Module 1: Reasoning with Shapes

TOPIC 3: CONGRUENCE THROUGH TRANSFORMATIONS

In this topic, students use formal reasoning to prove geometric theorems. They use what they know about rigid motions to prove triangle congruence theorems by construction. Students prove the Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle Congruence Theorems by construction. Each proof involves a sequence of transformations that maps one triangle onto another, given the congruence of three corresponding parts. Integrating their knowledge of geometry and algebra, students encounter triangles on the coordinate plane that require them to use the Distance Formula to apply the congruence theorems to triangles with given measurements on the plane.

Where have we been?

In middle school, students constructed triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. Through that hands-on exploration, they developed an intuition regarding the minimum criteria for determining whether triangles are congruent.

Where are we going?

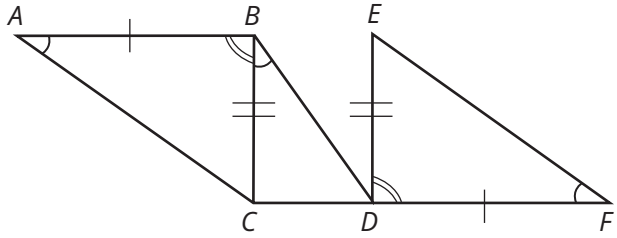
In addition to proof by construction, students will learn to write two-column, flowchart, and paragraph proofs. They will develop their deductive reasoning skills as they prove conjectures that they have made through investigation. The triangle congruence theorems proven in this topic will be used in many of these upcoming proofs. They are used to prove the properties of quadrilaterals and other polygons, the relationships between chords, secants, and tangents in circles, and line and angle relationships in triangles and polygons.

Congruence Markers

Marks are used in geometric drawings to indicate when two or more line segments are congruent. The markers on the diagram indicate congruent line segments.

$\overline{AB} \cong \overline{DF}$ and $\overline{BC} \cong \overline{ED}$

$\angle CAB \cong \angle EFD$ and $\angle ABC \cong \angle FDE$

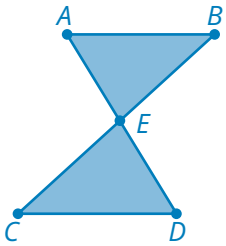


The Face That Launched a Thousand Triangles

How does chat software know where to apply the funny face filter, even as you move your face around on the screen? The software behind this face recognition may use congruent triangles, often in a mesh of triangles as shown. And, in order to work more quickly, the software wants to use the least amount of information it has to in order to identify congruent triangles.

Talking Points

Triangle congruence is an important topic to know about for college admissions tests. Here is a sample question:



In the figure, $\overline{AB}$ and $\overline{CD}$ are 8 cm apart, congruent, and parallel. If $AB = 4$ cm, what is the shaded area?

$\angle AEB \cong \angle CED$ because they are vertical angles. Alternate interior angles are also congruent, so the two triangles are congruent by Angle-Side-Angle.

Since the triangles are congruent, they have congruent altitudes, so each has an altitude of 4 cm. Thus, each triangle's area is $\frac{1}{2}(4)(4)$ or 8 square centimeters. The total area shaded, then, is 16 square centimeters.

Key Terms

postulate

A postulate is a statement that is accepted without proof.

theorem

A theorem is a statement that can be proven.

proof

A proof is a series of statements and corresponding reasons forming a valid argument that starts with a hypothesis and arrives at a conclusion.