5

Slide, Flip, Turn: The Latest Dance Craze?

Translations, Rotations, and Reflections on the Coordinate Plane

MATERIALS

scissors
patty paper

Lesson Overview

Students recall what they know about transformations of functions by examining the graph of the basic function, $f(x)$, and its transformed graph $g(x)$. Students then cut out a model of a trapezoid and translate, rotate, and reflect the model on a coordinate plane to determine how transformations affect the coordinates of the figure. Compositions of transformations are explored on the coordinate plane.

Geometry

Coordinate and Transformational Geometry

(3) The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(A) describe and perform transformations of figures in a plane using coordinate notation.
(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.
(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

ELPS

1.A, 1.C, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.D, 4.F, 4.J, 4.K, 5.B, 5.E, 5.F, 5.G

Essential Ideas

- When a horizontal translation occurs on a coordinate plane the x-coordinates of the pre-image change, but the y-coordinates remain the same.
- When a vertical translation occurs on a coordinate plane the y-coordinates of the pre-image change, but the x-coordinates remain the same.
- When a point or image on a coordinate plane is rotated 90° counterclockwise about the origin, its original coordinates (x, y) change to $(-y, x)$.

- When a point or image on a coordinate plane is rotated 180° counterclockwise about the origin, the original coordinates (x, y) change to $(-x, -y)$.
- When a point or image on a coordinate plane is rotated 270° counterclockwise about the origin, the original coordinates (x, y) change to $(y, -x)$.
- When a point or image on a coordinate plane is rotated 360° counterclockwise about the origin, the original coordinates (x, y) do not change.
- When a point or image on a coordinate plane is reflected over the x-axis, the original coordinates (x, y) change to $(x, -y)$.
- When a point or image on a coordinate plane is reflected over the y-axis, the original coordinates (x, y) change to $(-x, y)$.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: Basic Steps

Students recall what they know about transformations of functions by examining the graph of the basic function, $f(x)$ and its transformed graph $g(x)$.

Develop

Activity 5.1: Translating Geometric Figures on the Coordinate Plane

Students translate a trapezoid on a coordinate plane then explore how the coordinates would change given a transformation without graphing.

Day 2

Activity 5.2: Rotating Geometric Figures on the Coordinate Plane

Students begin by rotating a point around the origin to understand the effect of rotations on points on the coordinate plane. They then rotate the trapezoid and parallelogram about the origin.

Activity 5.3: Reflecting Geometric Figures on the Coordinate Plane

Students reflect points across the x- and y-axis. They make a generalization about the effect a reflection has on points on the coordinate plane. They then reflect the trapezoid and parallelogram across the x- and y-axis.

Day 3

Activity 5.4: Multiple Transformations

Students investigate determining images and pre-images of figures under compositions of transformations on the coordinate plane.

Demonstrate

Talk the Talk: Turn Yourself Around

Students make connections with what they have previously learned about slopes and rotations.

ENGAGE

Getting Started: Basic Steps

Facilitation Notes

In this activity, students recall what they learned in previous courses about function transformations. They examine the graph of the basic function $f(x)$ and describe a sequence of transformations.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- Students making a connection between their descriptions, function, and table. Even if students use a different sequence of transformations, they should all have the same corresponding coordinates in the table.
- Students are using the correct grouping symbols and signs when writing their function equation.

Misconception

- Students may assume there is only one sequence of transformations that will map $f(x)$ to $g(x)$.

Differentiation strategy

To scaffold support with writing a sequence of transformations,

- Direct students to use string, a pipe cleaner, or their pencil to model the sequence of transformations.

Questions to ask

- How would a different sequence of transformations change the function equation?
- If you simplified your function equation, would you be able to write a different sequence of transformations?

Summary

A sequence of function transformations maps a set of points onto another set of points.

Activity 5.1
Translating Geometric Figures on the Coordinate Plane

DEVELOP

Facilitation Notes

In this activity, students translate a trapezoid on a coordinate plane, then explore how the coordinates change given a transformation.

Have students cut out or trace the trapezoid onto patty paper.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- How did you determine the coordinates of the vertices of the trapezoid when it was translated left 15 units?
- How did you determine the coordinates of the vertices of the trapezoid when it was translated down 12 units?
- Could you have determined the coordinates of the vertices of the image without graphing? How?

Have students work with a partner or in a group to complete Question 3. Share responses as a class.

Questions to ask

- Is translating a parallelogram different than translating a trapezoid? How?
- How do you know which coordinate changes during the transformation?
- How do you know which coordinate does not change during the transformation?

Summary

Translation is described as a rigid motion that slides each point of a figure the same distance and direction.

Activity 5.2

Rotating Geometric Figures on the Coordinate Plane

Facilitation Notes

In this activity, students begin by rotating a point around the origin to understand the effect of rotations on points on the coordinate plane. They then rotate the trapezoid and parallelogram about the origin.

Ask a student to read the introduction and Worked Example aloud. Discuss as a class.

As students work, look for

Confusion about the counterclockwise direction.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- In which quadrant of the coordinate plane is point *A* located?
- If a point is rotated about the origin 90° counterclockwise, in which quadrant does the image appear?
- If a point is rotated about the origin 180° counterclockwise, in which quadrant does the image appear?
- How many degrees of rotation about the origin are necessary for an image in the first quadrant to appear in the fourth quadrant?
- What would be the coordinates if the point was rotated 270° counterclockwise about the origin?

Have students work with a partner or in a group to complete Question 5. Share responses as a class.

Questions to ask

- How would you describe the pattern represented in the table?
- Would this table change if a clockwise rotation occurred?
- How can you determine the coordinates of point (2, −3) after a 90 degree counterclockwise rotation?
- How can you determine the coordinates of point (2, −3) after a 90 degree clockwise rotation?
- Compare rotating the point (2, −3) 180 degrees counterclockwise to rotating (2, −3) 180 degrees clockwise.
- In general, how can you use the table in Question 5 to determine the coordinates after a clockwise rotation?

Have students work with a partner or in a group to complete Questions 6 and 7. Share responses as a class.

Questions to ask

- How did you determine the location of Q'? Q''? Q'''? Q''''?
- Do you think the information in the table will be true for any ordered pair?

Ask a student to read the narrative aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 8 through 10. Share responses as a class.

Questions to ask

- How is rotating a figure similar to rotating a point?
- How would the coordinates of trapezoid *ABCD* change if it was rotated 270 degrees counterclockwise?
- How would the coordinates of trapezoid *ABCD* change if it was rotated 360 degrees counterclockwise?

