3

I Never Forget a Face

Using Triangle Congruence to Solve Problems

MATERIALS

Rulers
Patty paper

Lesson Overview

Students determine whether triangles are congruent using SSS, SAS, and ASA. First, they explain how a triangle congruence theorem can be applied to a real-world situation. They then determine whether triangles in complex diagrams are congruent. Students then use the coordinate plane to assist in measurements or transformations to determine whether triangles are congruent. Finally, they apply transformations to create an original wallpaper design.

Geometry
Coordinate and Transformational Geometry

(G.2) The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines.

Proof and Congruence

(G.6) The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

(B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions.

(C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- The SSS, SAS, and ASA Congruence Theorems can be applied to solve real-world and mathematical problems.
- Congruent parts of triangles can be depicted from a diagram rather than stated. These can be instances where two triangles share a common side or angle.
- The SSS, SAS, and ASA Congruence Theorems can be applied to triangles on or off the coordinate plane.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: A Bridge Too Far

Students engage in a real-world scenario. They explain how triangle congruence can be used to indirectly measure the length of a bridge.

Develop

Activity 3.1: Using SSS, SAS, and ASA

Students deal with more complex diagrams including overlapping triangles and triangles sharing common sides or angles. They determine whether pairs of triangles are congruent and identify the triangle congruence theorem used. Students then solve problems where they must identify what additional information is needed to determine the congruence of a pair of triangles using a specific triangle congruence theorem.

Day 2

Activity 3.2: SSS, SAS, and ASA on the Coordinate Plane

Students determine whether pairs of triangles presented on the coordinate plane are congruent. First they use the coordinate plane to determine whether corresponding parts have the same measure, then they use transformations on the plane to describe how to prove triangles are congruent.

Day 3

Demonstrate

Talk the Talk: A Transformational Arteest

Students create their own design using a single geometric figure and repeated rigid motion transformations: a rotation, a translation, and a reflection. They share their designs with classmates and identify the rigid motions in their partners' designs.

ENGAGE

Getting Started: A Bridge Too Far

Facilitation Notes

In this activity, students engage in a real-world scenario. They explain how triangle congruence can be used to indirectly measure the length of a bridge.

Have the students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

- Whether or not students construct the congruent triangle on land.
- Measuring tools being used. Encourage them to use only construction tools to determine the length of the new bridge.

Misconception

Because of the sentence "The only thing you know is the length of the old bridge," students may not think they can reconstruct the line segment forming the side of the triangle connecting the old bridge and the new bridge on the lower river bank. Knowing that distance, or reproducing it, is an essential component of using the SAS Congruence Theorem. The same is true about the measure of the included angle formed by the old bridge and the lower river bank. It is not necessary to measure this angle to reconstruct it if students use vertical angles.

Differentiation strategy

To scaffold support, discuss strategies for reconstructing the triangle formed by the bridges and lower river bank on land. Review vertical angles and ask how they can be used to construct a triangle congruent to the original one.

Questions to ask

- Why can't the new bridge be measured?
- If the length of the new bridge can be constructed on land, could it easily be measured?
- Do the old bridge, the new bridge, and the river bank between the two bridges form a triangle?
- How can you move the length of the new bridge onto land by constructing a triangle congruent to the triangle formed by the old bridge, the new bridge, and the river bank?
- What side lengths of the original triangle do you know or can duplicate?
- How can vertical angles help in the construction of a congruent triangle?
- What three parts of the triangle are you using to create a congruent triangle?

- Which congruence theorem tells you that you will get a congruent triangle?
- Describe what parts of the context represent SAS.
- How did you use CPCTC in your explanation?

Summary

Triangle congruence theorems can be helpful when determining distances that cannot be measured directly.

Activity 3.1
Using SSS, SAS, and ASA

DEVELOP

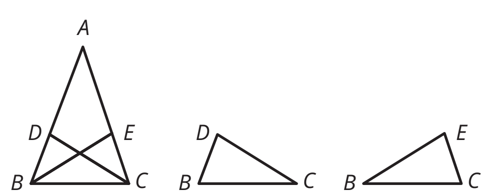
Facilitation Notes

In this activity, students deal with more complex diagrams including overlapping triangles and triangles sharing common sides or angles. They determine whether pairs of triangles are congruent and identify the triangle congruence theorem used. Students then solve problems where they must identify what additional information is needed to determine the congruence of a pair of triangles using a specific triangle congruence theorem.

