6

OKEECHOBEE

Reflectional and Rotational Symmetry

MATERIALS

Patty paper
Protractors

Lesson Overview

Students explore reflectional and rotational symmetry within a figure using patty paper before formal definitions are provided. They then analyze these symmetries in more depth as they relate the number of lines of symmetry and the measures of angles of rotation to specific types of figures. Students identify reflectional and rotational symmetry in letters of the alphabet and some titles in this lesson. They also identify the relationship between the rotational symmetries of a regular figure and the measure of each of its interior angles.

Geometry
Coordinate and Transformational Geometry

(3) The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(D) identify and distinguish between reflectional and rotational symmetry in a plane figure.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- A plane figure has reflectional symmetry if you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side.
- A plane figure has rotational symmetry if you can rotate the figure more than 0° and less than 360° and the resulting figure is the same as the original figure.
- An individual figure may have horizontal symmetry, vertical symmetry, and/or rotational symmetry.
- A regular polygon of *n*-sides has *n* lines of symmetry.
- The measure of the angle of rotation of a regular polygon with *n* sides is $\frac{360°}{n}$, which is the supplement of the measure of each of its interior angles.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: WOW MOM

Students trace six different figures on patty paper. They determine whether it is possible to fold each figure so that half of the figure lies exactly on the other half of the figure. Students then determine whether you can rotate the figure so that it looks exactly like it did before the rotation.

Develop

Activity 6.1: Reflectional and Rotational Symmetries

Students make sense of the definitions of *reflectional symmetry* and *rotational symmetry* as they investigate reflectional and rotational symmetries of different figures.

Activity 6.2: Identifying Symmetry

Students identify reflectional and rotational symmetry in letters of the alphabet.

Demonstrate

Talk the Talk: CHECK

Students identify the symmetries in the title of this lesson, OKEECHOBEE, along with the titles of the activity WOW MOM and this activity CHECK. They also identify the relationship between the rotational symmetries of a regular figure and the measure of each of its interior angles.

Facilitation Notes

In this activity, students trace six different figures on patty paper. They determine whether it is possible to fold each figure so that half of the figure lies exactly on the other half of the figure. Students then determine whether you can rotate the figure so that it looks exactly like it did before the rotation.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Misconception

Students may associate circles with rotations, but not reflections. They may assume that only polygons can have lines of reflection designated by folds in patty paper.

Questions to ask

- How is this question different than the questions in the previous lessons?
- Which shape is not a polygon? Why?
- Are all of the polygons convex? Which polygon is concave? How can you tell the difference between a convex polygon and a concave polygon?
- What special relationship exists within the hexagon? The trapezoid?
- How can you tell that you determined the total number of possible folds so that half of the figure lies exactly on the other half of the figure?
- What characteristic(s) of the figure determine whether there are any possible folds so that half of the figure lies exactly on the other half of the figure?
- Is the number of possible folds related to the number of angles, sides, or vertices?
- Do you think there are any parallelograms where there would be folds that place half of the figure exactly on the other half of the figure? Which ones?
- Why is there always at least one rotation that causes a figure to look exactly like it did before the rotation? What is the degree measure of that rotation?
- How can you tell if there are other rotations between 0° and 360° that have this same effect?

Differentiation strategies

To extend the lesson,

- Have students investigate whether there is a connection between the number of sides of regular figures and the number of possible lines of reflection. Students should conclude that a regular polygon of *n* sides has *n* lines of reflection.
- Suggest students investigate the relationship between the fold that maps half of the isosceles trapezoid exactly onto the other half of the isosceles trapezoid. The fold is the perpendicular bisector of both parallel sides of the trapezoid.

Summary

Some plane figures can be folded along a line so that half of the figure lies exactly on the other half of the figure. Any plane figure can be rotated 360° so that it looks like it did exactly before the rotation; some plane figures can be rotated other degree measures between 0° and 360° for this to occur as well.

DEVELOP

Activity 6.1

Reflectional and Rotational Symmetries

Facilitation Notes

In this activity, students make sense of the definitions of *reflectional symmetry* and *rotational symmetry* as they investigate reflectional and rotational symmetries of different figures.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

As students work, look for

An error when determining the lines of symmetry of the rectangle that is not a square. Students may think that the diagonals of the rectangle are lines of symmetry because each diagonal creates a pair of congruent triangles, but the orientation of the congruent triangles is not a reflection.