Have students work with a partner to complete Questions 11 and 12. Share responses as a class.

Questions to ask

- What is the difference between a clockwise rotation and a counterclockwise rotation?
- Is rotating the parallelogram different than rotating a trapezoid? If so, how?
- Is it possible to rotate a figure about a point other than the origin? How would that be different?
- Describe a clockwise rotation that produces the same result as a 90 degree counterclockwise rotation.
- Describe a clockwise rotation that produces the same result as a 180 degree counterclockwise rotation.
- Describe a clockwise rotation that produces the same result as a 270 degree counterclockwise rotation.
- Describe a clockwise rotation that produces the same result as a 360 degree counterclockwise rotation.

Summary

Rotation is described as a rigid motion that turns a figure about a fixed point for a given angle.

Activity 5.3

Reflecting Geometric Figures on the Coordinate Plane

Facilitation Notes

In this activity, students reflect points across the x- and y-axis. They make a generalization about the effect a reflection has on points on the coordinate plane. They then reflect the trapezoid and parallelogram across the x- and y-axis.

Ask a student to read the introduction and Worked Example aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- What are the coordinates of the point (2, 1) after reflecting over the x-axis?
- In general, how do the coordinates of a point change when reflected over the x-axis?
- What are the coordinates of the point (2, 1) after reflecting over the y-axis?
- In general, how do the coordinates of a point change when reflected over the y-axis?

Have students work with a partner or in group to complete Questions 5 through 8. Share responses as a class.

Questions to ask

- How would you determine the coordinates of the point (5, 7) after reflecting over a horizontal line other than the x-axis?
- How would you determine the coordinates of the point (5, 7) after reflecting over a vertical line other than the y-axis?

Have students work with a partner or in a group to complete Questions 9 through 11. Share responses as a class.

Questions to ask

- What are the coordinates of trapezoid *ABCD* after reflecting over the x-axis?
- What are the coordinates of trapezoid *ABCD* after reflecting over the y-axis?

Have students work with a partner or in group to complete Question 12. Share responses as a class.

Questions to ask

- How does the x-coordinate change when reflecting a point over the x-axis?
- How does the x-coordinate change when reflecting a point over the y-axis?
- How does the y-coordinate change when reflecting a point over the x-axis?
- How does the y-coordinate change when reflecting a point over the y-axis?

Summary

Reflection is described as a rigid motion that flips a figure over a reflection line.

Activity 5.4

Multiple Transformations

Facilitation Notes

In this activity, students investigate determining images and pre-images of figures under compositions of transformations on the coordinate plane.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- If point J is reflected across the diagonal line, at what location is its image?
- Can you always perform the reverse steps to determine a pre-image given an image and a sequence of transformations?

Have students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

Questions to ask

- What steps can you take to show a rotation of a point 90 degrees clockwise?
- Which transformation(s) preserve size, shape, and orientation?
- Which transformation(s) do not preserve size, shape, and orientation?

Summary

A composition of transformations can carry a pre-image onto an image.

DEMONSTRATE

Talk the Talk: Turn Yourself Around

Facilitation Notes

In this activity, students make connections with what they have previously learned about slopes and rotations.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What kind of slope is a negative reciprocal?
- Is there a rotation that will create a parallel slope?
- What is another word for rigid motion transformations?

Summary

The slope of a line that has been rotated 90 degrees is the negative reciprocal of the ratio for the original line. The image and pre-image have a perpendicular relationship.

5

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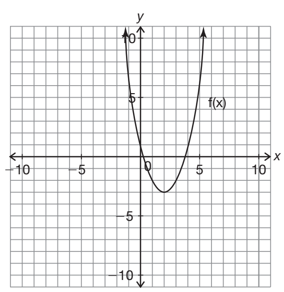
Translations, Rotations, and Reflections on the Coordinate Plane

Warm Up

Consider the graph of $f(x)$.



Complete the table.

$f(x)$	$-f(x)$	$f(-x)$
(−1, 6)		
(0, 1)		
(1, −2)		

Learning Goals

- Translate geometric figures on a coordinate plane.
- Rotate geometric figures on a coordinate plane.
- Reflect geometric figures on a coordinate plane.

You have learned that translations, reflections, and rotations behave like functions. What correlation can be drawn between transforming the points that make up a figure and the points that make up the graph of a function?

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Warm Up Answers

1.

$f(x)$	$-f(x)$	$f(-x)$
(−1, 6)	(−1, −6)	(1, 6)
(0, 1)	(0, −1)	(0, 1)
(1, −2)	(1, 2)	(−1, −2)

Answers

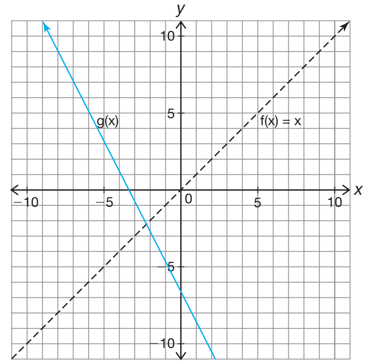
1. Reflection across the y-axis, vertical stretch by a factor of 2, and a horizontal translation to the left 3 units
2. $g(x) = 2(f(-x - 3))$
3.

$f(x)$	$g(x)$
(−1, −1)	(−2, −2)
(0, 0)	(−3, 0)
(1, 1)	(−4, 2)
(2, 2)	(−5, 4)

GETTING STARTED

Basic Steps

Consider the basic function $f(x) = x$ and the transformed graph $g(x)$.



1. Describe a sequence of transformations from the graph of $f(x)$ to the graph of $g(x)$.

2. Write the function equation for $g(x)$ in terms of $f(x)$.

3. Complete the table with the corresponding coordinates.

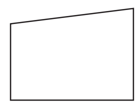
$f(x)$	$g(x)$
(−1, −1)	
(0, 0)	
(1, 1)	
(2, 2)	

2 • TOPIC 2: Rigid Motions on a Plane

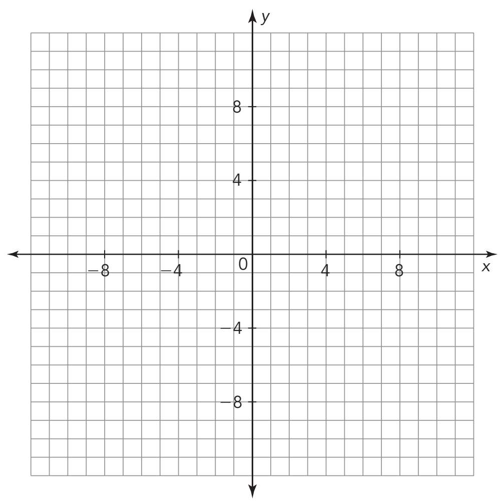
ACTIVITY
5.1

Translating Geometric Figures on the Coordinate Plane

Cut out a copy of the figure shown.