Have the students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

- To assist all students, suggest they sketch the pair of triangles they plan to prove congruent independent from the entire diagram. For example, in Question 2, draw $\triangle DBC$ and $\triangle ECB$.



- To scaffold support, suggest that students draw arc markers in the same color to mark congruent angles. Have them trace the entire side and use slash marks of the same color to label congruent sides.

As students work, look for

Use of the Reflexive Property. The Reflexive Property is valid reasoning for stating that a line segment is congruent to itself and an angle is congruent to itself. If two triangles share a common side or share a common angle, the Reflexive Property can be used as a reason for stating the part is congruent to itself.

Misconception

Students may not realize that the order of the parts of the triangle make a difference. For example, they may think that if two sides and an angle are congruent, that SAS can always be used. Use Question 3 to explain that the angle must be formed by the congruent sides.

Questions to ask for Question 1

- If $\overline{AD}$ bisects $\angle A$, which two angles are congruent?
- If $\overline{AD} \perp \overline{BC}$, what do you know to be true about $\angle ADB$ and $\angle ADC$?
- Are all right angles congruent? How do you know?
- Which part (side or angle) do both $\triangle ABD$ and $\triangle ADC$ have in common?
- What mathematical property supports $\overline{AD} \cong \overline{AD}$?
- What congruence theorem can be used to determine these triangles are congruent?
- What is the triangle congruence statement? What is another way to state it?

Questions to ask for Question 2

- Which triangle has $\angle DCB$ as an interior angle?
- Which triangle has $\angle EBC$ as an interior angle?
- Do $\triangle DCB$ and $\triangle EBC$ share a common angle or common side? What is it?
- What mathematical property supports that $\overline{BC}$ is congruent to itself?
- Which congruence theorem can be used to determine these triangles are congruent?
- What is the triangle congruence statement? What is another way to state it?
- How is this problem similar to Question 1? How is it different?

Questions to ask for Question 3

- Which two triangles appear to be congruent?
- Which three parts can you verify are congruent?
- How do you know $\overline{BC} \cong \overline{FE}$?
- Is Side-Side-Angle a valid triangle congruence theorem?
- How can you tell whether the parts form SAS?

Have the students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

Questions to ask for Question 4

- How are the two perpendicular statements helpful?
- What three parts of the triangles do you know are congruent according to the given statements and the diagram? Why aren't they enough to prove the triangles are congruent?
- What one additional pair of congruent parts would be helpful?
- Is there another set of congruent parts that could be used to demonstrate congruent triangles using ASA? If so, what are they?

- What corresponding parts do you know are congruent in $\triangle ABE$ and $\triangle DCE$?
- Why is $\angle AEB \cong \angle DEC$?
- What does the Vertical Angle Theorem state?
- Which two additional parts are needed to determine the triangles are congruent?
- Is there another set of congruent parts that could be used to demonstrate congruent triangles using SAS? If so, what are they?

Differentiation strategy for Question 5

To scaffold support, suggest that students sketch or trace the diagram so that they have a clean diagram to label for parts (a), (b), and (c).

Questions to ask for Question 5

- Which two parts do you already know are congruent? How do you know?
- How did you determine that part was needed?
- Is there more than one correct answer? If so, what is another response?
- Why are two parts needed for SSS? Why is there only one correct answer?
- What is the minimum number of additional parts needed to use SAS? What is the part needed?

Questions to ask for Question 6

- What is the other pair of corresponding sides that are not marked?
- Why can't you guarantee that the two triangles are congruent with these three pairs of corresponding congruent parts?
- Why isn't this a case of SAS?

Summary

Congruent parts of triangles can be depicted in a diagram rather than stated. There can be instances where two triangles share a common side or angle.

Activity 3.2

SSS, SAS, and ASA on the Coordinate Plane

Facilitation Notes

In this activity, students determine whether pairs of triangles presented on the coordinate plane are congruent. First they use the coordinate plane to determine whether corresponding parts have the same measure, then they use transformations on the plane to describe how to prove triangles are congruent.

