Rigid Motions on a Plane

Topic 2 Overview

How is *Rigid Motions on a Plane* organized?

Rigid Motions on a Plane begins by reminding students of what they know about functions via a function machine. Instead of having a numeric or algebraic input and output, a geometric function machine has a geometric figure as the input and output. An algebraic function transforms each input, mapping it according to an algebraic rule to an output value. A geometric function transforms each point on a pre-image onto exactly one point on the corresponding image. Students investigate simple geometric transformation machines, describing how each input shape is "carried" by geometric objects in the transformation machine to result in the output shape. Students then learn to construct parallel lines, which is important in understanding translations that occur along parallel lines.

Students investigate rigid motions before defining them formally with a transformation machine. This machine comprises a variety of lines and figures, each with specific characteristics that provide a path for input shapes to move from the start line through the machine and map back onto itself. This informal investigation is designed for students to consider how shapes can move on a plane. Some students will seek the shortest path from the start line to the target shape; other students will seek the most complicated path. Either of these paths—and everything in between—builds students' tactile understanding of rigid motions on the plane. As students engage in the exploration, they will likely make mistakes, break the rules, need to reorient their input shape, and/or need to start over. These experiences enrich their investigation, laying the groundwork for the formal definitions and rules that come in the lessons that follow.

Students then consider each of the rigid motions as functions. First, with translations, students investigate how a translation maps each point of a pre-image an equal distance along parallel lines onto the image. This verifies their intuitive understanding that translations preserve a figure's size and shape. Function notation is presented to represent a translation of a point a specific distance and direction using a directed line segment. Students use patty paper to complete transformations according to a given function. Throughout, they differentiate translations that map points equal distances along parallel lines with transformations that map points along non-parallel lines, which do not preserve distances and angle measures.

A reflection transformation is then similarly defined as a geometric function, mapping each point of a pre-image across a line onto the image so that each point is equidistant from the line of reflection. Students learn to construct a line of symmetry given two figures that are reflections of one another by connecting corresponding points in the two images and constructing the perpendicular bisector of the segment. They use this construction to determine whether given figures are reflections of one another. Because the line of reflection is the perpendicular bisector of the segments connecting each of the corresponding points in an image and pre-image, students can use isometries to show that the Perpendicular Bisector Theorem and its converse are true. Understanding this theorem is important in the next topic when students prove the Side-Angle-Side Congruence Theorem. Finally, students identify a sequence of translations and reflections that map one figure onto another.

Students use three concentric circles to create and rotate a triangle around the center of the circles at a rotation angle of their choosing. The formal definition of a rotation function is given and notation is provided. They examine how a rotation function rotates every point in a pre-image around arcs of concentric circles at a specific rotation angle. Finally, students identify a sequence of translations, reflections, and rotations that maps one figure onto another.

Students recall what they know about transformations of functions by examining the graph of the basic function, $f(x)$, and its transformed graph $g(x)$. Students then cut out a model of a trapezoid and translate, rotate, and reflect the model on a coordinate plane to determine how transformations affect the coordinates of the figure. Next, compositions of transformations are explored on the coordinate plane.

To close the topic, students identify geometric figures with line symmetry and rotational symmetry. They identify lines of reflection and angles of rotation that map given figures onto themselves.

x+y= What is the entry point for students?

Rigid Motions on a Plane builds on students' experiences in previous courses where they verified experimentally the properties of rotations, reflections, and translations. Students know that two figures are congruent if and only if there exists a sequence of one or more rigid motions that carries one of the figures onto the other. They have described the effect of rigid motions on two-dimensional figures using coordinates. However, most of this investigation about rigid motions occurred on the coordinate plane. *Rigid Motions on a Plane* builds upon these experiences, at first stripping away the coordinate plane and requiring students to rely on geometric reasoning. Instead of using distances on the coordinate plane to describe these transformations, students generalize their understanding to define these transformations on a plane in terms of parallel lines, perpendicular lines, and arcs of circles.