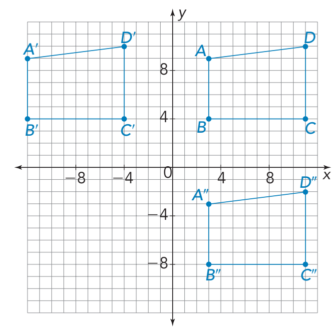


1. Graph trapezoid *ABCD* by plotting the points *A* (3, 9), *B* (3, 4), *C* (11, 4), and *D* (11, 10).



Answers

1.



ELL Tip

Before students perform transformations on figures, practice learning the terminology with points. Go over the different transformations: *translations*, *reflections*, and *rotations*. Then plot points on coordinate planes and ask students to perform different transformations. Be sure to have students verbalize the transformation as they demonstrate.

Answers

2. See table below.

   See coordinate plane in Question 1.

3a. The vertex coordinates of image *D′E′F′G′* are *D′* (−9, −7), *E′* (−12, −12), *F′* (−3, −12), and *G′* (0, −7).

2. Translate trapezoid *ABCD* on the coordinate plane. Graph the image and record the vertex coordinates in the table.

a. Translate trapezoid *ABCD* 15 units to the left to form trapezoid *A′B′C′D′*.

b. Translate trapezoid *ABCD* 12 units down to form trapezoid *A″B″C″D″*.

Coordinates of Trapezoid *ABCD*	Coordinates of Trapezoid *A′B′C′D′*	Coordinates of Trapezoid *A″B″C″D″*
A (3, 9)		
B (3, 4)		
C (11, 4)		
D (11, 10)		
Point on Trapezoid *ABCD* (*x*, *y*)		

Let's consider translations without graphing.

3. The vertices of parallelogram *DEFG* are *D* (−9, 7), *E* (−12, 2), *F* (−3, 2), and *G* (0, 7).

a. Determine the vertex coordinates of image *D′E′F′G′* if parallelogram *DEFG* is translated 14 units down.

2.

Coordinates of Trapezoid *ABCD*	Coordinates of Trapezoid *A′B′C′D′*	Coordinates of Trapezoid *A″B″C″D″*
***A* (3, 9)**	*A′* (−12, 9)	*A″* (3, −3)
***B* (3, 4)**	*B′* (−12, 4)	*B″* (3, −8)
***C* (11, 4)**	*C′* (−4, 4)	*C″* (11, −8)
***D* (11, 10)**	*D′* (−4, 10)	*D″* (11, −2)
Point on Trapezoid *ABCD* (*x*, *y*)	(*x* − 15, *y*)	(*x*, *y* − 12)

b. **How did you determine the image coordinates without graphing?**

c. **Determine the vertex coordinates of image *D″E″F″G″* if parallelogram *DEFG* is translated 8 units to the right.**

d. **How did you determine the image coordinates without graphing?**

Answers

3b. I determined the image coordinates by adding −14 to each of the *y*-coordinates. The *x*-coordinates stayed the same.

3c. The vertex coordinates of image *D″E″F″G″* are *D″* (−1, 7), *E″* (−4, 2), *F″* (5, 2), and *G″* (8, 7).

3d. I determined the image coordinates by adding 8 to each of the *x*-coordinates. The *y*-coordinates stayed the same.

ACTIVITY
5.2

Rotating Geometric Figures on the Coordinate Plane

Recall that a rotation is a rigid motion that turns a figure about a fixed point, called the point of rotation. The figure is rotated in a given direction for a given angle, called the angle of rotation. The angle of rotation is the measure of the amount the figure is rotated about the point of rotation. The direction of a rotation can either be clockwise or counterclockwise.

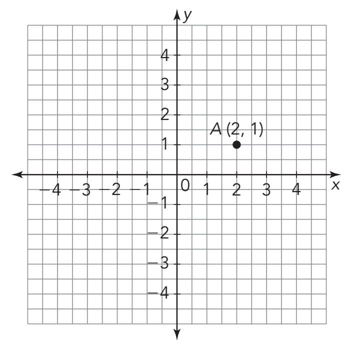
Let's rotate point *A* about the origin. The origin will be the point of rotation, and you will rotate point *A* 90°, 180°, and 270°.

First, let's rotate point *A* 90° counterclockwise about the origin.

Worked Example

Step 1: Plot a point anywhere in the first quadrant, but *not* at the origin.

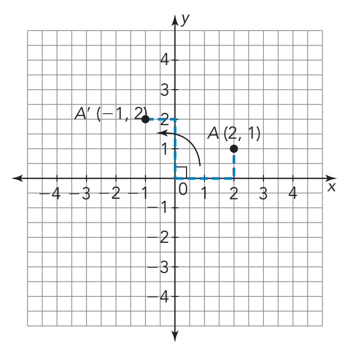
Point *A* is plotted at (2, 1).



Step 2: Next, draw a "hook" from the origin to point *A*, using the coordinates and horizontal and vertical line segments as shown.

Step 3: Rotate the "hook" 90° counterclockwise as shown.

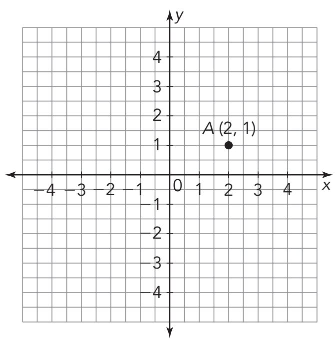
Point *A*′ is located at (–1, 2). Point *A* has been rotated 90° counterclockwise about the origin.



1. What do you notice about the coordinates of point *A* and the coordinates of point *A′*?

2. Predict what the coordinates of point *A″* will be if you rotate point *A′* 90° counterclockwise about the origin.

3. Graph and label point *A′*. Rotate point *A′* about the origin 90° counterclockwise on the coordinate plane shown. Label the point *A″*.



a. What are the coordinates of point *A″*? Was your prediction for the coordinates of point *A″* correct?

b. What do you notice about the coordinates of points *A* and *A″*? How are the two points related?