Have the students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To extend the lesson, have students use the coordinate plane to establish the congruence of the angles.

Questions to ask

- When using SAS, how do you know which angle is the included angle?
- Can you determine that two sides are congruent without completely solving using the Distance Formula? Explain your process.
- How does the coordinate plane help in determining whether triangles are congruent?

Have the students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

Questions to ask

- Which three parts of the triangles did you use to prove the triangles are congruent?
- What is another set of parts that could be used?
- When using the ASA Congruence Theorem, can any side of $\triangle ABC$ be identified as the included side?
- Why do you know the triangles are congruent just by the moves Emerson made?
- What is a sequence of transformations to map $\triangle ABC$ onto $\triangle XYZ$?
- What is an alternative sequence of transformations that map $\triangle ABC$ onto $\triangle XYZ$?
- How many transformations are necessary to map $\triangle MNP$ onto $\triangle ABC$?
- How would you describe the transformations necessary to map $\triangle MNP$ onto $\triangle ABC$?
- How would you describe an alternative set of transformations that map $\triangle MNP$ onto $\triangle ABC$?
- How was the coordinate plane used differently in Questions 3 through 5 as compared to Questions 1 and 2?

Differentiation strategy

To extend the lesson, ask students to use function notation to describe the sequences of transformations to determine congruent triangles.

Summary

The coordinate plane can be used to determine congruent corresponding parts of triangles in order to determine whether two triangles are congruent.

Talk the Talk: The Transformational Arteest

DEMONSTRATE

Facilitation Notes

In this activity, students create their own design using a single geometric figure and repeated rigid motion transformations: a rotation, a translation, and a reflection. They share their designs with classmates and identify the rigid motions in their partners' designs.

Have the students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What geometric shape did you use for your primary design?
- What transformation did you perform first? Second? Third?
- How does changing the order in which you perform the transformations affect the design?
- What is an example of a primary geometric shape that will not change in appearance when it undergoes a rotation? A translation? A reflection?
- Is there a primary geometric shape that will not change in appearance when it undergoes a rotation, translation, and reflection? What is it?

Differentiation strategies

To extend the lesson,

- Lead a classroom discussion focusing on which rigid motions create designs that are most pleasing to the eye. Sample questions: Do students generally prefer designs that are balanced and symmetric in nature? Do some students think patterns that produce random designs are more attractive and interesting? Are repeated patterns more calming or annoying? Which rigid motions are usually displayed on wallpaper? What is an example of a shape that would not look pleasing to the eye if it were reflected?
- Have students explore and identify the rigid motions used most often in some of Escher's popular tessellations.

Summary

Isometric transformations can be applied to geometric shapes to create designs in the real world.

NOTES

Warm Up Answers

1. 65°
2. 34°
3. 117°
4. 35°

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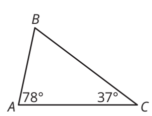
I Never Forget a Face

Using Triangle Congruence to Solve Problems

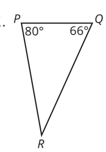
Warm Up

Determine the measure of the unknown angle in each triangle.

1.



2.



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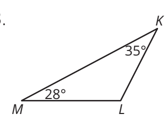

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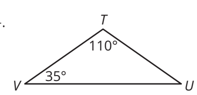


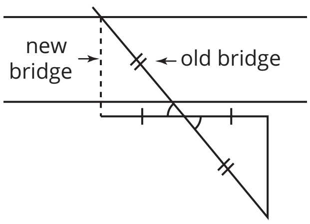
Learning Goals

- Use triangle congruence theorems to identify congruent triangles.
- Identify the information needed to conclude that two triangles are congruent by SSS, SAS, or ASA.

You have proven that Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle Congruence Theorems are valid criteria to determine triangle congruence. How can you apply these theorems to problems?

Answer

1. Create congruent triangles.
   Since you know the length of the old bridge, you can create a congruent triangle using SAS on the side of the river where you can measure a length for the new bridge.



