Rigid Motions on a Plane Summary

KEY TERMS

- collinear points
- angle
- ray
- rotation angle
- translation
- isometry
- reflection
- Perpendicular Bisector Theorem
- proof
- rotation
- reflectional symmetry
- rotational symmetry

LESSON 1

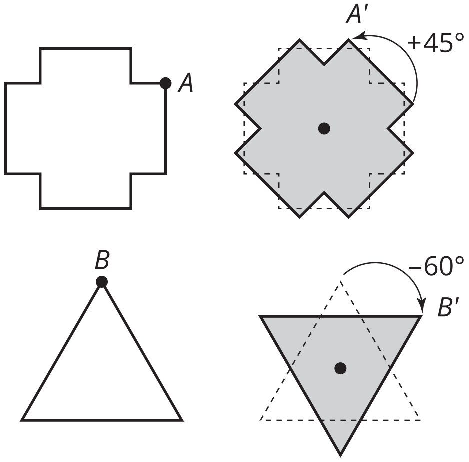
Put Your Input In, Take Your Output Out

Lines and circles form the basis of rigid motion transformations in geometry.

Points that lie along the same line are **collinear points**.

An **angle** is a set of points consisting of a vertex point and two rays extending from the vertex point. A **ray** is a portion of a line that begins with a single point and extends infinitely in one direction.

A **rotation angle** is a directed angle based on a circle. Positive rotation angles turn counterclockwise, and negative rotation angles turn clockwise.



LESSON 2

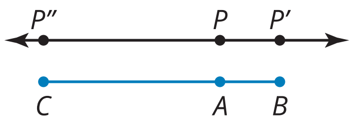
Bow Thai

Transformations are used frequently in web design and game animations and are often written as functions, which take points, distances, and angles as inputs. The functions output a new set of points after applying a transformation. These transformations move objects around on the screen.

A **translation** is a function, T, which takes as its input a set of pre-image points, P, and outputs a set of image points, $T_{AB}(P)$, or P'. The set of image points is a translation of the set of pre-image points. The pre-image points are translated a distance of AB in the direction AB. A translation is a rigid motion transformation, or **isometry**. An isometry preserves distances and angle measures.

A translation function can represent the distance and direction of the translation using a line or line segment, or a parallel line or line segment:

$T_{AB}(P) = P'$
$T_{AC}(P) = P''$

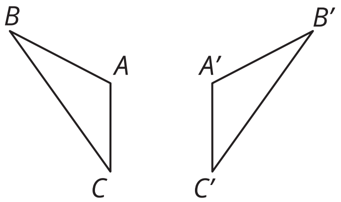


LESSON 3

Staring Back at Me

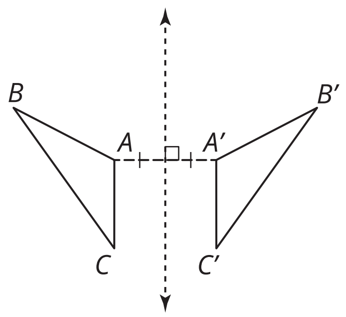
A **reflection** is a function, R_ℓ, which takes as its input, p, the location of a point with respect to some line of reflection, ℓ, and outputs $R_\ell(p)$, or the opposite of the location of p with respect to the line of reflection. A reflection is an isometry.

It is possible to construct the line of reflection when given two figures that are reflections of one another.

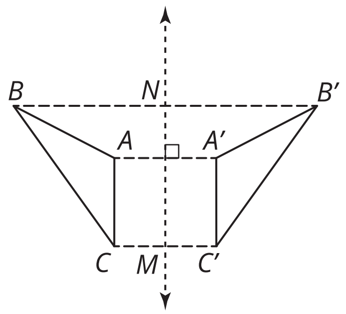


1. Label the vertices.

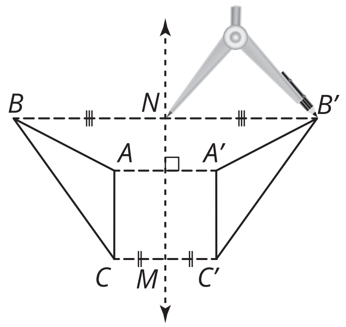
2. Connect two corresponding points. Construct the perpendicular bisector of the line segment connecting them.