Students have had extensive experience with functions, particularly linear and exponential functions. They understand that a function maps each element of the domain to exactly one element of the range. In the same way, students define rigid motions as transformations. Each rigid motion maps each point on the pre-image to exactly one element of the image. In the same way that algebraic functions have different characteristics based on their function type, geometric transformations have different characteristics based on their geometric function type.

In elementary school, students informally investigated reflectional symmetry in two-dimensional figures by folding line-symmetric figures and drawing lines of symmetry. They now combine this intuitive understanding of symmetry with their definition of reflections to determine which figures have reflectional symmetry.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Rigid Motions on a Plane* if they can:

- Define translations in terms of distances along parallel lines.
- Define reflections in terms of distances across perpendicular lines.
- Define rotations in terms of angles around a circle.
- Draw rigid motion transformations using patty paper or other construction tools.
- Recognize the difference between transformations that are rigid motions and those that are not.
- Understand that a translation maps all points in the pre-image equal distances along parallel lines to corresponding points in the image.
- Understand that the line of reflection is the perpendicular bisector of the line segments connecting each point in the pre-image to its corresponding point in the image.
- Determine the line of reflection for a given reflection.
- Understand that a rotation maps all points in the pre-image an equal angle around concentric circles to corresponding points in the image.
- Determine the center of rotation for a given rotation.
- Specify a sequence of transformations that maps one figure onto another in the plane.
- Illustrate the rotations and reflections that carry a figure onto itself.
- Describe rigid motion transformations using coordinate notation.
- Calculate the number of lines of symmetry and the degree of rotational symmetry of regular polygons.

Why is *Rigid Motions on a Plane* important?

Rigid Motions on a Plane begins the formal study of congruence and sets the stage for similarity and trigonometry. This topic is part of a long progression in understanding geometric and algebraic transformations. In the next topic, students define congruent figures in terms of rigid motion transformations. They then explain how the criteria for triangle congruence (SSS, SAS, and ASA) follow from the definition of congruence in terms of rigid motion. They will use the triangle congruence theorems in future topics to prove a wide range of geometric theorems.

Deepening their understanding of transformations to include geometric figures on the plane extends students' ability to transform both geometric and algebraic figures. Students will continue working with transformations in future courses as they transform quadratic, polynomial, rational, and radical functions.

How do the activities in *Rigid Motions on a Plane* promote student expertise in the mathematical practice standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Materials Needed

Compasses
Patty paper
Protractors
Rulers
Straightedges
Scissors
Tape

New Tools and Notation

A geometric transformation is a function that maps each point on the plane (input) to exactly one other point on the plane (output). Therefore, function notation can be used to describe each rigid motion transformation.

Translation

The notation $T_{AB}(P) = P'$ indicates that P' is a translation of point P the distance and direction of $\overline{AB}$.

Reflection

The notation $R_m(P) = P'$ indicates that P' is a reflection of point P across line m.

Rotation

The notation $R_{E,t}(P) = P'$ indicates that P' is a rotation of point P about center of rotation E of angle t.

Learning Together

ELPS: 1.A, 1.C, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.D, 4.F, 4.J, 4.K, 5.B, 5.E, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	Put Your Input In, Take Your Output Out: Geometric Components of Rigid Motions	G.3B G.3C	1	Students develop the concept that geometric rigid motion transformations can be considered as functions, with rotations, reflections, and translations as the operations. Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using rotation angles. The inputs and outputs are geometric shapes. Each input and its corresponding output have the same size and shape.
2	Bow Thai: Translations as Functions	G.3B G.3C G.6C	2	Students analyze transformation machines and conclude that translations along parallel lines always produce images that are congruent to their pre-image, while translations along rays with a common endpoint produce dilations or images that are similar to, but not congruent to, their pre-image. The term *isometry* is defined to label these differences, with the understanding that any rigid motion transformation that preserves size and shape is an isometry. Students then engage in a context involving an animated website where they learn and use function notation to represent geometric translations.
3	Staring Back at Me: Reflections as Functions	G.3B G.3C G.5B G.6A G.6C	2	Students analyze reflections as isometries. They construct a perpendicular bisector of a segment and then conclude that the perpendicular bisector is the line of reflection between the endpoints of the segment. Students investigate reflections as functions using the context from the previous lesson, use function notation to represent geometric reflections, and construct lines of reflection. They combine what they learned in this lesson and the previous lesson to identify sequences of translations and reflections to demonstrate that two figures are congruent.