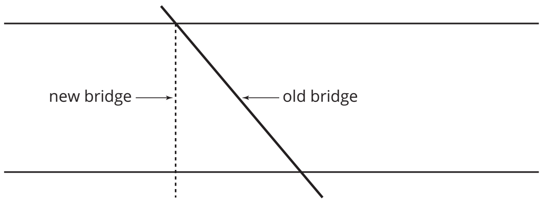
GETTING STARTED

A Bridge Too Far

Suppose you are planning to build a new bridge across a river to replace the old bridge, which has grown unusable. You need to know exactly how long to make the bridge, but you can't measure the width of the river. The only thing you know is the length of the old bridge.

Think about:

You can't assume that an angle is a right angle or that two figures are congruent, even if it seems like it.



1. Explain how you could use congruent triangles to determine the exact length of the new bridge needed to cross the river. Explain your thinking.

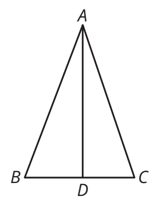
ACTIVITY 3.1

Using SSS, SAS, and ASA

You know that the Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle Congruence Theorems can be used as valid reasons to demonstrate triangles are congruent. Consider each theorem as you analyze the given statements and diagrams.

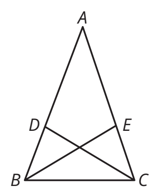
1. Suppose $\overline{AD}$ bisects $\angle A$, and $\overline{AD} \perp \overline{BC}$.

Are there congruent triangles in this diagram? Explain your reasoning.

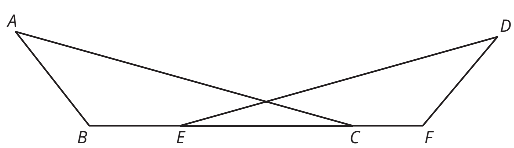


2. Suppose $\angle DBC \cong \angle ECB$, and $\angle DCB \cong \angle EBC$.

Are there congruent triangles in this diagram? Explain your reasoning.



3. Suppose $\overline{AB} \cong \overline{DF}$, $\angle A \cong \angle D$, and $\overline{BE} \cong \overline{FC}$.



Are there congruent triangles in this diagram? Explain your reasoning.

Answers

1. Yes, $\triangle ABD \cong \triangle ACD$ by ASA Congruence Theorem.

   Angles *BAD* and *CAD* are congruent because $\angle A$ is bisected.

   Side $\overline{AD}$ is congruent to itself (reflexive property).

   Angles *ADB* and *ADC* are congruent because they are right angles.

2. Yes, $\triangle DBC \cong \triangle ECB$ by ASA Congruence Theorem.

   It is given that $\angle DBC \cong \angle ECB$.

   Segment *BC* is a side of both triangles.

   It is given that $\angle DCB \cong \angle EBC$.

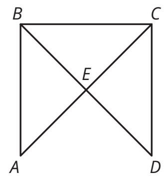
3. No. There is not enough information to determine that $\triangle ABC$ is congruent to $\triangle DFE$ because $\angle A$ and $\angle D$ are not the included angles.

Answers

4a. Sample answer.
I would need to know that $\angle BCA \cong \angle CBD$ in order to use the ASA Congruence Theorem.

4b. Sample answer.
I would need to know that $\overline{AE} \cong \overline{DE}$ and $\overline{BE} \cong \overline{CE}$.

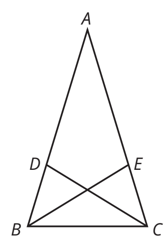
5a. Sample answer.
I would need to know that $\angle ABE \cong \angle ACD$ in order to use the ASA Congruence Theorem.



4. Suppose $\overline{AC} \cong \overline{DB}$, $\overline{AB} \perp \overline{BC}$, and $\overline{DC} \perp \overline{CB}$.

a. What information would you need to conclude $\triangle CAB$ is congruent to $\triangle BDC$ using the ASA Congruence Theorem?

b. What information would you need to conclude $\triangle ABE$ is congruent to $\triangle DCE$ using the SAS Congruence Theorem?