3. Connect remaining corresponding points. Label the intersection of each line segment with the perpendicular bisector.



4. Use a compass to determine whether each intersection point is the midpoint of the line segment connecting corresponding vertices. If that is the case, the perpendicular bisector is the line of reflection. If not, the figures are not reflections of one another.



$\overline{CM} \cong \overline{C'M}$
$\overline{BN} \cong \overline{B'N}$
$\overline{MN}$ is the line of reflection.

You know that the perpendicular bisector of a line segment is the line of reflection between the endpoints of the segment. The **Perpendicular Bisector Theorem** states: "Any point on the perpendicular bisector of a segment is equidistant from the endpoints of that segment." A **proof** is a series of statements and corresponding reasons forming a valid argument that starts with a hypothesis and arrives at a conclusion.

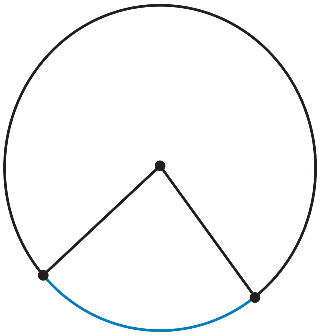
LESSON 4

Turn Yourself Around

A **rotation** is a function that maps its input, a point *P*, to another location, which is defined by a center of rotation, *E*, and a rotation angle, *t*. For this reason, a rotation function is written as $R_{E,t}(P)$. A rotation is an isometry.

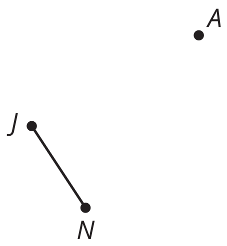
Because the rotation is defined about a point *E*, the movement of a specific point traces an arc that is part of the circumference of a circle with center *E*. The arc can be labeled by the starting point, *P*, and the endpoint, *P'*, or as $\overset{\frown}{PP'}$. The degree measure of this arc is equivalent to the degree rotation, *t*, that creates a central angle in Circle *E*.

A circle is a rotation of a point around a given center 360 degrees. An angle is the measure of the distance the point is rotated as measured by the central angle.

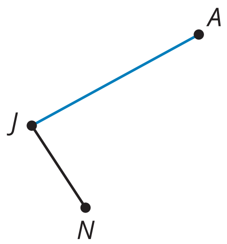


It is possible to apply the rotation function to a figure by using a protractor and a straightedge.

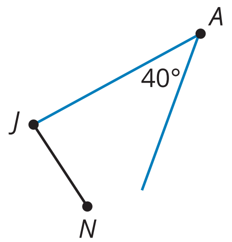
$R_{A,\,40}(\overline{JN})$



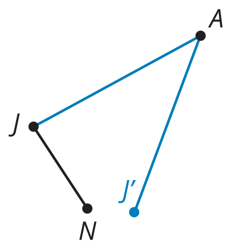
This means to rotate $\overline{JN}$ 40°, using point *A* as the center of rotation.



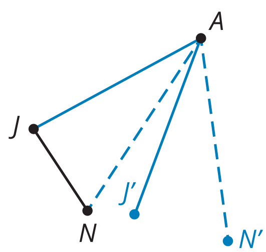
1. Draw a line segment from the center of rotation, *A*, to one endpoint of the line segment.



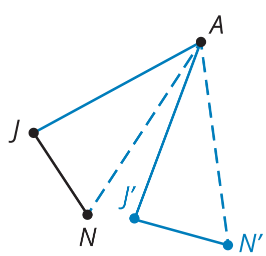
2. Using a protractor, draw a 40° angle. Use the center of rotation, *A*, as the vertex and the line segment drawn, $\overline{AJ}$, as one side of the angle. Since the angle measure is positive, place the angle in the counterclockwise direction of the line segment drawn.