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Answers

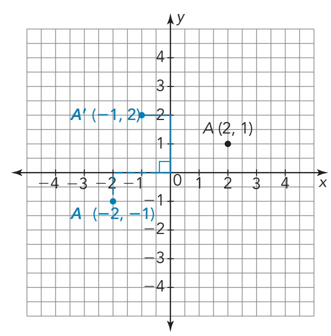
1 The *x*-coordinate of point *A* is 2, and the *y*-coordinate of point *A′* is 2.

The *y*-coordinate of point *A* is 1, and the *x*-coordinate of point *A′* is −1.

So, the *x*-coordinate of point *A* is the *y*-coordinate of point *A′*, and the opposite of the *y*-coordinate of point *A* is the *x*-coordinate of point *A′*.

2. If I follow the pattern, I believe point *A″* will have the coordinates (−2, −1).

3.



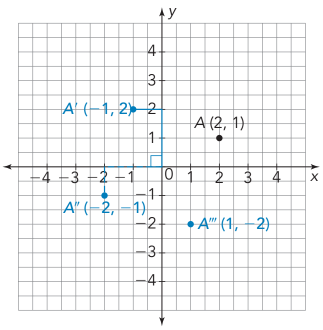
3a. The coordinates of point *A″* are (−2, −1).

My prediction was correct.

3b. The *x*-coordinate of point *A″* is the opposite of the *x*-coordinate of point *A*.

The *y*-coordinate of point *A″* is the opposite of the *y*-coordinate of point *A*.

4.



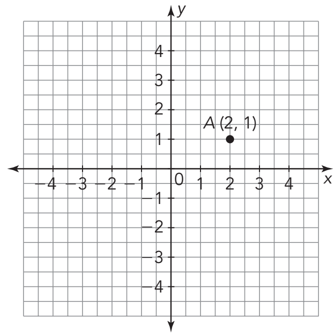
4a. The coordinates of point A''' are (1, −2).

4b. The *x*-coordinate of point A''' is the *y*-coordinate of point *A*.

The *y*-coordinate of point A''' is the opposite of the *x*-coordinate of point *A*.

5. See table below

4. Graph and label points *A′* and *A″*. Rotate point *A″* about the origin 90° counterclockwise on the coordinate plane shown. Label the point *A‴*.



a. What are the coordinates of point *A‴*?

b. What do you notice about the coordinates of point *A* and point *A‴*? How are the two points related?

You may have noticed that the values of the *x*- and *y*-coordinates seem to switch places for every 90° rotation about the origin. You may have also noticed that the rotation from point *A* to *A″* is a 180° counterclockwise rotation about the origin, and that the rotation from point *A* to *A‴* is a 270° counterclockwise rotation about the origin.

5. Determine the coordinates of point (*x*, *y*) after rotations of 90°, 180°, 270°, and 360°.

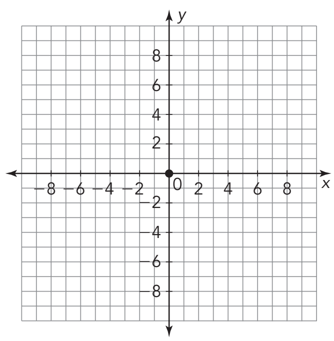
Original Point	Coordinates After a 90° Counterclockwise Rotation About the Origin	Coordinates After a 180° Counterclockwise Rotation About the Origin	Coordinates After a 270° Counterclockwise Rotation About the Origin	Coordinates After a 360° Counterclockwise Rotation About the Origin
(*x*, *y*)				

5.

Original Point	Coordinates After a 90° Counterclockwise Rotation About the Origin	Coordinates After a 180° Counterclockwise Rotation About the Origin	Coordinates After a 270° Counterclockwise Rotation About the Origin	Coordinates After a 360° Counterclockwise Rotation About the Origin
(*x*, *y*)	(−*y*, *x*)	(−*x*, −*y*)	(*y*, −*x*)	(*x*, *y*)

You can verify that the information in the table is correct by using a test point. Plot test point Q on the coordinate plane and rotate the point 90°, 180°, 270°, and 360° counterclockwise about the origin.

6. Graph and label point Q at (5, 7) on the coordinate plane.



Remember:

Remember that the table shows values for the coordinates, but coordinates for a plotted point are always in the form (x, y)!

7. Use the origin (0, 0) as the point of rotation.

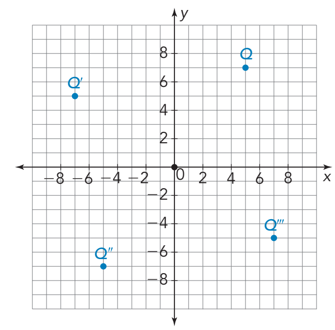
a. Rotate the pre-image Q 90° counterclockwise about the origin. Label the image Q'. Determine the coordinates of image Q', then describe how you determined the location of point Q'.

b. Rotate the pre-image Q 180° counterclockwise about the origin. Label the image Q''. Determine the coordinates of image Q'', then describe how you determined the location of image Q''.

c. Rotate the pre-image Q 270° counterclockwise about the origin. Label the image Q'''. Determine the coordinates of point Q''', then describe how you determined the location of image Q'''.

Answers

6.



7a. See coordinate plane in Question 6.

Answers will vary.

I know that after a 90° counterclockwise rotation about the origin, the x-coordinate of the pre-image is the y-coordinate of the image. I also know that the opposite of the y-coordinate of the pre-image is the x-coordinate of the image.

The location of image Q' is (−7, 5).

7b. See coordinate plane in Question 6.

Answers will vary.

I determined the location of image Q'' by using the information in the table. I know that after a 180° counterclockwise rotation about the origin, the coordinates of the image are the opposite of the coordinates of the pre-image.

The location of point Q'' is (−5, −7).

7c. See coordinate plane in Question 6.

Answers will vary.

I know that after a 270° counterclockwise rotation about the origin, the x-coordinate of the image is the y-coordinate of the pre-image. I also know that the y-coordinate of the image is the opposite of the x-coordinate of the pre-image.

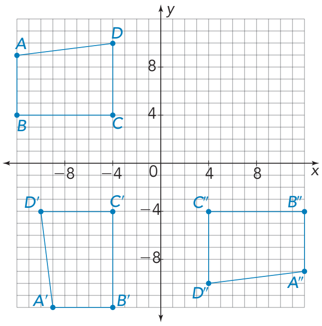
The location of Q''' is (7, −5).

