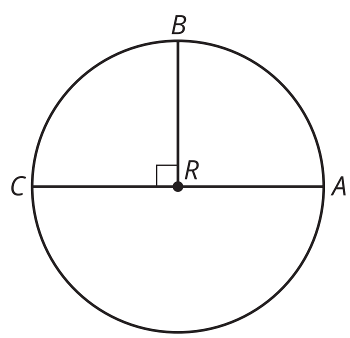
4

Turn Yourself Around

Rotations as Functions

Warm Up

Identify each central angle measure.



1. m∠*ARC*
2. m∠*ARB*
3. m∠*CRB*

Learning Goals

- Represent rotations in the plane.
- Describe rotation transformations as functions that take points as inputs and output rotated points.
- Specify a sequence of translations, reflections, and rotations that will carry a figure onto a congruent figure.

Key Term

- rotation

You have studied circles and rotations. How can you use circles to define and describe rotations on the plane?

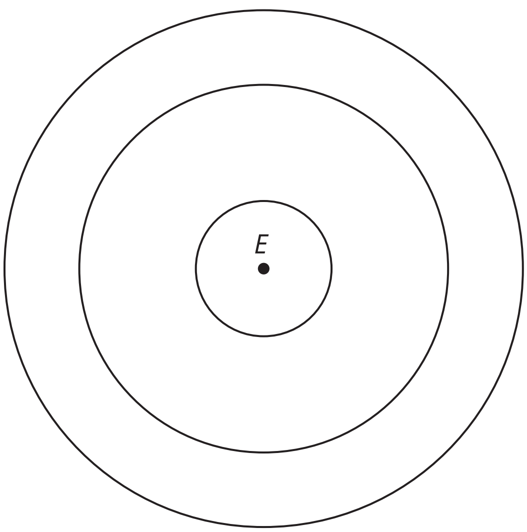
GETTING STARTED

Concentric Circles

Concentric circles are circles with a common center point.

You have constructed circles and identified parts of circles in previous lessons. Circles are important for understanding rotations.

1. Consider three concentric circles with center at point *E*.



a. Draw one point on each circle so that you can connect the points to form $\triangle ABC$. Draw the sides of the triangle.

b. Choose a positive or negative rotation angle, *t*.

c. Draw *A'* as an endpoint of *EA'* such that $\angle AEA'$ has a measure of *t* degrees.

d. Repeat part (c) to draw points *B'* and *C'*. Connect the transformed points to form $\triangle A'B'C'$.

Remember:

An isometry preserves distances and angle measures.

2. What do you notice about the pairs of line segments $\overline{EA}$ and $\overline{EA'}$, $\overline{EB}$ and $\overline{EB'}$, and $\overline{EC}$ and $\overline{EC'}$? Explain your observations.

3. Is the transformation you created an isometry? Explain your thinking.

ACTIVITY 4.1 Rotation Functions

A **rotation** is a function that maps its input, a point *P*, to another location, *f*(*P*). This movement to a new location is defined by a center of rotation, *E*, and a rotation angle, *t*. For this reason, a rotation function is written as $R_{E,t}(P)$.

Because the rotation is defined about a point *E*, the movement of a specific point traces an arc that is part of the circumference of a circle with center *E*. The arc can be labeled by the starting point, *P*, and the endpoint, *P′*, or as, $\overset{\frown}{PP'}$. The degree measure of this arc is equivalent to the degree rotation, *t*, that creates a central angle in Circle *E*.

Remember:

A circle is named by the point that is its center.

1. Consider the rotation you created in the previous activity.

a. Write a function of the form $R_{E,t}(\triangle ABC)$ to describe the rotation of $\triangle ABC$.

b. Explain how your function represents the rotation of every point of $\triangle ABC$.

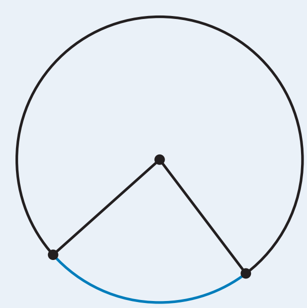
c. Do the arcs you created all have the same measure? Explain your answer.

2. Write equality and congruence statements to compare the corresponding sides of the pre-image and the image.

Think about:

A circle is a rotation of a point around a given center 360 degrees.

A circle is a locus of points that are all a given distance from a center point.



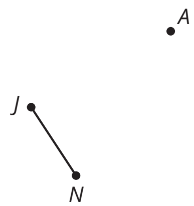
An angle is the measure of the distance the point is rotated as measured by the central angle.

It is possible to apply the rotation function to a figure by using a protractor and ruler.

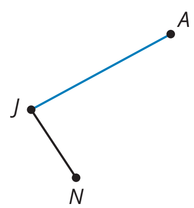
Worked Example

$R_{A,40}(\overline{JN})$

This means to rotate $\overline{JN}$ 40°, using point A as the center of rotation.