Misconception

Students may think that a line of reflection and a line of symmetry are two different concepts; however, they are essentially the same. A figure is symmetric about a line if the reflection of every point through that line is another point on the figure. The result of a reflection is a figure that is mapped onto itself.

Questions to ask

- How is Question 2 related to the folds you made in the Getting Started activity?
- How do you determine whether a figure has reflectional symmetry?
- What is an example of a figure that has no lines of reflectional symmetry?
- What is an example of a figure that has exactly one line of reflectional symmetry?
- Do lines of symmetry always pass through the vertices of the figure?
- Does any polygon have an infinite number of lines of symmetry?
- Why do circles have an infinite number of lines of symmetry?
- Why doesn't this rectangle have diagonal lines of symmetry?
- Why do squares have more lines of reflection than rectangles that are not squares?

Differentiation strategy

To extend the activity, have students investigate the placement of the lines of reflection in a regular polygon with an even number of sides, such as the regular hexagon in the Getting Started, and the equilateral triangle in Question 3. Have them explain the similarities and differences and why they occur.

Have students work with a partner or in a group to complete Questions 6 through 9. Share responses as a class.

As students work, look for

Appropriate use of vocabulary identifying the lines of reflection and centers of rotation for Question 9.

Misconception

Students may make an incorrect assumption because they determined earlier that all figures can be mapped onto themselves after performing a 360° rotation. However, this does not imply that every figure has rotational symmetry. In this activity, the definition of rotational symmetry states that you must be able to rotate the figure more than 0° but less than 360°.

Differentiation strategy

To extend the lesson, encourage students to investigate how to describe the location of the center of rotation for each figure in Question 9. While a response that the center of rotation is the center of the figure is sufficient, a more detailed response would include the fact that the center of rotation is the point of intersection of the lines of reflection.

Questions to ask

- What is the difference between reflectional symmetry and rotational symmetry?
- How do you determine if a figure has rotational symmetry?

- If a figure has rotational symmetry, how is the center of rotation determined?
- How did you determine the measure of the angle of rotation for the regular octagon?

Differentiation strategies

To extend the lesson,

- Have students investigate Clark's thinking in more depth. Ask them to determine if there are any cases where a series of reflections and rotations yield the same result. If a figure has two lines of reflection that are perpendicular, this is equivalent to rotational symmetry of 180°.
- Have students explore the relationship between the number of sides of a regular polygon and the measure of the angles of rotation. Students should conclude that a regular polygon of *n* sides will have an angle of rotation of $\frac{360°}{n}$. Then, the other angles of rotation can be generated by multiplying $\frac{360°}{n}$ by integer values between 1 and $(n - 1)$. Students will explore this relationship and connect it to the measure of the interior angles of a regular polygon in the Talk the Talk at the end of this lesson.

Summary

A plane figure has reflectional symmetry if you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side of the line. A plane figure has rotational symmetry if you can rotate the figure more than 0° but less than 360°, and the resulting figure is the same as the original figure in the original position. A plane figure can have both reflectional and rotational symmetry.

Activity 6.2
Identifying Symmetry

Facilitation Notes

In this activity, students identify reflectional and rotational symmetry in letters of the alphabet.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

As students work, look for

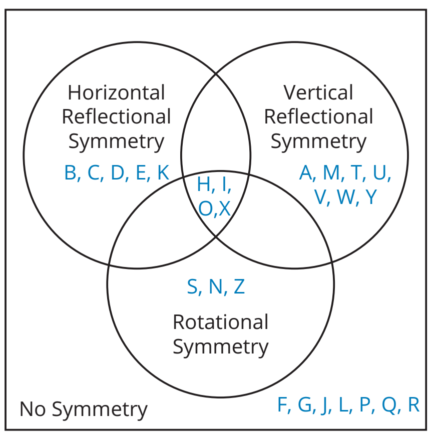
Errors when reflecting the letters N, S, and Z. These letters do not have a vertical line of reflection.