3. Use a straightedge to extend the side of the angle so that is the same length as $\overline{AJ}$. Label the other endpoint *J*′.



4. Repeat steps 1, 2, and 3 using the other endpoint of the original line segment.



5. Connect endpoints *J*′ and *N*′.

   $R_{A,\,40}(\overline{JN}) = \overline{J'N'}$
   Segment $\overline{J'N'}$ is the result of a 40° rotation of $\overline{JN}$ about point *A*.

LESSON 5

Slide, Flip, Turn: The Latest Dance Craze?

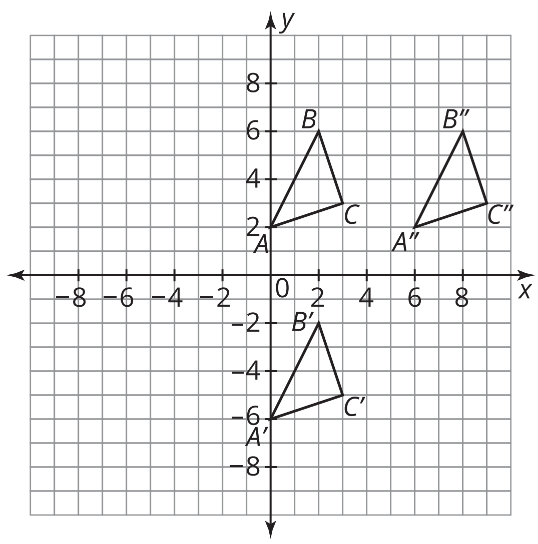
A translation slides an image on the coordinate plane. When an image is horizontally translated c units on the coordinate plane, the value of the x-coordinates change by c units. When an image is vertically translated c units on the coordinate plane, the value of the y-coordinate changes by c units. The coordinates of an image after a translation are summarized in the table.

Original Point	Horizontal Translation to the Left	Horizontal Translation to the Right	Vertical Translation Up	Vertical Translation Down
(x, y)	$(x - c, y)$	$(x + c, y)$	$(x, y + c)$	$(x, y - c)$

For example, the coordinates of $\triangle ABC$ are A (0, 2), B (2, 6), and C(3, 3).

When $\triangle ABC$ is translated down 8 units, the coordinates of the image are A' (0, −6), B' (2, −2), and C' (3, −5).

When $\triangle ABC$ is translated right 6 units, the coordinates of the image are A'' (6, 2), B'' (8, 6), and C'' (9, 3).



A rotation is a rigid motion that turns a figure about a fixed point called the *point of rotation*. The figure is rotated in a given direction for a given angle called the *angle of rotation*. The angle of rotation is the measure of the amount the figure is rotated about the point of rotation. The direction of a rotation can either be clockwise or counterclockwise. When the center of rotation is at the origin (0, 0), and the angle of rotation is 90°, 180°, 270°, or 360°, the coordinates of an image can be determined using the rules summarized in the table.

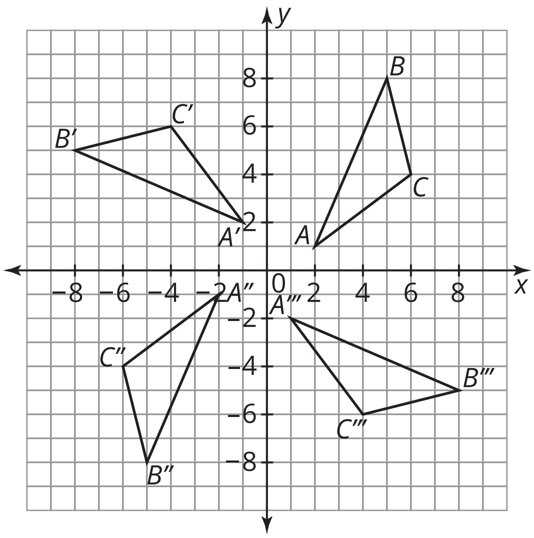
Original Point	Coordinates After a 90° Counterclockwise Rotation About the Origin	Coordinates After a 180° Counterclockwise Rotation About the Origin	Coordinates After a 270° Counterclockwise Rotation About the Origin	Coordinates After a 360° Counterclockwise Rotation About the Origin
(x, y)	$(-y, x)$	$(-x, -y)$	$(y, -x)$	(x, y)

For example, the coordinates of △*ABC* are *A* (2, 1), *B* (5, 8), and *C* (6, 4).