7d. Answers will vary.

I know that after a 360° counterclockwise rotation about the origin, the image and the pre-image have the same coordinates.

The location of Q'''' is (5, 7).

8. See coordinate plane.



d. Rotate the pre-image *Q* 360° counterclockwise about the origin. Label the image Q''''. Determine the coordinates of point Q'''', then describe how you determined the location of image Q''''.

Remember:

Make sure you have the trapezoid that you cut out earlier.

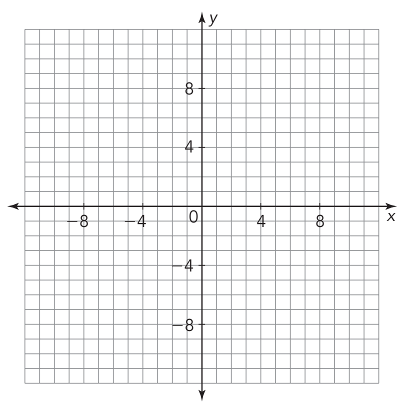
You have been rotating points about the origin on a coordinate plane. However, do you think polygons can also be rotated on the coordinate plane?

You can use models to help show that you *can* rotate polygons on a coordinate plane. However, before we start modeling the rotation of a polygon on a coordinate plane, let's graph the trapezoid to establish the pre-image.

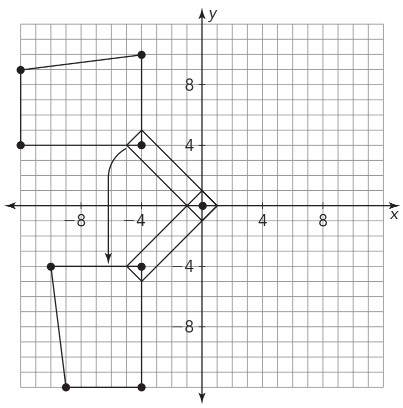
8. Graph trapezoid *ABCD* by plotting the points *A* (−12, 9), *B* (−12, 4), *C* (−4, 4), and *D* (−4, 10).

Now that you have graphed the pre-image, you are ready to model the rotation of the polygon on the coordinate plane.

- **First, fold a piece of tape in half and tape it to both sides of the trapezoid you cut out previously.**
- **Then, take your trapezoid and set it on top of trapezoid *ABCD* on the coordinate plane, making sure that the tape covers the origin (0, 0).**
- **Finally, put a pin or your pencil point through the tape at the origin and rotate your model counterclockwise.**



The 90° counterclockwise rotation of trapezoid *ABCD* about the origin is shown.



9. Rotate trapezoid *ABCD* about the origin for each given angle of rotation. Graph and label each image on the coordinate plane and record the coordinates in the table.

a. Rotate trapezoid *ABCD* 90° counterclockwise about the origin to form trapezoid *A′B′C′D′*.

b. Rotate trapezoid *ABCD* 180° counterclockwise about the origin to form trapezoid *A″B″C″D″*.

Coordinates of Trapezoid *ABCD*	Coordinates of Trapezoid *A′B′C′D′*	Coordinates of Trapezoid *A″B″C″D″*
A (−12, 9)		
B (−12, 4)		
C (−4, 4)		
D (−4, 10)		
Point on Trapezoid *ABCD*		

Answers

9a. See coordinate plane in Question 8. and table for answers.

9b. See coordinate plane in Question 8. and table for answers.

9b.

Coordinates of Trapezoid *ABCD*	Coordinates of Trapezoid *A′B′C′D′*	Coordinates of Trapezoid *A″B″C″D″*
A **(−12, 9)**	*A′* (−9, −12)	*A″* (12, −9)
B **(−12, 4)**	*B′* (−4, −12)	*B″* (12, −4)
C **(−4, 4)**	*C′* (−4, −4)	*C″* (4, −4)
D **(−4, 10)**	*D′* (−10, −4)	*D″* (4, −10)
Point on Trapezoid *ABCD*	(−*y*, *x*)	(−*x*, −*y*)

Answers

10. Because a polygon is made of multiple points, the same methods and algebraic rule can be used when rotating the point(s) of the polygon about the origin.

11a. The vertex coordinates of image *D′E′F′G′* are *D′* (−7, −9), *E′* (−2, −12), *F′* (−2, −3), and *G′* (−7, 0).

11b. I determined the image coordinates by following the pattern for rotating points 90° counterclockwise about the origin on a coordinate plane. Each *x*-coordinate of the pre-image became the *y*-coordinate of the image. The opposite of each *y*-coordinate of the pre-image became the *x*-coordinate of the image.

11c. The vertex coordinates of image *D″E″F″G″* are *D″* (9, −7), *E″* (12, −2), *F″* (3, −2), and *G″* (0, −7).

11d. I determined the image coordinates by following the pattern for rotating points 180° about the origin on a coordinate plane. The image coordinates are the opposite values of the *x*- and *y*-coordinates of the pre-image.

10. What similarities do you notice between rotating a single point about the origin and rotating a polygon about the origin?

Let's consider rotations without graphing.

11. The vertices of parallelogram *DEFG* are *D* (−9, 7), *E* (−12, 2), *F* (−3, 2), and *G* (0, 7).

a. Determine the vertex coordinates of image *D′E′F′G′* if parallelogram *DEFG* is rotated 90° counterclockwise about the origin.

b. How did you determine the image coordinates without graphing?

c. Determine the vertex coordinates of image *D″E″F″G″* if parallelogram *DEFG* is rotated 180° counterclockwise about the origin.

d. How did you determine the image coordinates without graphing?

e. **Determine the vertex coordinates of image *D‴E‴F‴G‴* if parallelogram *DEFG* is rotated 270° counterclockwise about the origin.**

f. **How did you determine the image coordinates without graphing?**

12. **Dante claims that if he is trying to determine the coordinates of an image that is rotated 180° about the origin, it does not matter which direction the rotation occurred. Desmond claims that the direction is important to know when determining the image coordinates. Who is correct? Explain why the correct student's rationale is correct.**

ACTIVITY 5.3 Reflecting Geometric Figures on the Coordinate Plane

Recall that figures that are mirror images of each other are called reflections. A reflection is a rigid motion that reflects, or "flips," a figure over a given line called a line of reflection. A line of reflection is a line over which a figure is reflected so that corresponding points are the same distance from the line.