Questions to ask

- Does the letter A have any line of symmetry? Explain.
- What other letter has symmetry similar to the letter A?
- What is another letter of the alphabet that has a horizontal line of symmetry, but not a vertical line of symmetry?
- What is another letter of the alphabet that has a vertical line of symmetry, but not a horizontal line of symmetry?
- What is another letter of the alphabet that has both a horizontal and vertical line of symmetry?
- Will you get the same result when you test to see if a letter has a vertical line of symmetry and when you reflect a letter across a vertical line of reflection to see if you get the identical letter? Explain why or why not.

Differentiation strategy

To extend the activity, present students with the entire alphabet and ask them to complete a Venn diagram to identify the different types of symmetry.



Summary

An individual figure may have horizontal reflectional symmetry, vertical reflectional symmetry, and/or rotational symmetry.

Talk the Talk: CHECK

DEMONSTRATE

Facilitation Notes

In this activity, students identify the symmetries in the title of this lesson, OKEECHOBEE, along with the titles of the activity WOW MOM and this activity CHECK. They also identify the relationship between the rotational symmetries of a regular figure and the measure of each of its interior angles.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Does your name contain any letters that have a horizontal line of symmetry?
- Does your name contain any letters that have a vertical line of symmetry?
- Does your name contain any letters that have both horizontal and vertical lines of symmetry?
- Does your name contain any letters that have rotational symmetry?

Differentiation strategy

To extend the activity, have students investigate palindromes.

- A palindrome is a word or sentence that reads the same forward as it does backward, like MOM, WOW, and RACECAR. Names like LIL, HANNAH, and BOB are also palindromes. Have students list a few palindromes and identify the symmetries of each palindrome.
- A palindrome can also be a number that reads the same forward and backward. Have students investigate how many three-digit and four-digit palindromes exist. Then ask students to create a formula to determine the number of palindromes that exist for any *n*-digit number. Note that 0220 is considered a four-digit palindrome.

Summary

The measure of the angle of rotation of a regular polygon with n sides is $\frac{360°}{n}$, which is the supplement of the measure of each of its interior angles.

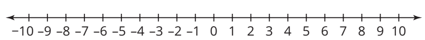
6

OKEECHOBEE

Reflectional and Rotational Symmetry

Warm Up

Identify the opposite of each number on the number line.



1. -8

2. $-|-2|$

3. 9

4. $-(-7)$

Learning Goals

- Identify geometric figures with line symmetry and rotational symmetry.
- Identify lines of symmetry for different geometric figures.
- Describe rotations that carry a figure onto itself.
- Describe reflections that carry a figure onto itself.

Key Terms

- reflectional symmetry
- rotational symmetry

You have learned that pre-images are congruent to images after rigid motion transformations. How can you use transformations to show that a figure can be carried onto itself?

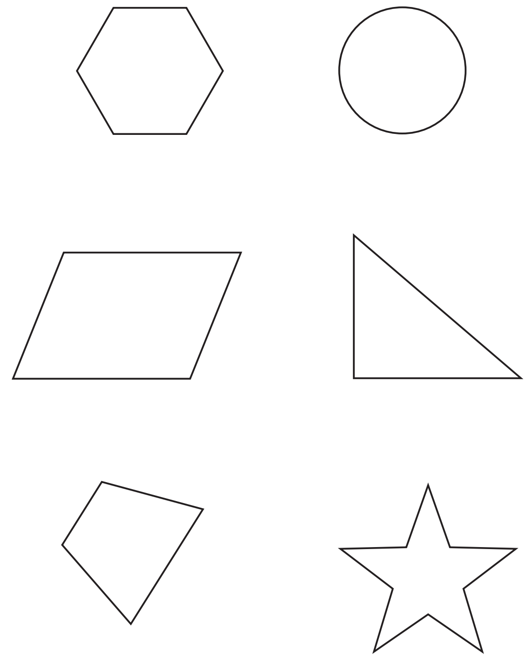
Warm Up Answers

1. 8
2. 2
3. −9
4. −7

GETTING STARTED

WOW MOM

Consider the different shapes shown.



