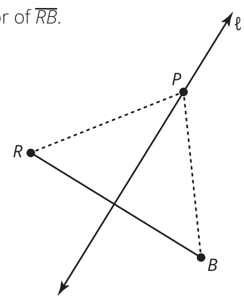
Given: Line ℓ is the perpendicular bisector of $\overline{RB}$.
Point *P* is on ℓ.

Prove: $PR = PB$

The reflection of point *P* across line ℓ is point *P* by the definition of reflection.

Because line ℓ is the perpendicular bisector of $\overline{RB}$, the reflection of point *R* across line ℓ is point *B*.

Therefore, $PR = PB$.



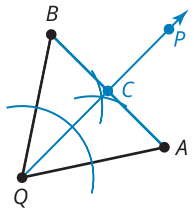
2. Provide a reason why $PR = PB$.

Answer

1. Answers may vary. Sample answer. Point *P* appears to be the same distance from points *R* and *B*.
2. $\overline{PR}$ is a reflection across line ℓ of $\overline{PB}$, and rigid motions preserve size.

Answers

3a.



3b. $\overline{QB'}$ will lie on top of $\overline{QA}$, with point *Q* aligning with itself, and point *B'* lying on top of point *A*. The distances *BC* and *CA* are equal.

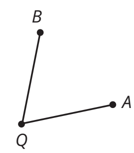
3c. Points *A* and *B* are equidistant from point *C* and also equidistant from point *Q*. That means that points *C* and *Q* must lie on the perpendicular bisector. Because points *Q*, *C*, and *P* lie on the same ray, $\overrightarrow{QP}$ is a perpendicular bisector.

The **Perpendicular Bisector Theorem** states: "Any point on the perpendicular bisector of a segment is equidistant from the endpoints of that segment." Remember that a theorem is a statement that can be demonstrated to be true by accepted mathematical operations and arguments.

Let's consider the relationship between endpoints of a line segment and any point on the perpendicular bisector from another perspective.

Suppose you have two points that are equidistant from a third point. You can show that the third point lies on the perpendicular bisector of the segment connecting the points.

3. Consider point *Q*, which is equidistant from points *A* and *B*.



a. Draw $\overrightarrow{QP}$ so that it bisects $\angle BQA$. This makes $\angle BQP$ and $\angle PQA$ congruent angles. Label the intersection of $\overrightarrow{QP}$ and $\overline{AB}$ as point *C*.

b. Describe the location of $\overline{QB'}$ if $\overline{QB}$ is reflected across $\overrightarrow{QP}$. What does this tell you about the distances *BC* and *CA*? Explain your thinking.

c. Explain how you know that $\overrightarrow{QP}$ is a perpendicular bisector of $\overline{AB}$.

ACTIVITY **3.3**

Sequences of Isometries Using Translations and Reflections

You have learned that reflections and translations are isometries, which means that they preserve distances and angle measures.

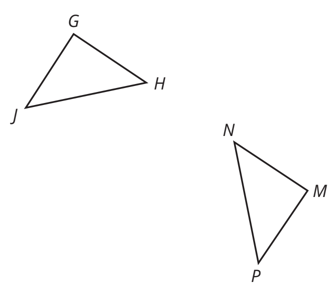
1. Sunita made a conjecture about reflections. She said that you can always map one congruent line segment onto the other using at most two reflections.

Do you think Sunita is correct? Draw examples or counterexamples to justify your reasoning.

Remember:

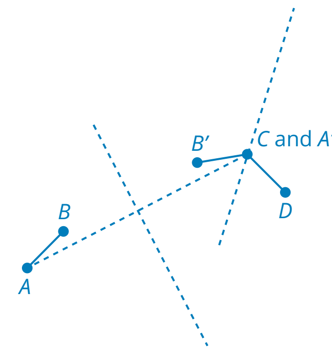
Any point that is equidistant from two other points lies on the perpendicular bisector between the two points.

2. Triangle *GHJ* is a reflection of $\triangle MNP$. Describe the locations of the midpoints of $\overline{GM}$, $\overline{HN}$, and $\overline{JP}$.



Answers

1.



Sunita is correct.