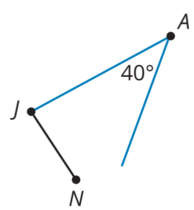


1. Draw a line segment from the center of rotation, A, to one endpoint of the line segment.

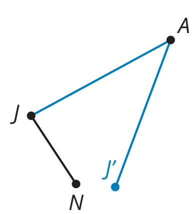


2. Using a protractor, draw a 40° angle. Use the center of rotation, A, as the vertex and the line segment drawn, $\overline{AJ}$, as one side of the angle.

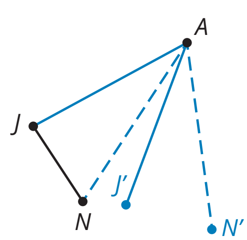
   Since the angle measure is positive, place the angle in the counter-clockwise direction of the line segment drawn.



3. Use a ruler or compass to extend the side of the angle so that it is the same length as $\overline{AJ}$. Label the other endpoint J'.



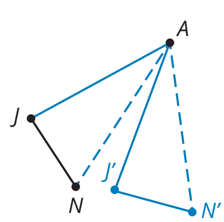
4. Repeat steps 1, 2, and 3 using the other endpoint of the original line segment.



5. Connect endpoints J' and N'.

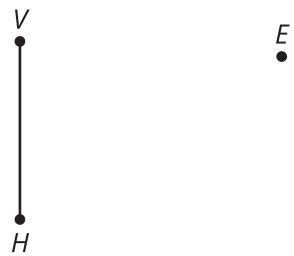
   $R_{A,40}(\overline{JN}) = \overline{J'N'}$

   Segment $\overline{J'N'}$ is the result of a 40° rotation of $\overline{JN}$ about point A.

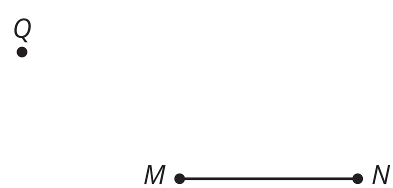


3. Complete each rotation using the given function.

a. $R_{E,\,75}(\overline{VH})$



b. $R_{Q,\,-30}(\overline{MN})$



Remember:

When you construct, you use only a compass and straightedge. Here you are drawing, so you can use those tools as well as measuring tools, such as a protractor and ruler.

ACTIVITY

4.2

Determining the Center of Rotation

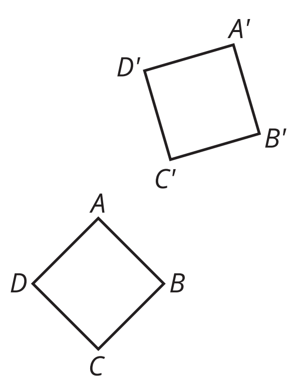
You have seen that corresponding points on rotated figures are equidistant from the center of rotation.

1. Draw an example to explain why Tori is correct.

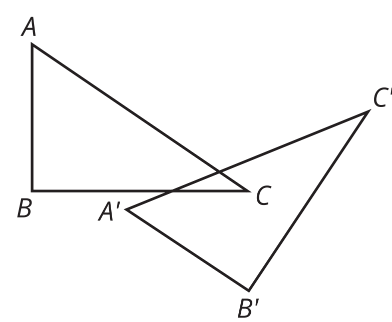
> Tori
>
> If two points Q and Q′ are equidistant from the center, then the perpendicular bisector of $\overline{QQ'}$ passes through the center.

2. Use what you know to determine the center of rotation and rotation angle for the transformation of each figure. Write each rotation as a function.

a.



b.



ACTIVITY

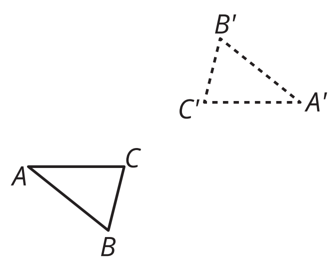
4.3

Sequences of Isometries

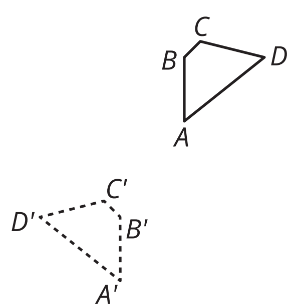
You have learned that translations, reflections, and rotations are isometries, which means that they preserve distances and angle measures.

1. Describe and sketch the sequence of isometries that shows that the two figures in each pair are congruent. Images are shown with dashed lines.

a.



b.



NOTES

TALK the TALK

That's What It's All About!

1. **Complete the graphic organizers to summarize the characteristics of each.**

   a. **Translation**
   b. **Reflection**
   c. **Rotation**

Graphic Organizer

Geometric Object

Function Notation

Translation

Example

Description

Graphic Organizer

Geometric Object

Function Notation

Reflection

Example

Description

Graphic Organizer

Geometric Object

Function Notation

Rotation

Example

Description