Lesson	Lesson Name	TEKS	Days	Highlights
4	Turn Yourself Around: Rotations as Functions	G.3B G.3C G.5B G.6C	3	Students analyze rotations. First they use concentric circles to rotate a triangle and determine that rotations are isometries. They then are introduced to the notation for the rotation function and use it to rotate any figure using only a protractor and ruler. Students then reverse the process and identify the center of rotation and angle of rotation given a pre-image and image of a figure. As in the previous lesson, they identify sequences of transformations to demonstrate that two figures are congruent. Students then use a graphic organizer to summarize what they have learned about translation, reflection, and rotation isometries.
5	Slide, Flip, Turn: The Latest Dance Craze?: Translations, Rotations, and Reflections on the Coordinate Plane	G.3A G.3B G.3C	3	Students recall what they know about transformations of functions by examining the graph of the basic function, $f(x)$, and its transformed graph $g(x)$. Students then cut out a model of a trapezoid and translate, rotate, and reflect the model on a coordinate plane to determine how transformations affect the coordinates of the figure. Compositions of transformations are explored on the coordinate plane.
6	OKEECHOBEE: Reflectional and Rotational Symmetry	G.3D	1	Students explore reflectional and rotational symmetry within a figure using patty paper prior to formal definitions being provided. They then analyze these symmetries in more depth as they relate the number of lines of symmetry and the measures of angles of rotation to specific types of figures. Students identify reflectional and rotational symmetry in letters of the alphabet and some titles in this lesson. They also identify the relationship between the rotational symmetries of a regular figure and the measure of each of its interior angles.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: G.3B, G.3C **LESSON 1** **Put Your Input In, Take Your Output Out** GETTING STARTED ACTIVITY 1 TALK THE TALK	TEKS: G.3B, G.3C, G.6C **LESSON 2** **Bow Thai** GETTING STARTED ACTIVITY 1	**LESSON 2** continued ACTIVITY 2 TALK THE TALK	MATHia Use LiveLab and Reports to monitor students' progress	**MID-TOPIC ASSESSMENT**

Day 6	Day 7	Day 8	Day 9	Day 10
TEKS: G.3B, G.3C, G.5B, G.6A, G.6C **LESSON 3** **Staring Back at Me** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 3** continued ACTIVITY 3 TALK THE TALK	MATHia Use LiveLab and Reports to monitor students' progress	TEKS: G.3B, G.3C, G.6C, G.5B **LESSON 4** **Turn Yourself Around** GETTING STARTED ACTIVITY 1	**LESSON 4** continued ACTIVITY 2

Day 11	Day 12	Day 13	Day 14	Day 15
LESSON 4 continued ACTIVITY 3 TALK THE TALK	MATHia Use LiveLab and Reports to monitor students' progress	TEKS: G.3A, G.3B, G.3C **LESSON 5** **Slide, Flip, Turn: The Latest Dance Craze?** GETTING STARTED ACTIVITY 1	**LESSON 5** continued ACTIVITY 2 ACTIVITY 3	**LESSON 5** continued ACTIVITY 4 TALK THE TALK

Day 16	Day 17	Day 18	Day 19
MATHia Use LiveLab and Reports to monitor students' progress	TEKS: G.3D **LESSON 6** **OKEECHOBEE** GETTING STARTED ACTIVITY 1 ACTIVITY 2 TALK THE TALK	MATHia Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**

Assessments

There are two assessments aligned to this topic: Mid-Topic Assessment and End of Topic Assessment.