Let's reflect point *A* over the *y*-axis.

Answers

11e. The vertex coordinates of image *D‴E‴F‴G‴* are *D‴* (7, 9), *E‴* (2, 12), *F‴* (2, 3), and *G‴* (7, 0).

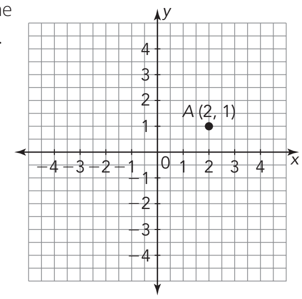
11f. I determined the image coordinates by following the pattern for rotating points 270° counterclockwise about the origin on a coordinate plane. Each *y*-coordinate of the pre-image became the *x*-coordinate of the image. The opposite of each *x*-coordinate of the pre-image became the *y*-coordinate of the image.

12. Dante is correct. Because the rotation is 180°, the rotation will always end up at the same location whether it is rotated clockwise or counterclockwise. The direction is not important for a 180° rotation.

Worked Example

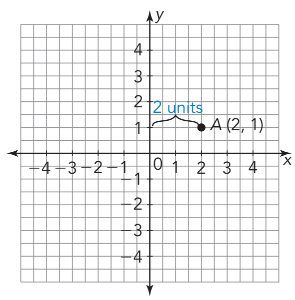
Step 1: Plot a point anywhere in the first quadrant, but *not* at the origin.

Point A is plotted at (2, 1)



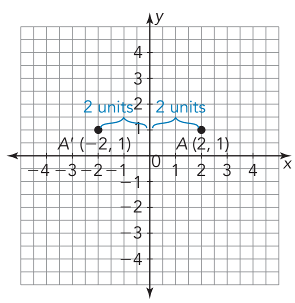
Step 2: Next, count the number of x-units from point A to the y-axis.

Point A is 2 units from the y-axis.



Step 3: Then, count the same number of x-units on the opposite side of the y-axis to locate the reflection of point A. Label the point A'.

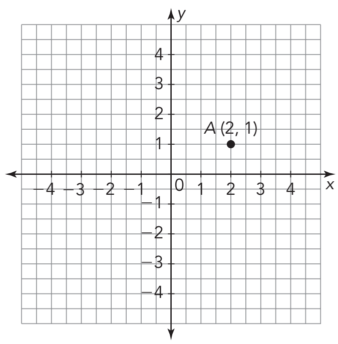
Step 4: Point A' is located at (−2, 1). Point A has been reflected over the y-axis.



1. **What do you notice about the coordinates of point *A* and the coordinates of image *A*′?**

2. **Predict the coordinates of *A*″ if point *A* is reflected over the *x*-axis. Explain your reasoning.**

3. **Reflect point *A* over the *x*-axis on the coordinate plane shown. Verify whether your prediction for the location of the image was correct. Graph the image and label it *A*″.**



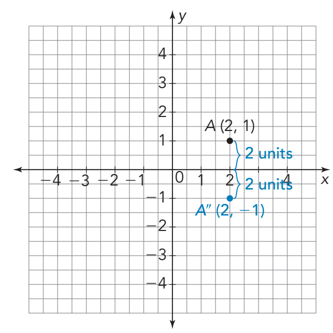
4. **What do you notice about the coordinates of *A* and *A*″?**

Answers

1. The *x*-coordinates are opposites while the *y*-coordinates remained the same.

2. The coordinates of A″ will be (2, −1).

   To reflect point *A* over the *x*-axis, I would count the number of *y*-units from point *A* to the *x*-axis. Then, I would count the same number of *y*-units on the opposite side of the *x*-axis to determine the location of the reflected point.

3. 

   The location of *A*″ after the reflection is (2, −1).

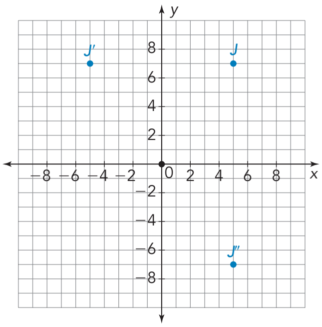
   Yes. My prediction was correct.

4. The *x*-coordinates are the same, but the *y*-coordinates are opposite.

Answers

5. See table below

6. See coordinate plane.



7. See coordinate plane in Question 6.

   The coordinates of *J*′ are (−5, 7). Because I am reflecting *J* over the *y*-axis, I determined the location of point *J*′ using the information in the table. I knew that by reflecting a point over the *y*-axis, the *x*-coordinate of the image would be the opposite of the pre-image and the *y*-coordinate would stay the same.

The coordinates of a pre-image reflected over either the *x*-axis or the *y*-axis can be used to determine the coordinates of the image.

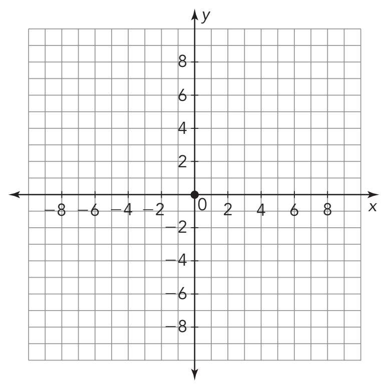
Remember:

Does this table still make sense if the line of reflection is not the *x*- or *y*-axis?

5. Determine the coordinates of point (*x*, *y*) after reflections about the *x*-axis or *y*-axis.

Original Point	Coordinates of Image After a Reflection Over the *x*-axis	Coordinates of Image After a Reflection Over the *y*-axis
(*x*, *y*)		

6. Graph point *J* at (5, 7) on the coordinate plane shown.



7. Reflect point *J* over the *y*-axis on the coordinate plane. Label the image *J*′. Determine the coordinates of *J*′. Then, describe how you determined the location of image *J*′.

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5.

Original Point	Coordinates of Image After a Reflection Over the *x*-axis	Coordinates of Image After a Reflection Over the *y*-axis
(*x*, *y*)	(*x*, −*y*)	(−*x*, *y*)

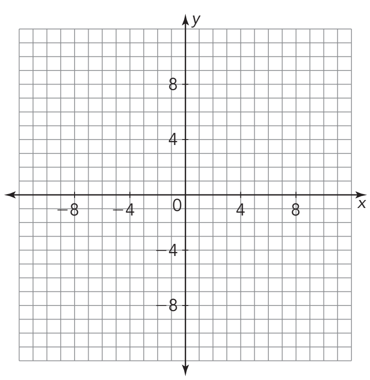
8. Reflect point *J* over the *x*-axis on the coordinate plane. Label the image *J*″. Determine the coordinates of *J*″. Then, describe how you determined the location of image *J*″.