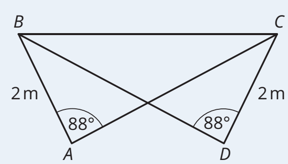
5. Suppose $\overline{AB} \cong \overline{AC}$.

a. What additional information would be needed to conclude $\triangle ABE$ is congruent to $\triangle ACD$ using the ASA Congruence Theorem?

b. What additional information would be needed to conclude △*ABE* is congruent to △*ACD* using the SSS Congruence Theorem?

c. What additional information would be needed to conclude △*ABE* is congruent to △*ACD* using the SAS Congruence Theorem?

6. Simone says that since △*ABC* and △*DCB* have two pairs of congruent corresponding sides and congruent corresponding angles, then the triangles are congruent by SAS. Is Simone correct? Explain your reasoning.



Answers

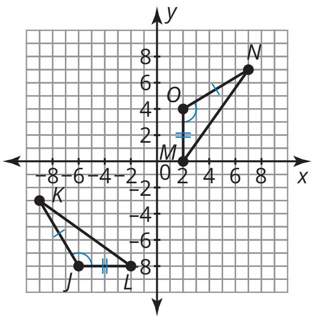
5b. Sample answer. I would need to know that $\overline{AE} \cong \overline{AD}$ and $\overline{BE} \cong \overline{CD}$ in order to use the SSS Congruence Theorem.

5c. Sample answer. I would need to know that $\overline{AE} \cong \overline{AD}$ in order to use the SAS Congruence Theorem.

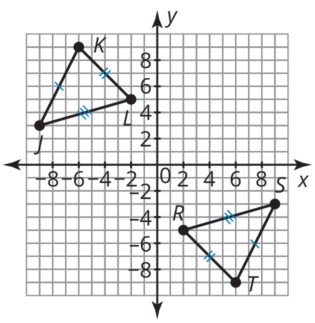
6. Simone is not correct. The congruent angles are not formed by two pairs of congruent sides, so they are not the included angles. There is not enough information to determine if the triangles are congruent by SAS or SSS.

Answers

1. Sample answer.
   Check students' work.



2. Sample answer.
   Check students' work.



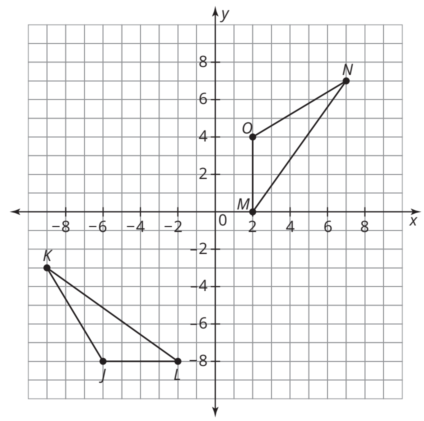
ACTIVITY **3.2**

SSS, SAS, and ASA on the Coordinate Plane

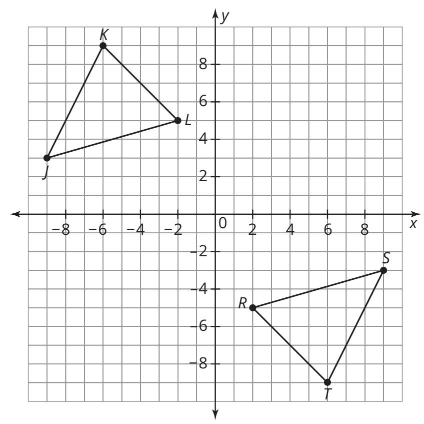
Think

How does your reasoning change when figures are represented on a coordinate plane?

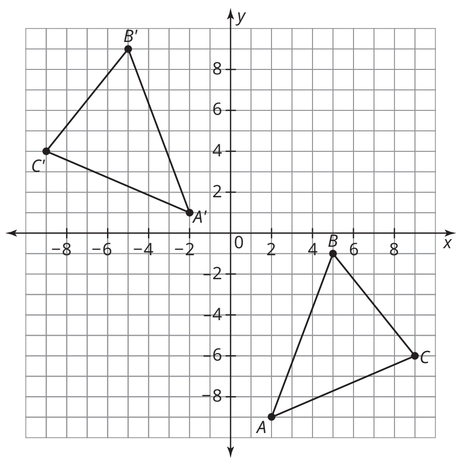
1. Use the Distance Formula and patty paper to show that the two triangles on the coordinate plane are congruent by SAS. Show your work.