Given $\overline{AB} \cong \overline{CD}$, if $\overline{AB}$ is reflected across a line to map point A to point C, $\overline{A'B'} \cong \overline{CD}$. Then another reflection across a line through C will map $\overline{A'B'}$ to $\overline{CD}$.

2. The midpoints of $\overline{GM}$, $\overline{HN}$, and $\overline{JP}$ all lie on the perpendicular bisector, or the line of reflection for the two triangles.

ELL Tip

Ask students to identify what the word *counter* means in the term *counterexample*. If they are unfamiliar with the meaning, define *counter* as *to go against or follow the opposite direction*. Follow up with additional examples of words that begin with *counter*, including *counterintuitive, counteract*, and *counterpart*. Define these words and then ask students to explain why *counterexample* means *an example that disproves an idea, theory, or proposition*. Provide an example of a statement, such as “All triangles are isosceles.” Discuss why a scalene triangle is a *counterexample* to that statement.

Answers

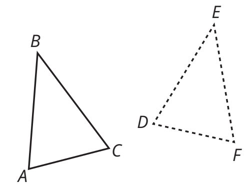
3a. Reflect $\triangle ABC$ across a vertical line and then translate it up.

3b. Reflect $\triangle ADC$ across a horizontal and a vertical line and then translate it.

4. Lina is correct. Sometimes a sequence of transformations is required to map a figure onto itself. In this case, a translation up was also required.

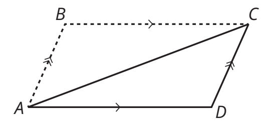
3. Describe and sketch the sequence of translations and reflections that shows that the two figures in each pair are congruent. Images are shown with dashed lines.

a.

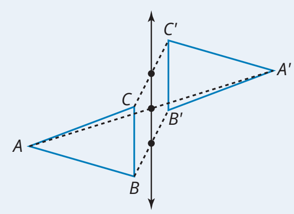


Sides that have the same number of arrowhead markings are parallel to one another.

b.



4. Miguel was investigating a transformation of $\triangle ABC$ to $\triangle A'B'C'$ and discovered that the midpoints of the segments connecting corresponding points were collinear, but the line was not a perpendicular bisector of each segment. He thought that this must not be a reflection.



Lina disagreed. She said that you can translate first to show that the midpoints lie on a perpendicular bisector.
Who is correct? Explain your reasoning.

10 • TOPIC 2: Rigid Motions on a Plane

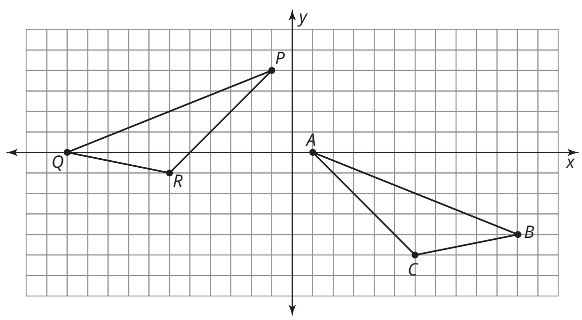
NOTES

TALK the TALK

But We're Off the Grid

In this activity, you translated and reflected figures in the plane. Let's compare those transformations with the same ones performed on a coordinate plane.

1. **Describe the sequence of translations and reflections which maps the pre-image triangle, △*PQR*, to the image triangle, △*ABC* on the coordinate plane.**



2. **How are translating and reflecting geometric figures in the plane different from performing these transformations on the coordinate plane? How are they the same?**

3. **What similarities and differences are there between a geometric reflection function and an algebraic equation that shows the reflection of a function?**

LESSON 3: Staring Back at Me • 11

Answers

1. Triangle *PQR* can be reflected across the *y*-axis and then translated down 4 units.
2. Sample answer. Translating and reflecting in the plane and on a coordinate plane both produce the desired results. When reflecting in a plane and on a coordinate plane, a line of reflection must be identified. When translating on the coordinate plane, the movements can be quantified by units on the grid. When translating in the plane, the movements are identified by directed line segments.
3. An algebraic equation which shows the reflection of a function, such as $g(x) = -f(x)$ or $g(x) = f(-x)$, is similar to a reflection function in that it reflects a set of points across a line. When reflecting such an algebraic function, the line of reflection is horizontal or vertical. When using a geometric reflection function, the line of reflection can be any line in the plane.