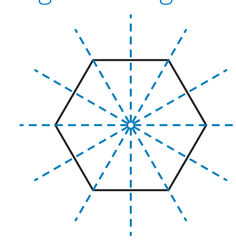
1. **Copy each shape onto patty paper. For each shape:**

    a. **Determine whether you can fold the shape so that half of the figure lies exactly on the other half of the figure. If so, are there any other folds that will do this?**

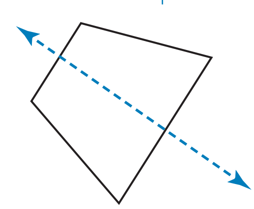
    b. **Determine whether you can rotate the figure so that it looks exactly like it did before the rotation. If so, are there other rotations that will do this?**

Answers

1a. The regular hexagon, circle, isosceles trapezoid, and star can be folded so that one half of the figure lies exactly on the other half of the figure. This is not possible for the parallelogram and scalene right triangle. There are six possible folds for the regular hexagon.



There are an infinite number of folds for the circle.
There is only one possible fold for the isosceles trapezoid.



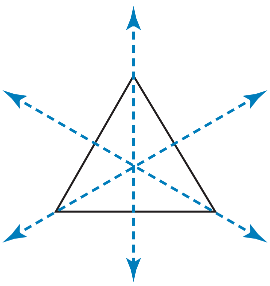
There are five possible folds for the star.

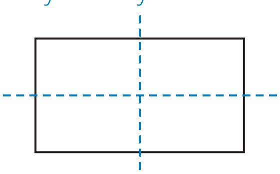
1b. Each figure can be rotated 360° so that it looks like it did exactly before the rotation. There are other rotations between 0° and 360° that will do this for the hexagon, circle, parallelogram, and star.

There are a total of 6 possible rotations for the hexagon.
There are an infinite number of rotations for the circle.
There are a total of two possible rotations for the parallelogram.

There are a total 5 possible rotations for the star.

Answers

1. Regular hexagon
   Circle
   Isosceles trapezoid
   Parallelogram
   Scalene right triangle
   Star
2. The regular hexagon, circle, isosceles trapezoid, and star can be folded so that one half of the figure lies exactly on the other half of the figure.
3. 
4. The rectangle has 2 lines of symmetry.



ACTIVITY 6.1

Reflectional and Rotational Symmetries

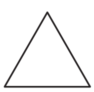
Consider the shapes from the Getting Started.

1. Name the shapes.

A plane figure has **reflectional symmetry** if you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side of the line. Recall that the line that you drew on each shape is called the line of symmetry. A figure may have more than one line of symmetry.

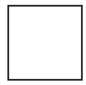
2. Which shapes have reflectional symmetry?

3. Consider the equilateral triangle shown. It has 3 lines of symmetry. Draw these lines of symmetry.



4. How many lines of symmetry does the rectangle shown have? Explain your reasoning.

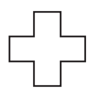
5. How many lines of symmetry are there in a square? Show the line(s) of symmetry.



A plane figure can also have **rotational symmetry** if you can rotate the figure more than 0° but less than 360° and the resulting figure is the same as the original figure in the original position.

6. Which shapes in the Getting Started have rotational symmetry?

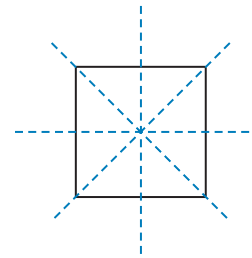
7. Do you think that the given shape has rotational symmetry? Why or why not?



8. Can a shape have both reflectional and rotational symmetry? Explain your reasoning.

Answers

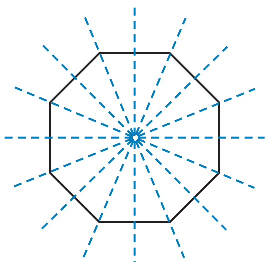
5. A square has 4 lines of symmetry.



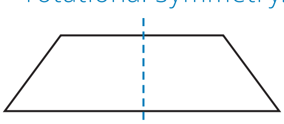
6. The regular hexagon, circle, parallelogram, and star have rotational symmetry.
7. Yes. The shape has rotational symmetry. It can be rotated 90°, 180°, and 270° to match itself exactly.
8. Yes. A shape can have both reflectional and rotational symmetry. In this activity, we have shown that regular hexagons, circles, and the star have both reflectional and rotational symmetry.