2. Use the Distance Formula and patty paper to show that the two triangles on the coordinate plane are congruent by SSS. Show your work.



3. **Emerson wants to translate $\triangle ABC$ and then reflect it across the *y*-axis to form a new triangle in Quadrant II. She uses what she knows about transformations to determine the vertices of $\triangle A'B'C'$ before performing the transformations.**



a. **Describe how Emerson can use the ASA Congruence Theorem to determine whether or not she transformed $\triangle ABC$, such that the image is congruent to the pre-image.**

b. **Did Emerson perform the transformations on $\triangle ABC$ so that the image is congruent to the pre-image? Explain your reasoning.**

Answers

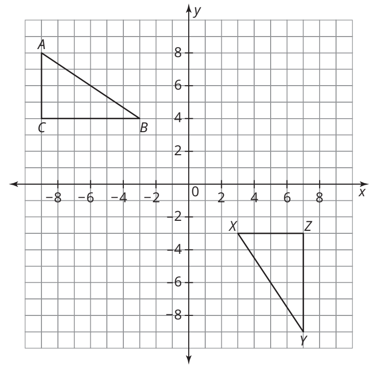
3a. Emerson can determine the measure of two corresponding angles in both triangles and the length of the included sides. If the angles are congruent and the sides are congruent, then the triangles are congruent by the ASA Congruence Theorem.

3b. Yes. Emerson performed the transformations on $\triangle ABC$ so that the image and pre-image are congruent. I know that the triangles are congruent by the ASA Congruence Theorem.

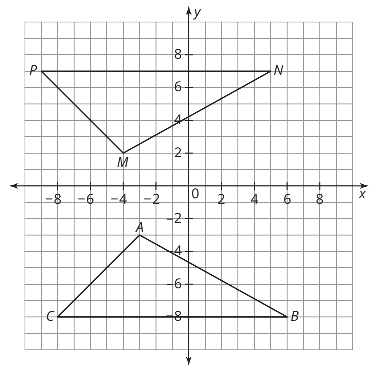
Answers

4. Sides $\overline{AC}$ and $\overline{XZ}$ are congruent because they each have measures of 4 units. Sides $\overline{CB}$ and $\overline{ZY}$ are congruent because they each have measures of 6 units. The included angles *C* and *Z* are congruent because they are both right angles. Using SAS, $\triangle ABC \cong \triangle XYZ$.
5. Angles *P* and *C* are congruent.
   Angles *N* and *B* are congruent.
   The included sides $\overline{PN}$ and $\overline{CB}$ are congruent.
   Using ASA,
   $\triangle PMN \cong \triangle CAB$.

4. Describe how to prove the given triangles are congruent. Use the key terms *included angle* and *Side-Angle-Side Congruence Theorem* in your answer.



5. Describe how to prove the given triangles are congruent. Use the key terms *included side* and *Angle-Side-Angle Congruence Theorem* in your answer.



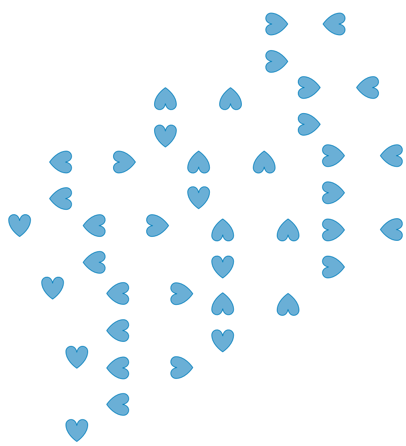
NOTES

TALK the TALK

A Transformational Arteest

Congruence is an important concept, not only in technology but also in the world of art and design.

The image shown uses all three rigid motions—a rotation, a translation, and a reflection—and a single shape repeated to create a wallpaper design.



1. **Use the space on the next page to create your own wallpaper design using a single geometric figure and all three rigid motions.**
2. **Share your designs with your classmates. Determine the repeated pattern of transformations used to create each design.**

Answers

1. Answers will vary.
2. Answers will vary.