You can also reflect polygons on the coordinate plane. You can model the reflection of a polygon across a line of reflection. Just as with rotating a polygon on a coordinate plane, you will first need to establish a pre-image.

9. Graph trapezoid *ABCD* by plotting the points *A* (3, 9), *B* (3, 4), *C* (11, 4), and *D* (11, 10).

Now that you have graphed the pre-image, you are ready to model the reflection of the polygon on the coordinate plane. For this modeling, you will reflect the polygon over the *y*-axis.

- **First, take your trapezoid that you cut out previously and set it on top of trapezoid *ABCD* on the coordinate plane.**
- **Next, determine the number of units point *A* is from the *y*-axis.**
- **Then, count the same number of units on the opposite side of the *y*-axis to determine where to place the image in Quadrant II.**
- **Finally, physically flip the trapezoid over the *y*-axis like you are flipping a page in a book.**

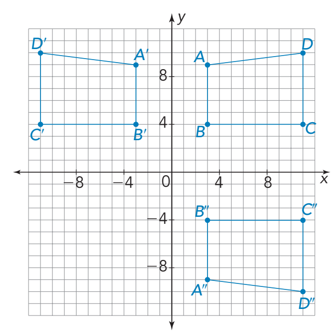


Answers

8. See coordinate plane in Question 6.

   The coordinates of *J*″ are (5, −7). Because I am reflecting *J* over the *x*-axis, I determined the location of point *J*″ using the information in the table. I knew that by reflecting a point over the *x*-axis, the *x*-coordinate of the image would be the same as the pre-image and the *y*-coordinate would be the opposite.

9. See coordinate plane.

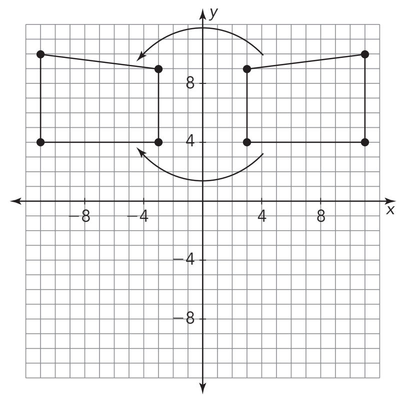


Answers

10. See table below.

See Coordinate plane in Question 9.

The reflection of trapezoid *ABCD* over the *y*-axis is shown.



10. Reflect trapezoid *ABCD* over each given line of reflection. Graph and label each image on the coordinate plane and record each image's coordinates in the table.

a. Reflect trapezoid *ABCD* over the *y*-axis to form trapezoid *A′B′C′D′*.

b. Reflect trapezoid *ABCD* over the x-axis to form trapezoid *A″B″C″D″*.

Coordinates of Trapezoid *ABCD*	Coordinates of Trapezoid *A′B′C′D′*	Coordinates of Trapezoid *A″B″C″D″*
A (3, 9)		
B (3, 4)		
C (11, 4)		
D (11, 10)		
Point on Trapezoid *ABCD* (*x*, *y*)		

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10.

Coordinates of Trapezoid *ABCD*	Coordinates of Trapezoid *A′B′C′D′*	Coordinates of Trapezoid *A″B″C″D″*
***A* (3, 9)**	*A′* (−3, 9)	*A″* (3, −9)
***B* (3, 4)**	*B′* (−3, 4)	*B″* (3, −4)
***C* (11, 4)**	*C′* (−11, 4)	*C″* (11, −4)
***D* (11, 10)**	*D′* (−11, 10)	*D″* (11, −10)
Point on Trapezoid *ABCD* (*x*, *y*)	(−*x*, *y*)	(*x*, −*y*)

11. **What similarities do you notice between reflecting a single point over the *x*- or *y*-axis and reflecting a polygon over the *x*- or *y*-axis?**

Let's consider reflections without graphing.

12. **The vertices of parallelogram *DEFG* are *D* (−9, 7), *E* (−12, 2), *F* (−3, 2), and *G* (0, 7).**
    a. **Determine the vertex coordinates of image *D′E′F′G′* if parallelogram *DEFG* is reflected over the *x*-axis.**

    b. **How did you determine the image coordinates without graphing?**

    c. **Determine the vertex coordinates of image *D″E″F″G″* if parallelogram *DEFG* is reflected over the *y*-axis.**

    d. **How did you determine the image coordinates without graphing?**

Answers

11. The similarities I notice are that even though a polygon consists of multiple points, the same methods and algebraic rule can be used when reflecting the point(s) over the *x*- or *y*-axis.

12a. The vertex coordinates of the image *D′E′F′G′* are *D′* (−9, −7), *E′* (−12, −2), *F′* (−3, −2), and *G′* (0, −7).

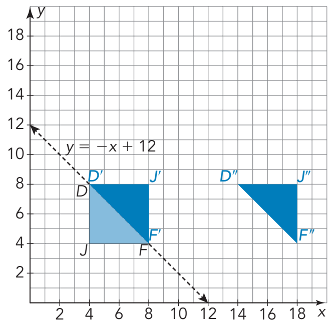
12b. I determined the image coordinates by following the pattern for reflecting over the *x*-axis. The *x*-coordinate of each point remained the same. The opposite of the *y*-coordinate of the pre-image became the *y*-coordinate of the image.

12c. The vertex coordinates of image *D″E″F″G″* are *D″* (9, 7), *E″* (12, 2), *F″* (3, 2), and *G″* (0, 7).

12d. I determined the image coordinates by following the pattern for reflecting over the *y*-axis. The opposite of the *x*-coordinate of the pre-image became the *x*-coordinate of the image. The *y*-coordinate of each point remained the same.

Answers

1.



1a. *J* (4, 4), *D* (4, 8), *F* (8, 4)

1b. See graph.

J′ (8, 8), *D′* (4, 8), *F′* (8, 4)

1c. See graph.

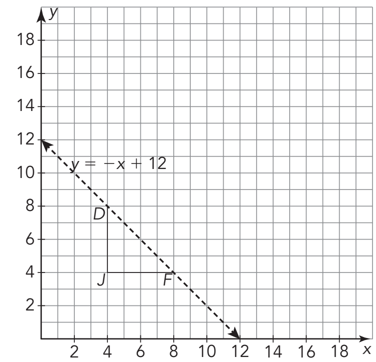
J″ (18, 8), *D″* (14, 8), *F″* (18, 4)

ACTIVITY **5.4**

Multiple Transformations

You can also perform multiple transformations, or a composition of transformations, on the coordinate plane.