When △*ABC* is rotated 90° counterclockwise about the origin, the coordinates of the image are *A′* (−1, 2), *B′* (−8, 5), and *C′* (−4, 6).

When △*ABC* is rotated 180° about the origin, the coordinates of the image are *A″* (−2, −1), *B″* (−5, −8), and *C″* (−6, −4).

When △*ABC* is rotated 270° counterclockwise about the origin, the coordinates of the image are *A‴* (1, −2), *B‴* (8, −5), and *C‴* (4, −6).



When △*ABC* is rotated 360° counterclockwise about the origin, the coordinates of the image and the pre-image have the same coordinate.

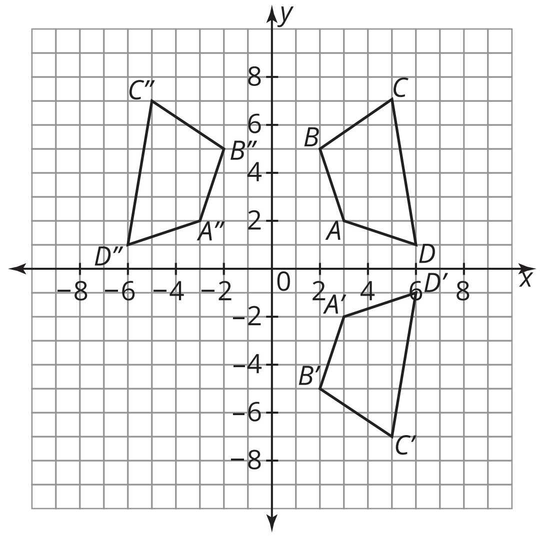
Figures that are mirror images of each other are called reflections. A reflection is a rigid motion that reflects, or "flips," a figure over a given line called a line of reflection. A line of reflection is a line over which a figure is reflected so that corresponding points are the same distance from the line. When an image on the coordinate plane is reflected across the *y*-axis, the value of the *x*-coordinate of the image is opposite the *x*-coordinate of the pre-image. When an image on the coordinate plane is reflected across the *x*-axis, the value of the *y*-coordinate of the image is opposite the *y*-coordinate of the pre-image. The coordinates of an image after a reflection on the coordinate plane are summarized in the table.

Original Point	Coordinates of Image After a Reflection Over the *x*-axis	Coordinates of Image After a Reflection Over the *y*-axis
(x, y)	$(x, -y)$	$(-x, y)$

For example the coordinates of Quadrilateral *ABCD* are *A* (3, 2), *B* (2, 5), *C* (5, 7), and *D* (6, 1).

When Quadrilateral *ABCD* is reflected across the *x*-axis, the coordinates of the image are *A*′ (3, −2), *B*′ (2, −5), *C*′ (5, −7), and *D*′ (6, −1).

When Quadrilateral *ABCD* is reflected across the *y*-axis, the coordinates of the image are *A*″ (−3, 2), *B*″ (−2, 5), *C*″ (−5, 7), and *D*″ (−6, 1).



LESSON 6

OKEECHOBEE

A plane figure has **reflectional symmetry** if you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side of the line. So, both the octagon and the trapezoid have reflectional symmetry. The line that you can draw on each shape is called the line of symmetry. A figure can have more than one line of symmetry.

A plane figure can also have **rotational symmetry** if you can rotate the figure more than 0° but less than 360° and the resulting figure is the same as the original figure in the original position.

A shape, such as a square, can have both reflectional and rotational symmetry. The standard alphabet has many letters that have a variety of symmetries, including reflectional and rotational symmetry. Some letters have a vertical line of symmetry. Other letters have a horizontal line of symmetry.