Answers

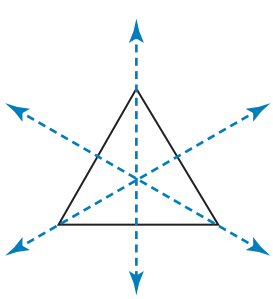
9a. The regular octagon has 8 lines of reflection that either pass through a pair of vertices or a pair of midpoints of opposite sides. It also has 7 angles of rotation between 0° and 360° degrees: 45°, 90°, 135°, 180°, 225°, 270°, and 315° with a center of rotation that is the intersection of all the lines of symmetry.



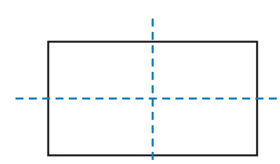
The isosceles trapezoid has one line of reflection that is the perpendicular bisector of the parallel sides. It does not have any rotational symmetry.



The equilateral triangle has three lines of reflection, each one passing through a vertex and the midpoint of the opposite side. It also has 2 angles of rotation between 0° and 360° degrees: 120° and 240° with a center of rotation that is the intersection of all the lines of symmetry.



The rectangle has 2 lines of symmetry, each one passing through the midpoints of a pair of parallel sides. It also has one angle of rotation between 0° and 360° degrees: 180° with a center of rotation that is the intersection of its two lines of symmetry.



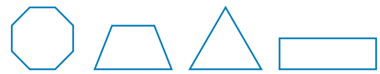
9b. See next page.

You have also identified transformations that carry a figure onto itself. Reflectional and rotational symmetry are properties of figures that can be carried onto themselves by reflections and rotations.

Think about:

For reflections, identify the line of reflection. For rotations, describe the rotation angle and center of rotation.

9. Consider the 4 shapes shown.



a. Describe the reflections and rotations that can carry each figure onto itself.

b. Clark says that the horizontal line of symmetry in the rectangle means that a reflection across that line carries the figure onto itself. He also says that it means that a 180° rotation will carry the figure onto itself.

Is Clark correct? Does his reasoning apply to other figures? Explain your thinking using the shapes from the Getting Started.

6 • TOPIC 2: Rigid Motions on a Plane

ACTIVITY **6.2**

Identifying Symmetry

The standard alphabet has many letters that have a variety of symmetries, including reflectional and rotational symmetry. Some letters have a vertical line of symmetry. Other letters have a horizontal line of symmetry.

1. Which letter(s) have a horizontal but not a vertical line of symmetry?

A **C** **H** **M**

2. Which letter(s) have a vertical but not a horizontal line of symmetry?

M **B** **H** **X**

3. Which letter(s) have both a horizontal and a vertical line of symmetry?

Z **E** **H** **M**

4. Which letter(s) have rotational symmetry?

Z **W** **K** **M**

Answers

9b. Clark is correct about the rectangle; however, his reasoning does not apply to every shape. For example, the isosceles trapezoid has one line of symmetry which carries the figure onto itself, but it does not indicate anything about the figure's rotational symmetry because an isosceles trapezoid has no rotational symmetry. Clark seems to be confusing the fact that a line is an angle with a measure of 180° with an angle of rotation of 180°.

1. C
2. M
3. H
4. Z

Answers

1. The title OKEECHOBEE has a horizontal line of symmetry through the middle of the word, since each letter has a horizontal line of symmetry through its center.

   The title WOW MOM has rotational symmetry. You can rotate it 180° to match itself exactly.

   The title CHECK has a horizontal line of symmetry through the middle of the word.

2a. The measure of the angle of rotation of a regular polygon with n sides is $\frac{360°}{n}$, which is the supplement of the measure of each of its interior angles.

2b. The pattern is the same. The measure of each interior angle of a regular pentagon is 108°, and the measure of the angle of rotation is 72°. The measure of each interior angle of a regular hexagon is 120°, and the measure of the angle of rotation is 60°. In each case, the sum of the measure of an interior angle and the angle of rotation is 180°.

NOTES

TALK the TALK

CHECK

The title of this lesson, OKEECHOBEE, is the name of a city, a county, and a lake in Florida. That title, along with the title of the Getting Started activity, WOW MOM, and the title of this activity, CHECK, all have rotational and/or reflectional symmetry.

1. Identify the symmetries in each title. Explain your thinking.

2. Consider the rotational symmetries of an equilateral triangle, square, and regular hexagon.

   a. What relationship exists between the rotational symmetries of each figure and its interior angle measures?

   b. Test the pattern you noticed on a regular pentagon and regular hexagon. What do you notice?