1. Consider Δ*JDF* on the coordinate plane shown.



a. Write the coordinates of the vertices of the triangle.

b. Show a reflection of Δ*JDF* across the line *y* = −*x* + 12 shown. Label the new image as Δ*J′D′F′*. Write the coordinates of the vertices of the image.

c. Show a translation of the image to the right 10 units. Label the new image as Δ*J″D″F″*. Write the coordinates of the vertices of the image.

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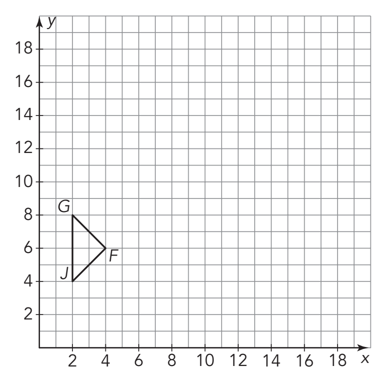
2. **Bianca says that there are many different ways to use multiple transformations to transform ΔJDF to create $\Delta J''D''F''$.**

   Is Bianca correct? If so, identify a sequence of rigid motions that can transform ΔJDF to create $\Delta J''D''F''$.

3. **Describe how you could determine the coordinates of the pre-image, ΔJDF, if you were given the coordinates of the image, $\Delta J''D''F''$, and the sequence of rigid motions.**

4. **Consider ΔJGF on the coordinate plane shown. Determine the coordinates of the vertices of the image, $\Delta J'G'F'$, under the composition of transformations given:**
   - **a rotation of 90° clockwise about point F**
   - **a translation 4 units down**



Answers

2. Answers will vary.

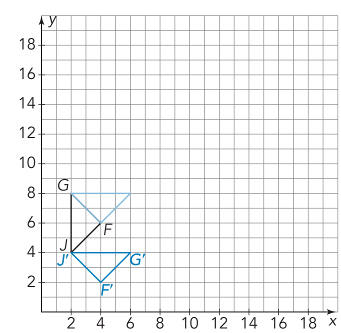
   I can reflect the pre-image over the horizontal line $y = 8$ and then reflect again over the vertical line $x = 8$. Then I can translate the triangle 6 units to the right and then 4 units down to create $\Delta J''D''F''$.

3. Answers may vary.

   I can work backwards from the coordinates of $\Delta J''D''F''$ and the sequence of rigid motions to arrive at the coordinates of the pre-image, ΔJDF.

   Using the transformations from Question 1, first I can translate the triangle to the left 10 units and then I can reflect the triangle over the line $y = -x + 12$.

4.

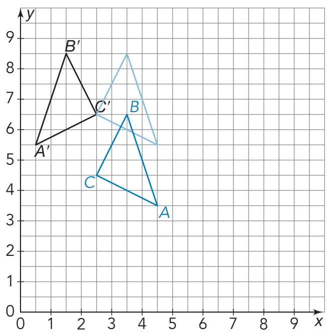


The coordinates of the image under the composition of transformations are J' (2, 4), G' (6, 4), and F' (4, 2).

Answers

5. A 90° clockwise rotation about point *J* would produce the same result as the composition of rigid motions.

6.



I can work backwards through the list of transformations.

First, I reflect triangle $A'B'C'$ over the line $x = 2\frac{1}{2}$. The coordinates of the new triangle are $A'\left(4\frac{1}{2}, 5\frac{1}{2}\right)$, $B'\left(3\frac{1}{2}, 8\frac{1}{2}\right)$, $C'\left(2\frac{1}{2}, 6\frac{1}{2}\right)$.

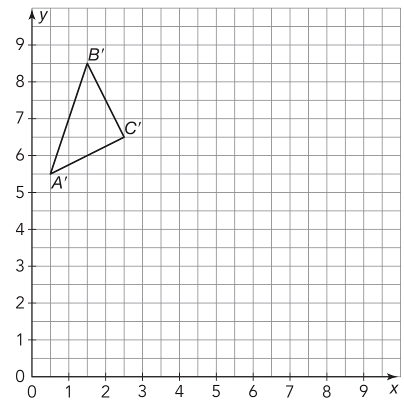
Then, I translate the triangle 2 units down. The coordinates of pre-image, ΔABC, are $A\left(4\frac{1}{2}, 3\frac{1}{2}\right)$, $B\left(3\frac{1}{2}, 6\frac{1}{2}\right)$, $C\left(2\frac{1}{2}, 4\frac{1}{2}\right)$.

5. Describe a single rigid motion that could produce the same result as the composition of transformations in Question 4. Explain your reasoning.

6. Triangle *A′B′C′* is the result of this composition of rigid motions:

- **a translation up 2 units**
- **a reflection over the line $x = 2\frac{1}{2}$**

Determine the coordinates of the vertices of the pre-image, triangle *ABC*. Explain your reasoning.



TALK the TALK

Turn Yourself Around

You know that a line is determined by two points. The slope of any line represented on a coordinate plane can be given by $\frac{y_2 - y_1}{x_2 - x_1}$.

You also now know that when rotating a point (x, y) 90° counterclockwise about the origin, the x-coordinate of the original point maps to the y-coordinate of the transformed point and the y-coordinate of the original point maps to the opposite of the x-coordinate of the transformed point.

1. Rewrite the slope ratio above to describe the slope of a line that has been rotated 90° counterclockwise. What do you notice? Explain your reasoning.

2. Complete the sentence using *always, sometimes,* or *never.*

Images that result from a translation, rotation, or reflection are ________________ congruent to the original figure.

NOTES

Think about:

When two lines are perpendicular to each other, how can you describe their slopes?

Answers

1. $\frac{y_2 - y_1}{x_2 - x_1}$

   Replace x with y and replace y with $-x$.

   $\frac{-x_2 - (-x_1)}{y_2 - y_1}$

   $-\frac{(x_2 - x_1)}{(y_2 - y_1)}$

   The slope ratio for a line that is rotated 90° counterclockwise is the negative reciprocal of the ratio for the original line.

   When a line is rotated 90° counterclockwise, it is perpendicular to the original line. A line that is perpendicular to another line has a slope that is the negative reciprocal of the slope of the original line.

2. always